A LINEARIZATION SYSTEM FOR PARAMETRIC ARRAY LOUDSPEAKERS USING THE PARALLEL CASCADE VOLTERRA FILTER

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ABSTRACT

The parametric array loudspeaker (PAL) is well known for its ability to radiate a narrow sound beam from a relatively small ultrasonic emitter. Nonlinear distortions commonly occur in the self-demodulated sound of the PAL. Based on the Volterra filter modeling the self-demodulation process of the PAL, a linearization system can be developed for the PAL. However, the computational complexity of the Volterra filter increases dramatically with the tap length. In this paper, the parallel cascade structure is adopted to implement the Volterra filter. The experiment results demonstrate that the computational complexity of the Volterra filter is significantly reduced by using the parallel cascade structure, and based on such an implementation of the Volterra filter, the performance of the linearization system is not compromised.

Index Terms— Volterra filter, Parallel cascade structure, Parametric array loudspeaker, Nonlinear distortion

1. INTRODUCTION

Recently, the parametric array loudspeaker (PAL) has been widely studied as a novel type of sound reproduction device in a variety of applications where the directional sound is preferred [1]. For example, a pair of PALs have been employed in the active noise control to create individual quite zones near of the left and right ears of a person [2]. The PALs have also been examined for the three-dimensional audio reproduction [3, 4]. However, the sound quality of the PAL is not satisfactory due to the nonlinear distortion, which is by-produced by the self-demodulation process. Hence, there have been many preprocessing methods derived based on the modulation technique to reduce the nonlinear distortion [5,6]. But the actual performance of a preprocessing method is often limited by the discrepancy between the real environments and the theoretical assumptions made in the derivation of the preprocessing method.

Therefore, the linearization method based on the Volterra filter identification is more favorable [7–11]. This is because
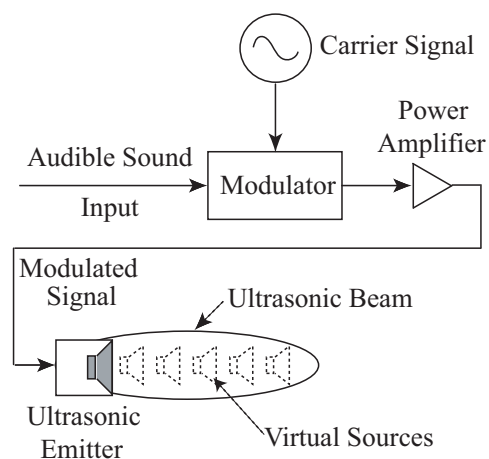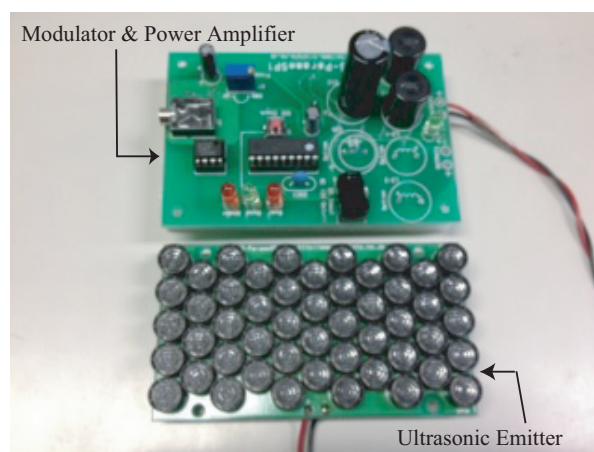
Fig. 1. Block diagram of the PAL.

Fig. 2. Picture of the PAL.

that the Volterra filter that models the self-demodulation process of the PAL is able to adapt to different environments. The earliest work on such a linearization method can be traced back to 2002 [7]. The major difficulty in practice is that the computational complexity of the Volterra filter increases

dramatically with the tap length [8, 9]. One of the existing solutions is to adopt the one dimensional Volterra filter that keeps only the diagonal elements of the 2nd–order Volterra kernel [10]. It leads to significant reduction of the computational complexity, but sacrifices the ability to compensate for the intermodulation distortion. In another recent work, the sparse normalized least mean squares (NLMS) algorithm has been used in the identification of the Volterra filter [11]. But it is found that the sparsity in the 2nd–order Volterra kernel may be trivial. Therefore, the Volterra filter is proposed to be implemented using the parallel cascade structure in this paper. The parallel cascade structure explores the factorization of the 2nd–order Volterra kernel to reduce the computational complexity of the Volterra filter [12]. The effectiveness of the parallel cascade structure is validated through the comparison with the conventional implementation of the Volterra filter, in terms of the compensation amount of the 2nd–order nonlinear distortion in the PAL.

2. PARAMETRIC ARRAY LOUDSPEAKER

When the amplitude of a sound wave is sufficiently small, the linear acoustic model is generally accurate and concise. However, when the amplitude of a sound wave is large, the nonlinear acoustic effects become noticeable. In this case, the sound wave is referred to as the finite amplitude wave. When two finite amplitude waves at close frequencies are radiated in the same direction, intermodulation frequencies, such as the sum and difference frequencies, are generated along with other harmonics. The intermodulation frequency waves travel in a similarly narrow beam as the finite amplitude waves. In the PAL, the finite amplitude waves are radiated in the ultrasonic frequency range, and the difference frequency is produced in the audible range of human beings. Therefore, the sound beam of the PAL is much narrower than the conventional sound devices with the same emitter size.

Figures 1 and 2 show the block diagram and picture of the PAL, respectively. The audible sound input is modulated on an ultrasonic carrier so that the waveform of the audible sound input is embedded in the sideband of the modulated signal. After the modulated signal is radiated into air, the waveform of the audible sound input is cumulatively recovered from the interaction between the sideband frequencies and the carrier frequency. This phenomenon is known as the self-demodulation process. The sound reproduced by the PAL is thus referred to as the self-demodulated sound.

3. LINEARIZATION SYSTEM BASED ON THE VOLTERRA FILTER

3.1. Volterra Filter

Nonlinear systems with memory are able to be modeled using Volterra filters [13]. When the Volterra filter is utilized to model the nonlinear response of the PAL, it is allowed to be truncated at the 2nd–order and use a finite memory length of N to balance between the computational complexity and model accuracy. Hence, the nonlinear response of the PAL is given by

$$
\begin{aligned}
y(n) = & \sum_{k_1=0}^{N-1} h_1(k_1)x(n-k_1) \\
& + \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} h_2(k_1,k_2)x(n-k_1)x(n-k_2),
\end{aligned} \tag{1}
$$

where $x(n)$ and $y(n)$ are the discrete input and output signals in the audible frequency range; h_1 and h_2 are the coefficients of the 1st–order and 2nd–order Volterra kernels, respectively.

3.2. Nonlinear System Identification

A nonlinear system identification approach to determine the coefficients of the Volterra kernels is provided by the frequency response method [14]. The coefficients of the Volterra kernels are computed from the inverse Fourier transform of the frequency responses obtained in measurements. Firstly, a sine sweep signal is applied to the nonlinear system to be identified. The frequency response of the 1st–order Volterra kernel is obtained from the ratio of the output spectrum $Y(\omega)$ and the input spectrum $X(\omega)$, which is given by

$$
\hat{H}_1(\omega) = \frac{Y(\omega)}{X(\omega)}. \tag{2}
$$

Secondly, two sine sweep signals are applied to the nonlinear system to obtain the frequency response of the 2nd–order Volterra kernel, which is determined by dividing the output spectrum $Y(\omega_1+\omega_2)$ by the input spectra $X(\omega_1)$ and $X(\omega_2)$, i.e.

$$
\hat{H}_2(\omega_1,\omega_2) = \frac{Y(\omega_1+\omega_2)}{X(\omega_1)X(\omega_2)}\frac{N}{\alpha}, \tag{3}
$$

where α denotes the number of symmetries in the frequency response of the 2nd–order Volterra kernel.

If the nonlinear system to be identified contains only the 2nd–order nonlinearity, the frequency response method is able to obtain the 2nd–order Volterra kernel accurately. However, in practice, the nonlinear system usually contains more than 2nd–order nonlinearity. Therefore, intermodulation frequencies resultant from higher-order nonlinearity overlap in the output spectrum $Y(\omega_1+\omega_2)$. This introduces errors in the identified 2nd–order Volterra kernel. Based on our past experience, the errors introduced by higher-order nonlinearity is likely to be tolerable in the identification of the 2nd–order Volterra kernel of the PAL.

3.3. Linearization System Design

The block diagram of the linearization system compensating for the 2nd–order nonlinear distortion of the PAL is shown in Fig. 3. In this linearization system, $\hat{H}_2(z_1, z_2)$ is the identified 2nd–order Volterra kernel and $H_1^{-1}(z)$ is the linear inverse filter that fulfills the following condition:

$$H_1(z) \cdot H_1^{-1}(z) = z^{-\Delta}. \tag{4}$$

The 2nd–order nonlinear output of the PAL after the linearization system consists of two terms. The first term is provided by the combination of the 1st–order path of the linearization system and the 2nd–order nonlinear response of the PAL. The second term is resultant from the combination of the 2nd–order path of the linearization system and the linear response of the PAL. Therefore, the 2nd–order nonlinear output of the overall system is written as

$$z^{-\Delta}H_2(z_1, z_2) - \hat{H}_2(z_1, z_2)H_1^{-1}(z)H_1(z)$$
$$= z^{-\Delta}H_2(z_1, z_2) - \hat{H}_2(z_1, z_2)z^{-\Delta}$$
$$= z^{-\Delta}\{H_2(z_1, z_2) - \hat{H}_2(z_1, z_2)\}. \tag{5}$$

If there is no discrepancy between the actual 2nd–order nonlinear response of the PAL and the identified 2nd–order Volterra kernel, *i.e.* $\hat{H}_2(z_1, z_2) = H_2(z_1, z_2)$, the 2nd–order nonlinear distortion of the PAL should be completely eliminated by the linearization system.

3.4. Parallel Cascade Volterra Filter

According to the parallel cascade structure of the Volterra filter [12], the 2nd–order nonlinear component in (1) is rewritten in a matrix form as
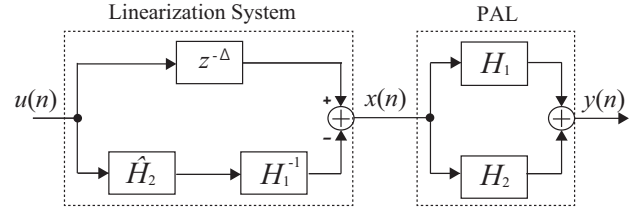
$$y_2(n) = \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} h_2(k_1, k_2)x(n - k_1)x(n - k_2)$$
$$= \mathbf{X}^T(n)\mathbf{H}_2\mathbf{X}(n), \tag{6}$$

where $\mathbf{X}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]$ is the input signal vector; and $\mathbf{H}_2$ is the matrix form of the 2nd–order Volterra kernel, *i.e.*

$$\mathbf{H}_2 = \begin{bmatrix} h_2(0,0) & h_2(1,0) & \cdots & h_2(N-1,0) \\ h_2(0,1) & h_2(1,1) & \cdots & h_2(N-1,1) \\ \vdots & \vdots & \ddots & \vdots \\ h_2(0,N-1) & h_2(1,N-1) & \cdots & h_2(N-1,N-1) \end{bmatrix}. \tag{7}$$

Without loss of generality, H_2 is assumed to be a symmetric Volterra kernel. It can be decomposed as

$$\mathbf{H}_2 = \sum_{i=0}^{N-1} \lambda_i \mathbf{L}_i \mathbf{L}_i^T, \tag{8}$$

$$
\begin{aligned}
H_1(z) &: \quad 1^{st}\text{-order Volterra Kernel} \\
H_2(z_1, z_2) &: \quad 2^{nd}\text{-order Volterra Kernel} \\
\hat{H}_2(z_1, z_2) &: \quad \text{Identified } 2^{nd}\text{-order Volterra Kernel} \\
H_1^{-1}(z) &: \quad \text{Linear Inverse Filter} \\
z^{-\Delta} &: \quad \text{Delay}
\end{aligned}
$$

Fig. 3. Block diagram of the linearization system.
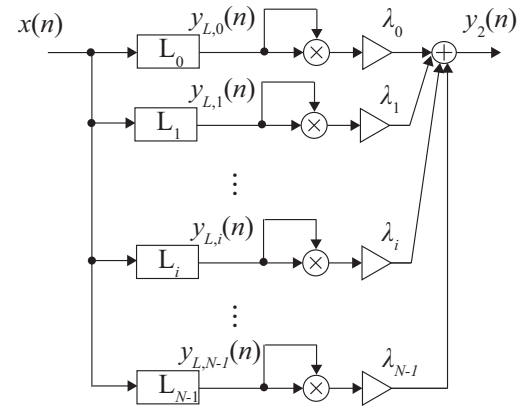

Fig. 4. Parallel cascade structure of the 2nd–order Volterra filter.

where λ_i is ith eigenvalue and $\mathbf{L}_i$ is ith eigenvector given by

$$\mathbf{L}_i = \begin{bmatrix} l_{i,0} & l_{i,1} & \cdots & l_{i,N-1} \end{bmatrix}^T. \tag{9}$$

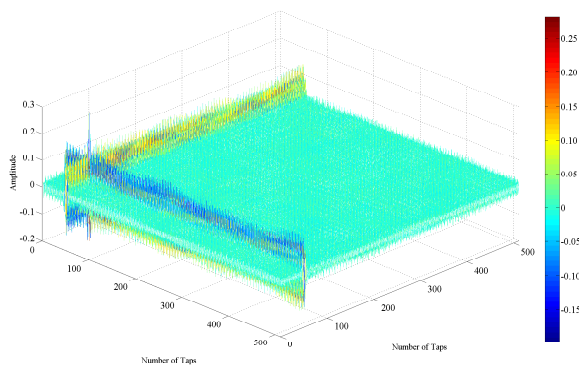Substituting (8) to (6) yields the parallel cascade expression of the 2nd–order nonlinear component:

$$y_2(n) = \sum_{i=0}^{N-1} \lambda_i[\mathbf{X}(n)^T\mathbf{L}_i][\mathbf{L}_i^T\mathbf{X}(n)]$$
$$= \sum_{i=0}^{N-1} \lambda_i y_{L,i}^2(n), \tag{10}$$

where $y_{L,i}(n) = \mathbf{X}^T(n)\mathbf{L}_i = \mathbf{L}_i^T\mathbf{X}(n)$ provides the ith parallel path.

Based on (10), the parallel cascade structure of the 2nd–order Volterra filter is shown in Fig. 4. The advantage of the parallel cascade structure over the conventional implementation of the Volterra filter is that the computational complexity of using the parallel cascade structure is controllable by computing only the significant eigenvalues. The computational

(a) 1st–order Volterra kernel.
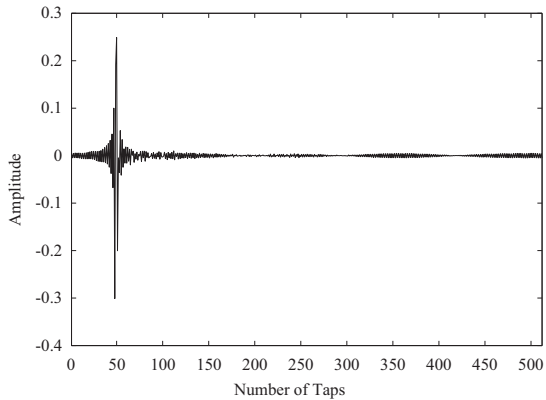
Fig. 6. Eigenvalues of the 2nd–order Volterra kernel.

(b) 2nd–order Volterra kernel.

Fig. 5. Identified 1st–order and 2nd–order Volterra kernels.

(a) 2nd harmonic distortion

(b) Sum frequency distortion

(c) Difference frequency distortion

(d) Before compensation

Fig. 7. Compensation results.

complexity of the conventional implementation of 2nd–order Volterra filter is given by $N^2 + N$, while the computational complexity of using the parallel cascade structure is reduced to
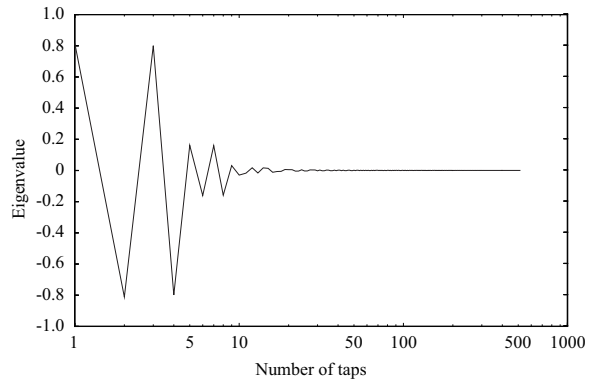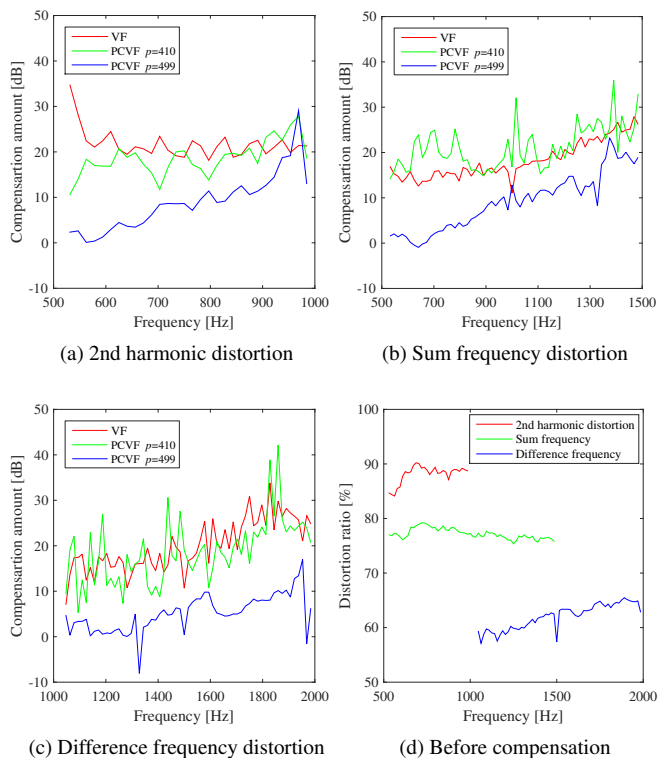
$$(N + 2)(N - p) = N^2 - N(p - 2) - 2p, \qquad (11)$$

where p is the number of the discarded trivial eigenvalues. The computational complexity and the model accuracy can be well balanced by choosing the threshold of the eigenvalues.

4. EXPERIMENT RESULTS

Experiments were carried out to compare the linearization systems of the PAL implemented conventionally and using the parallel cascade structure. The PAL was placed 2.0 m from the microphone. The sampling frequency was set at 8000 Hz. The sine sweeps were generated from 500 Hz to 2000 Hz for the frequency response method that determines the 1st–order and 2nd–order Volterra kernels. The tap length of the 1st–order Volterra kernel was 512 and the dimension of the 2nd–order Volterra kernel was 512×512.

The identified Volterra kernels were plotted in Fig. 5. It is found that there are plenty of trivial coefficients in the 1st–

order and 2nd–order Volterra kernels, which can be taken advantage of to reduce the computational complexity. It is also observed that the one dimensional Volterra filter may lead to compromised performance, since the significant coefficients are distributed on a cross instead of the diagonal.

Figure 6 shows the eigenvalues of the identified 2nd–order Volterra kernel. The significant eigenvalues only occur in the first 20 numbers. Hence, the number of discarded trivial eigenvalues is set to 410 and 499 in turn for comparison. When $p = 499$, the remaining eigenvalues for computing the 2nd–order nonlinear component is only 13, which is less

Table 1. Average of compensation amount.

Kind of distortions	VF	PCVF ($p = 410$)	PCVF ($p = 499$)
Harmonic distortion	21.71 dB	18.61 dB	8.83 dB
Sum frequency	18.28 dB	21.10 dB	9.10 dB
Difference frequency	19.99 dB	18.61 dB	5.12 dB

than the number of significant eigenvalues observed in Fig. 6. Therefore, compromised performance is expected for this extreme setting.

The 2nd–order harmonic distortion and intermodulation distortion were measured before and after the linearization systems. The input signal of the PAL consists of a sine sweep from 500 Hz to 2000 Hz and a fixed sine tone at 500 Hz. The experiment results were plotted in Fig. 7. In Fig. 7(d), the distortion curves before the linearization system have different frequency ranges, which is due to the use of a fixed bandpass filter ranging from 500 Hz to 2000 Hz. This limitation of measurement equipment is also reflected on the frequency ranges in Figs. 7(a)-(c).

For simplicity, the linearization systems based on the conventional Volterra filter and parallel cascade structure are referred to as the VF and PCVF, respectively. The average compensation amounts are listed in Table 1. When $p = 410$, the PCVF achieves equivalent performance as the VF and the computational complexity is reduced by almost 80%. When the extreme setting $p = 499$ is used, the computational complexity is reduced by 97%. However, the compensation performance is also scarified since some significant eigenvalues are not computed. Moreover, the VF, PCVF ($p = 401$), and PCVF ($p = 499$) take 101 s, 40 s, and 8 s to run on an Intel Core i7 3.10 GHz processor.

5. CONCLUSION

In this paper, the parallel cascade structure has been adopted to implement the 2nd–order Volterra filter in the linearization system of the PAL. The computational complexity is remarkably reduced by using the parallel cascade structure, while the compensation performance in terms of the reduction of the 2nd–order harmonic distortion and intermodulation distortion is not compromised. Therefore, the parallel cascade structure is highly recommended for the Volterra filter based linearization system of the PAL, whereby the real-time implementation is possible to be carried out.

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