TARGET: A DIRECT AOA-TDOA ESTIMATION FOR BLIND BROADBAND GEOLOCALIZATION

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ABSTRACT

In this paper, a new robust and low computationally algorithm is proposed for broadband geolocalization. Recent work have demonstrated the superiority of the geolocalization in 1-step over the 2-steps algorithms. However this superiority is obtained at the price of a bandwidth slicing which is unfortunately limited for computational reasons and leads to an asymptotic bias due to the residual broadband effect. This paper we propose an alternative approach fully exploiting the total bandwidth and consequently suppressing the slicing drawbacks. The proposed method is named TARGET and exploits the rank deficiency of a temporal shift dependent covariance matrix after a multichannel synchronization. Our analysis and simulations prove the performance advantage of proposed method over recently introduced ones.

Index Terms— joint AoA and TDoA estimation; broadband geolocalization; direct geolocalization

1. INTRODUCTION

The context of this work is the blind geolocalization of multiple radiating sources with multiple separated antenna arrays (also called stations). For example, these sources stem from telecommunication transmissions such as GSM, 4G, *etc*. Traditional technics [1] rely on a two steps strategy where measurements such as Angle of Arrival (AoA), Time Differential of Arrival (TDoA), Frequency Differential of Arrival (FDoA), *etc.* are obtained from each antenna array in a first step and combined in a second step to estimate the position of the sources. The sources impinging on each station are assumed to be narrowband and far-field. For instance, the AoAs of sources are estimated by each station independently in the first step and, in a second step, the locations of sources are computed from the AoAs (*i.e.* by triangulation) [1].

However, such 2-steps methods present drawbacks [3] and are generally less efficient than the direct algorithms (1-step methods) [5]-[10]. The most efficient direct geolocalization algorithms use the array composed of all the stations (global array) to directly estimate the sources locations [3], [8], [9] and [10]. Unfortunately, the sources are generally wideband with respect to that global array [11]-[13]. To overcome this difficulty, recent algorithms such as DPD [8] for Direct Position Determination, and more recently LOST-FIND [10] for LOcalization by Space-Time with Frequency Identification in Narrowband Decomposition, struggle against the broadband effect in their own way. They are based on a bandwidth decomposition of the received signal allowing the use of high resolution narrowband algorithms on the resulting narrowband signals. Although these algorithms are more efficient than the conventional 2-steps methods in a wide scope of scenarios, the technical implementation constraints (cost calculation, number of snapshot, *etc.*) lead to a residual broadband effect which was studied for the DPD [12] algorithm and an other 1-step algorithm named LOST [9] for LOcalization by Space-Time in [13].

The purpose of this paper is to propose a new broadband approach. This new algorithm is named TARGET for Time and Angle Retrieval for Geolocation Estimation Technic and is based on a completely different strategy. Indeed, TARGET strategy does not counteract the broadband effect (unlike all the 1-step algorithms working on the global array previously cited), but composes with it. TARGET exploits the rank deficiency of a temporal shift dependent covariance matrix after a multichannel synchronization, which allows a joint estimation of AoA and TDoA.

In this paper, for sake of simplicity we will consider that the system is composed of two antenna arrays. However, the TARGET algorithm is also applicable for more than 2 stations.

Notations: $\mathbf{A}$ or $(a_{ij})_{1\leq i\leq I,1\leq j\leq J}\ \forall (I,J)\in\mathbb{N}_*^2$ is a matrix of dimension $I\times J$, $\mathbf{a}$ or $(a_i)_{1\leq i\leq I}\ \forall I\in\mathbb{N}_*$ is a column vector of dimension I, $\mathbf{I}_I$ is the identity matrix of dimension I, a or A is a scalar, $(\cdot)^H$ is the Hermitian of a matrix or a vector, $(\cdot)^T$ is the transpose of a matrix or vector, $(\cdot)^*$ is the conjugate of a scalar, $\mathbb{E}[\cdot]$ is mathematical expectation, $[\![a,b]\!]$ is the set defined by $\left\{x\in\mathbb{Z}: a\leq x\leq b, \forall(a,b)\in\mathbb{Z}^2\right\}$, for all commutative ring or semiring $\mathbb{K}$ we have $\mathbb{K}_* = \mathbb{K}\backslash\{0\}$ and $\mathbb{K}_+ = \{x\in\mathbb{K}: 0\leq\Re\{x\}<+\infty\}$.

2. ASSUMPTIONS AND MODEL

2.1. Signal and system modeling

The global geolocalization system is composed of two remote stations located at $\mathbf{p}_{b_1}$ and $\mathbf{p}_{b_2}$ in a Cartesian coordinate. Each station is composed of M_1 and M_2 sensors respectively where $M = M_1 + M_2$ is the number of sensors in the global system. All the stations have the same reception bandwidth B and are perfectly synchronized in time. In this paper, the number Q of sources is assumed to be known. The observation vector of the l-th station is $\mathbf{x}_l(t)$ whose components $x_m^l(t)$ for $m \in [\![1, M_l]\!]$ and $l \in \{1, 2\}$ are the complex envelopes of the signal received on the l-th antenna. Assuming a Line of Sight (LoS) propagation, we have:

$$\mathbf{x}_l(t) = \sum_{q=1}^{Q} \rho_{l,q} \mathbf{a}_l\left(\theta_l(\mathbf{p}_q)\right) s_q\left(t - \tau_l(\mathbf{p}_q)\right) + \mathbf{n}_l(t) \quad (1)$$

$$= \mathbf{A}_l \boldsymbol{\Omega}_l \mathbf{s}_l(t) + \mathbf{n}_l(t) \quad (2)$$

where $s_q(t)$ is the complex envelope of the q-th source at location $\mathbf{p}_q$ and $\mathbf{s}_l(t) = \left(s_q(t - \tau_l(\mathbf{p}_q))\right)_{1 \le q \le Q}$. The sources are statistically independent and have a bandwidth B_q such that $(\max_{q \in [\![1,Q]\!]} B_q \le B)$. The additional noise $\mathbf{n}_l(t)$ is spatially white and independent between the stations. The l-th station steering vector $\mathbf{a}_l\left(\theta_l(\mathbf{p})\right)$ is noted $\mathbf{a}_l(\mathbf{p})$ in the remainder of the paper and $\mathbf{A}_l = [\mathbf{a}_l(\mathbf{p}_q), ..., \mathbf{a}_l(\mathbf{p}_Q)]$. The parameters, $\rho_{l,q}$, $\theta_l(\mathbf{p}_q)$ and $\tau_l(\mathbf{p}_q)$ are the complex attenuation, the AoA and the Time of Arrival (ToA) of the q-th source arriving on the l-th station respectively. Let us note the diagonal matrix $\boldsymbol{\Omega}_l$ as $\boldsymbol{\Omega}_l = (\delta_{j,q} \rho_{l,q})_{1 \le (q,j) \le Q}$ and $\Delta\tau_{ij}(\mathbf{p}) = \tau_j(\mathbf{p}) - \tau_i(\mathbf{p})$ the TDoAs of a source of location $\mathbf{p}$.

2.2. Problem formulation

The direct geolocalization algorithms [8]-[10] use the global array on which the associated observation vector is:

$$\mathbf{x}(t) = \left[\mathbf{x}_1^T(t), \mathbf{x}_2^T(t)\right]^T \quad (3)$$

Such the global observation is generally broadband because $|s_q(t - \tau_1(\mathbf{p}_q))| \ne |s_q(t - \tau_2(\mathbf{p}_q))|$ in the general case, [11]-[13]. More precisely, the narrowband hypothesis is verified on the global array if and only if:

$$\max_{q \in [\![1,Q]\!]} |\Delta\tau_{12}(\mathbf{p}_q) \times B_q| \ll 1 \quad (4)$$

The DPD, LOST and LOST-FIND algorithms cope with the broadband effect in two different ways. The DPD approach decomposes the signal into K regular narrowband signals of bandwidth $\frac{B}{K}$ with a filters bank. The LOST and LOST-FIND approaches are similar to the DPD one in the time domain thanks to the use of a space-time observation with K temporal shifts $\mathbf{x}(t - kT_e)$ of the observation $\mathbf{x}(t)$. These algorithms then make use of conventional narrowband algorithms such as MUSIC [2] on the resulting narrowband signals. On the one hand, if K is not large enough the estimation of the sources location is biased [12], [13]. On another hand, the computational complexity increases with K.

3. NEW APPROACH: TARGET

In order to overcome the limitations of DPD, LOST and LOST-FIND, we propose in this paper a broadband algorithm based on a new observation of the signals $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$. According to Eq.(1), $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t - \tau)$ are temporally synchronized with respect to the q-th source for $\tau = \Delta\tau_{12}(\mathbf{p}_q)$. This is the reason why we consider the following observations instead of Eq.(3):

$$\mathbf{x}(t, \tau) = \left[\mathbf{x}_1^T(t), \mathbf{x}_2^T(t - \tau)\right]^T \quad (5)$$

where the observation of the second station is $\mathbf{x}_2(t - \tau)$ instead of $\mathbf{x}_2(t)$. Using the algebraic properties of the covariance matrix of the multiple observations $\mathbf{x}(t, \tau)$, the purpose is the joint estimation of the AoAs $(\theta_1(\mathbf{p}_q), \theta_2(\mathbf{p}_q))$ and TDoA $(\Delta\tau_{12}(\mathbf{p}_q))$ of each source. It is also useful to note that the values τ of the observations $\mathbf{x}(t, \tau)$ are bounded. Indeed, thanks to the triangle inequality we have:

$$|\tau| \le \tau_{max} = \frac{|\mathbf{p}_{b_1} - \mathbf{p}_{b_2}|}{c} \quad (6)$$

where c is the speed of light in vacuum.

3.1. Spectral behavior of the new covariance matrix

We will study in this part the algebraic properties of the covariance matrix formed by the observation Eq.(5). Indeed, the TARGET algorithm will exploit the rank deficiency of this covariance matrix after a broadband multichannel synchronization. According to Eq.(1), the observation $\mathbf{x}(t, \tau)$ is:

$$\mathbf{x}(t, \tau) = \sum_{q=1}^{Q} \mathbf{U}\left(\mathbf{p}_q, \boldsymbol{\rho}_q\right) \mathbf{s}_q\left(t, \tau\right) + \mathbf{n}(t, \tau) \quad (7)$$

where $\boldsymbol{\rho}_q = [\rho_{1,q}, \rho_{2,q}]^T$,

$$\mathbf{U}\left(\mathbf{p}_q, \boldsymbol{\rho}_q\right) = \begin{pmatrix} \rho_{1,q}\mathbf{a}_1\left(\theta_1\left(\mathbf{p}_q\right)\right) & \mathbf{0} \\ \mathbf{0} & \rho_{2,q}\mathbf{a}_2\left(\theta_2\left(\mathbf{p}_q\right)\right) \end{pmatrix}, \quad (8)$$

$$\mathbf{s}_q\left(t, \tau\right) = \begin{bmatrix} s_q(t) \\ s_q(t + \Delta\tau_{12}(\mathbf{p}_q) - \tau) \end{bmatrix} \quad (9)$$

and $\mathbf{n}(t, \tau) = \left[\mathbf{n}_1^T(t), \mathbf{n}_2^T(t - \tau)\right]^T$. In presence of Q broadband signals and according to Eq.(7), the observation $\mathbf{x}(t, \tau)$ is in the general case a mixing of $2Q$ equivalent signals if $\tau \ne \Delta\tau_{12}(\mathbf{p}_q)$. However, if $\tau = \Delta\tau_{12}(\mathbf{p}_q) = \Delta\tau_q$, the observation $\mathbf{x}(t, \tau)$ becomes:

$$\mathbf{x}(t, \Delta\tau_q) = \mathbf{u}_q s_q(t) + \sum_{j \ne q}^{Q} \mathbf{U}\left(\mathbf{p}_j, \boldsymbol{\rho}_j\right) \mathbf{s}_j\left(t, \Delta\tau_q\right) + \mathbf{n}(t, \Delta\tau_q) \quad (10)$$

where $\mathbf{u}_q = \mathbf{U}\left(\mathbf{p}_q, \boldsymbol{\rho}_q\right) \times \mathbf{1}$ and $\mathbf{1} = [1, 1]^T$.

The TARGET approach exploits the rank deficiency of the covariance matrix of $\mathbf{x}(t, \Delta\tau_q)$. More precisely:

$$\mathbf{R}_\mathbf{x}(\tau) = \mathbb{E}\left[\mathbf{x}(t,\tau)\mathbf{x}^H(t,\tau)\right] \tag{11}$$

$$\mathrm{rank}\left(\mathbf{R}_\mathbf{x}(\tau) - \mathbf{R}_n\right) = \begin{cases} 2Q & \text{if } \tau \neq \Delta\tau_q \\ 2Q-1 & \text{if } \tau = \Delta\tau_q \end{cases} \tag{12}$$

where

$$\mathbf{R}_n = \mathbb{E}\left[\mathbf{n}(t,\tau)\mathbf{n}^H(t,\tau)\right] = \begin{pmatrix} \sigma_1^2\mathbf{I}_{M_1} & \mathbf{0} \\ \mathbf{0} & \sigma_2^2\mathbf{I}_{M_2} \end{pmatrix} \tag{13}$$

Thus, the q-th source in the observation $\mathbf{x}(t, \Delta\tau_q)$ is temporally synchronized according to Eq.(10) and Eq.(12). The observation $\mathbf{x}(t, \Delta\tau_q)$ is consequently a mixing of $2Q-1$ equivalent signals.

The estimation of the σ_1^2 and σ_2^2 needs at least one sensor per antenna array. Therefore, the identifiability of the sources number Q of the system is:

$$Q < \min\{M_1, M_2\} \tag{14}$$

with $M_l \neq 0$. This is the same as for the identifiability of the conventional triangulation algorithm. However if we reasonably consider a "good" SNR, we can say that $\sigma_1^2 = \sigma_2^2 = \epsilon$ with $\epsilon \to 0$. Let $\lambda_1(\tau) \geq \lambda_2(\tau) \cdots \geq \lambda_{2Q}(\tau) \geq \lambda_{2Q+1}(\tau) = \cdots = \lambda_M(\tau) = \epsilon$ be the eigenvalues of the matrix $\mathbf{R}_\mathbf{x}(\tau) - \epsilon\mathbf{I}_M$ ranked in decreasing order, we have:

$$Q < \frac{M}{2} = \frac{M_1 + M_2}{2} \tag{15}$$

This means that, in the case of a "good" SNR (*e.g.* 10dB or more), TARGET releases the identifiability initial conditions and permits to be more flexible with the sensors number of each antenna array.

Thus, a first approach to estimate the TDoAs $\Delta\tau_q$ of the sources is searching the zeros of the criterion $\lambda_{2Q}(\tau)$ where $|\tau| \leq \tau_{max}$ according to Eq.(6). However, the computation cost of $\lambda_{2Q}(\tau)$ is high and the criterion does not exploit the sources direction of arrival on each station. Thus, in the following sections we propose an alternative to these problems.

3.2. Low computation cost TDoA estimator

To understand of the temporal synchronization functioning of the TARGET algorithm, we propose here to study this synchronization, resulting in a new TDoA estimator. We will use these results in the next section.

Assuming that $\mathbb{E}\left[\mathbf{n}(t,\tau)\mathbf{n}^H(t,\tau)\right] = \mathbf{R}_n$, the covariance matrix of Eq.(11) can be rewritten as:

$$\mathbf{R}_\mathbf{x}(\tau) = \tilde{\mathbf{R}}_\mathbf{x}(\tau) + \mathbf{R}_n \tag{16}$$

with

$$\tilde{\mathbf{R}}_\mathbf{x}(\tau) = \begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12}(\tau) \\ \mathbf{R}_{12}^H(\tau) & \mathbf{R}_{22} \end{pmatrix} \tag{17}$$

and $\forall(l, v) \in \{1, 2\}^2$

$$\mathbf{R}_{lv}(\tau) = \begin{cases} \mathbb{E}\left[\mathbf{x}_l(t-\tau)\mathbf{x}_v^H(t-\tau)\right] - \sigma_l^2\mathbf{I}_{M_l} & \text{if } l = v \\ \mathbb{E}\left[\mathbf{x}_l(t)\mathbf{x}_v^H(t-\tau)\right] & \text{otherwise} \end{cases} \tag{18}$$

To simplify the mathematical manipulations we will write in the remainder of this paper $\mathbf{R}_{12}^H(\tau) = \mathbf{R}_{21}(-\tau)$. The noise variances σ_1^2 and σ_2^2 can be estimated from an eigenvalues decomposition of $\mathbf{R}_\mathbf{x}(\tau)$. According to Eq.(2), the matrix $\mathbf{R}_{lv}(\tau)$ is:

$$\mathbf{R}_{lv}(\tau) = \mathbf{A}_l\mathbf{\Omega}_l\mathbf{S}_{lv}(\tau)\mathbf{\Omega}_v^H\mathbf{A}_v^H \tag{19}$$

where $\mathbf{S}_{lv}(\tau) = \mathbb{E}\left[\mathbf{s}_l(t)\mathbf{s}_v^H(t-\tau)\right]$ are diagonal matrices.

Let $\mathbf{R}_{ll}^{1/2}$ be the $M_l \times Q$ whitening inverse matrix of l-th station defined from the eigenvalue decomposition of $\mathbf{R}_{ll}$ as:

$$\mathbf{R}_{ll}^{1/2} = \mathbf{E}_l\mathbf{\Lambda}_l^{1/2} \tag{20}$$

where the columns of the $M_l \times Q$ matrix $\mathbf{E}_l$ are composed by the eigenvectors associated to the nonzero eigenvalues of $\mathbf{R}_{ll}$ and the diagonal matrix $\mathbf{\Lambda}_l$ is composed by the associated eigenvalues. Then, the $M \times M$ matrix $\tilde{\mathbf{R}}_\mathbf{x}(\tau)$ can be reduced into the following $2Q \times 2Q$ covariance matrix of the whitened observations:

$$\tilde{\mathbf{R}}_\mathbf{w}(\tau) = \mathbf{W}\tilde{\mathbf{R}}_\mathbf{x}(\tau)\mathbf{W}^H \tag{21}$$

$$\mathbf{W} = \begin{pmatrix} \left(\mathbf{R}_{11}^{1/2}\right)^+ & \mathbf{0} \\ \mathbf{0} & \left(\mathbf{R}_{22}^{1/2}\right)^+ \end{pmatrix} \tag{22}$$

where $\mathbf{A}^+$ is the Moore-Penrose pseudoinverse of $\mathbf{A}$ such that $\mathbf{A}^+ = \left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H$. The matrix $\tilde{\mathbf{R}}_\mathbf{w}(\tau)$ is then:

$$\tilde{\mathbf{R}}_\mathbf{w}(\tau) = \begin{pmatrix} \mathbf{I}_Q & \tilde{\mathbf{R}}_{12}(\tau) \\ \tilde{\mathbf{R}}_{21}(-\tau) & \mathbf{I}_Q \end{pmatrix} \tag{23}$$

$$\tilde{\mathbf{R}}_{lv}(\tau) = \left(\mathbf{R}_{ll}^{1/2}\right)^+\mathbf{R}_{lv}(\tau)\left(\mathbf{R}_{vv}^{1/2}\right)^{+H} \tag{24}$$

$\forall(l, v) \in \{1, 2\}^2$ and $l \neq v$. It can be shown that the matrix $\tilde{\mathbf{R}}_\mathbf{w}(\tau)$ has at least one null eigenvalue if and only if $\tilde{\mathbf{R}}_\mathbf{w}(\tau)\mathbf{v} = \mathbf{0}$, where $\mathbf{v} = \left[\mathbf{v}_1^T, \mathbf{v}_2^T\right]^T$ is the eigenvector projecting the matrix onto the null space. After resolution of this matrix system, one has:

$$\mathbf{v}_l^H\left(\mathbf{I}_Q - \tilde{\mathbf{R}}_{lv}(\xi_v \times \tau)\tilde{\mathbf{R}}_{vl}(\xi_l \times \tau)\right)\mathbf{v}_l = 0 \tag{25}$$

$$\mathbf{v}_l^H\tilde{\mathbf{G}}_l(\tau)\mathbf{v}_l = 0 \tag{26}$$

with $\xi_l = (-1)^l$. Thanks to Rayleigh's quotient properties [14] and according to Eq.(6), the optimum value of TDoA is given by:

$$\Delta\hat{\tau}_q = \min_\tau \lambda_{\min}\left\{\tilde{\mathbf{G}}_l(\tau)\right\} \tag{27}$$

where $\lambda_{\min}\{\mathbf{A}\}$ is the smallest eigenvalue of $\mathbf{A}$.

We can do here a useful comparison between this method and conventional TDoA estimation methods which consist in measuring the cross-correlation between one and only one sensor of each station. Indeed, if we take $M_1 = M_2 = 1$ then we can observe that the TDoA estimator of Eq.(27) is identical to the usually used one:

$$\Delta\hat{\tau}_q = \min_\tau\left(1 - \frac{|r_{12}(\tau)|^2}{r_{11}(0)r_{22}(0)}\right) \tag{28}$$

where, according to Part 2.1, r_{11}, r_{22} and r_{12} are the intra-station and inter-station correlation:

$$r_{lv}(\tau) = \mathbb{E}\left[x_1^l(t)x_1^{v*}(t-\tau)\right] \quad (29)$$

3.3. A new AoA-TDoA joint estimator

In this section a joint AoA-TDoA estimator is derived from the algebraic properties of the matrices $\tilde{\mathbf{G}}_l(\tau)$ at Eq.(26). The matrix of interest is more precisely the following $M_l \times M_l$ matrix $\mathbf{F}_l(\tau)$. According to Eq.(24) and Eq.(26), $\mathbf{F}_l(\tau)$ is:

$$\begin{aligned}\mathbf{F}_l(\tau) &= \left(\mathbf{R}_{ll}^{1/2}\right)^{+H}\tilde{\mathbf{G}}_l(\tau)\left(\mathbf{R}_{ll}^{1/2}\right)^{H} && (30)\\ &= \mathbf{\Pi}_l - \mathbf{R}_{ll}^{+}\mathbf{R}_{lv}(\xi_v \times \tau)\mathbf{R}_{vv}^{+}\mathbf{R}_{vl}(\xi_l \times \tau) && (31)\end{aligned}$$

where $\mathbf{\Pi}_l = \mathbf{E}_l\mathbf{E}_l^H$ is the projector onto signal subspace of $\mathbf{R}_{ll}$. The highest eigenvalues of the matrix $\mathbf{F}_l(\tau)$ are the same than the matrix $\tilde{\mathbf{G}}_l(\tau)$, and the smallest are equal to zero. To guarantee that the eigenvalues of $\tilde{\mathbf{G}}_l(\tau)$ are the smallest eigenvalues of $\mathbf{F}_l(\tau)$, we add $\mathbf{F}_l(\tau)$ to the projector onto orthogonal signal subspace:

$$\begin{aligned}\mathbf{G}_l(\tau) &= \mathbf{F}_l(\tau) + \mathbf{\Pi}_l^{\perp} && (32)\\ &= \mathbf{I}_{M_l} - \mathbf{R}_{ll}^{+}\mathbf{R}_{lv}(\xi_v \times \tau)\mathbf{R}_{vv}^{+}\mathbf{R}_{vl}(\xi_l \times \tau) && (33)\end{aligned}$$

Then, according to Eq.(19), the matrix $\mathbf{R}_{ll}^{+}$ is:

$$\mathbf{R}_{ll}^{+} = \mathbf{A}_l^{+H}\left(\mathbf{\Omega}_l^{-1}\right)^H \mathbf{S}_{ll}^{-1}(0)\mathbf{\Omega}_l^{-1}\mathbf{A}_l^{+} \quad (34)$$

In the following, the algebraic structure of $\mathbf{G}_l(\tau)$ is analyzed in order to establish the TARGET criterion. According to Eq.(19), Eq.(33) and Eq.(34), we have:

$$\mathbf{G}_l(\tau) = \mathbf{I}_{M_l} - \mathbf{A}_l\left(\mathbf{A}_l^H\mathbf{A}_l\right)^{-1}\mathbf{\Sigma}_l(\tau)\mathbf{A}_l^H \quad (35)$$

$$\mathbf{\Sigma}_l(\tau) = \mathbf{S}_{ll}^{-1}(0)\mathbf{S}_{lv}(\xi_v \times \tau)\mathbf{S}_{vv}^{-1}(0)\mathbf{S}_{vl}(\xi_l \times \tau) \quad (36)$$

where $\mathbf{\Sigma}_l(\tau)$ is a diagonal matrix of components:

$$\left(\mathbf{\Sigma}_l(\tau)\right)_{q,q} = \frac{|r_q(\tau - \Delta\tau_q)|^2}{|r_q(0)|^2} \quad (37)$$

where $r_q(\tau) = \mathbb{E}\left[s_q(t)s_q^*(t-\tau)\right]$, such that $0 \leq (\mathbf{\Sigma}_l(\tau))_{q,q} \leq 1$ and reaches the upper bound for $(\mathbf{\Sigma}_l(\Delta\tau_q))_{q,q}$. In addition, the q-th column of $\mathbf{A}_l$ is $\mathbf{a}_l(\mathbf{p}_q)$ such that $\mathbf{a}_l^H(\mathbf{p}_q)\mathbf{A}_l = \mathbf{1}_q$ where the vector $\mathbf{1}_q$ is the q-th column of the identity matrix $\mathbf{I}_{M_l}$ and $\mathbf{a}_l(\mathbf{p}_q) = \mathbf{\Pi}_l\mathbf{a}_l(\mathbf{p}_q)$. According to Eq.(35), we obtain:

$$\begin{aligned}J_{T_l}(\theta_{lq}, \tau) &= \frac{\mathbf{a}_l^H(\theta_{lq})\mathbf{G}_l(\tau)\mathbf{a}_l(\theta_{lq})}{\mathbf{a}_l^H(\theta_{lq})\mathbf{a}_l(\theta_{lq})} && (38)\\ &= \left(1 - \frac{|r_q(\tau - \Delta\tau_q)|^2}{|r_q(0)|^2}\right) && (39)\end{aligned}$$

where $\Delta\tau_q = \Delta\tau_{12}(\mathbf{p}_q)$ and $\theta_{lq} = \theta_l(\mathbf{p}_q)$. It is important to note that the angle θ_l depends on the TDoA τ. Indeed the angle domain is conditioned by τ as for a fixed τ the whole angle set of the l-th station is not reachable. According to Eq.(38), $J_{T_l}(\theta_{lq}, \Delta\tau_q) = 0$ and $0 \leq J_{T_l}(\theta_l, \tau) \leq 1$. Thus, a first joint AoA-TDoA estimator of TARGET is:

$$\left(\hat{\theta}_{lq}, \Delta\hat{\tau}_q\right) = \min_{(\theta_l,\tau)} J_{T_l}(\theta_l, \tau) \quad (40)$$

The source location $\mathbf{p}_q$ is then estimated from $(\hat{\theta}_{lq}, \Delta\hat{\tau}_q)$ for $q \in [\![1, Q]\!]$ from the equations $\Delta\tau_q = \Delta\tau_{12}(\mathbf{p}_q)$ and $\theta_{lq} = \theta_l(\mathbf{p}_q)$. The criterion Eq.(38) only depends on the AoA of the l-th station θ_l and the TDoA τ. The exploitation of the AoAs on both stations leads to the following TARGET criterion:

$$J_{TARGET}(\theta_1, \theta_2, \tau) = \frac{J_{T_1}(\theta_1, \tau) + J_{T_2}(\theta_2, \tau)}{2} \quad (41)$$

where $J_{TARGET}(\theta_{1q}, \theta_{2q}, \Delta\tau_q) = 0$ for $\tau = \Delta\tau_{12}(\mathbf{p}_q)$, $\theta_1 = \theta_1(\mathbf{p}_q)$ and $\theta_2 = \theta_2(\mathbf{p}_q)$. As the AoAs and TDoA depend on a location $\mathbf{p}$, one could define a function such that $\theta_1(\mathbf{p}) = \Psi\left(\theta_2(\mathbf{p}), \Delta\tau_{12}(\mathbf{p})\right)$. Thus, the implementation of TARGET can be written as:

$$\left(\hat{\theta}_{2q}, \Delta\hat{\tau}_q\right) = \min_{(\theta_2,\tau)} J_{TARGET}(\Psi(\theta_2, \tau), \theta_2, \tau) \quad (42)$$

where we recall that θ_2 is conditioned by τ and according to Eq.(6) τ is bounded.

4. SIMULATIONS

In this part we compare the TARGET algorithm to DPD, LOST, LOST-FIND, the classical triangulation [1] (AoA/AoA) and the localization in 2-steps combining the AoA of one station and the TDoA with 1 sensor of each station as described in Eq.(28) [1] (AoA/TDoA). We will consider two received stations. In a Cartesian coordinate system, the first one is located at (-200m,0), and the second one at (+200m,0). Both received arrays are composed of six sensors where five are in a circular formation around a sixth in the center. The array radius is 0.8m, they have a received bandwidth of 2MHz and all sources have the same bandwidth. We consider $K = 5$ temporal shifts for the space-time processing (LOST and LOST-FIND) and $K = 5$ decompositions of the stations bandwidth for DPD.

First of all, we compare the performance of the TARGET TDoA estimator of Eq.(27) and the traditional one described in Eq.(28). For this we consider a two sources case where the first source is located at (+50m,+100m) and the second at (-50m,+100m). One observe the RMSE performance of the TDoA estimate of the first source on Fig.1 as a function of the SNR. For the traditional TDoA estimator, the propagation time between the sources ($\approx$ 330ns) is higher than the temporal resolution limit ($\approx$ 250ns) but smaller than the limit from which the estimator is not biased anymore ($\approx$ 500ns). Therefore it is biased and much less efficient compared to the estimated TDoA resulting from TARGET. We clearly see that the performance of the TARGET TDoA estimator is better and converges very quickly to the theoretical values of TDoA.

Then, we study the TARGET algorithm's performance given in Eq.(42). We consider in a first time a single source

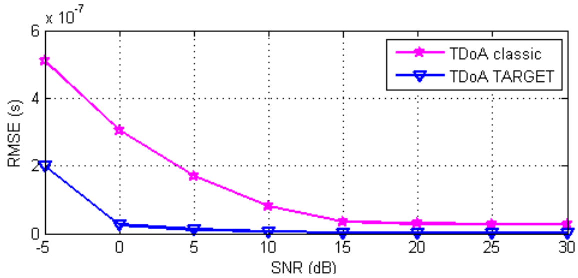

Fig. 1. Classic TDoA versus TARGET TDoA.

case. The source is located at (0,+5m). One could note that such a scenario is very severe for algorithms which only exploit the AoA as their performances will be strongly deteriorated. In Fig.2, we plot the RMSE of the estimated source position as a function of the SNR. We observe that, for the two algorithms which do not exploit the TDoA (*i.e.* AoA/AoA and LOST), the performance is poor. We also observe that in the context of a low SNR, TARGET has the lowest RMSE. Globally, TARGET, DPD and LOST-FIND algorithms have good performance.

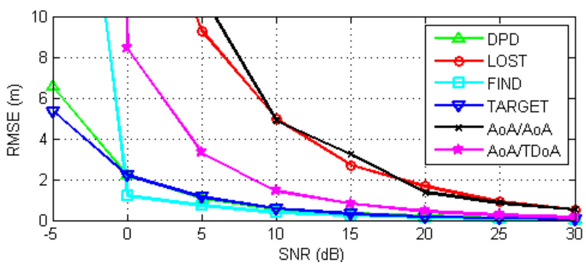

Fig. 2. One source at position (0,+5m).

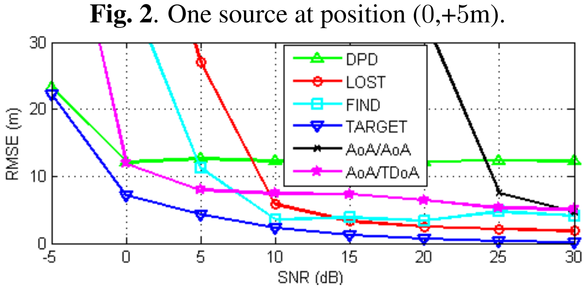

Fig. 3. Two sources of positions (+140m,+5m), (+150,-5m), estimation of the first source position.

In Fig.3 we consider a two sources case where the two sources are very more closely spaced than previously. We position the first source at (+140m,+5m) and the second at (+150m,-5m). The two sources are located very close to the axis formed by the two stations (which is always a complicated context to estimate the position with only the AoAs parameters) and are very close to each other with respect to the system dimension. Furthermore, the sources are shifted away from the center of the arrays to reveal the bias due to the residual broadband effect defined in [12] and [13]. We plot the RMSE of the estimate of the first source position as a function of the SNR. The AoA/TDoA algorithm malfunctions due to the lack of temporal resolution of the TDoA estimation (the TDoA estimator of Eq.(28) does not distinguish the two sources). We can observe that both algorithms only exploiting the AoA are in difficulty and that 1-step methods (DPD, LOST and LOST-FIND algorithms) are biased. As the TARGET algorithm is an algorithm "naturally" operating with broadband signals, it is not biased and has the best performance.

5. CONCLUSION

A new algorithm TARGET was proposed for blind geolocalization. This algorithm is designed to operate in a broadband context (broadband signals and remote stations) and simultaneously exploits the TDoA and AoA of the sources. This new algorithm fully exploits the broadband context unlike the recent 1-step algorithms such as DPD, LOST and LOST-FIND. We observed that it has no bias due to the residual broadband effect and the performance of TARGET are good, even in unfavorable contexts for 1-step methods previously mentioned.

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