MUSIC-like Processing of Pulsed Continuous Wave Signals in Active Sonar Experiments

Hock Siong LIM
Thales Research and Technology, Singapore
Thales Solutions Asia Pte Ltd
Singapore
hocksiong.lim@asia.thalesgroup.com

Boon Poh NG
School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore
EBPNG@ntu.edu.sg

***Abstract*—This paper proposes a novel processing approach for range-bearing and range-doppler processing of pulsed continuous wave signals used in active sonar applications. The proposed algorithm, termed the MUSIC-Capon algorithm in this paper, has MUSIC-like spectral estimation performance and the motivation is to capitalize on its super-resolution capability to minimize spectral leakage in range-bearing and range-doppler spectra so as to mitigate energy cluttering due to underwater reverberation. Encouraging results are obtained with real data processing in a proof of concept, where very distinct peaks can be observed in target cells with flat spectral floor at non-target cells; very little spectral leakage is also observed in range-Doppler spectrum especially near zero-Doppler region. Hence the results demonstrate some advantages of our proposed approach.**

***Index Terms*—active sonar; MUSIC-like; super-resolution**

I. Introduction

It is well known that one of the challenges faced by active sonar in shallow-water environments is the effects of reverberation. These effects will cause high energy clutter in Range-Bearing and Range-Doppler spectra and have adverse impact on target detection and parameter estimation. Adaptive array [4] and Doppler processing with low sidelobes are some of the popular measures used to mitigate such effects. Therefore it is intuitive that super-resolution processing could also help in mitigating some effects of reverberation.

Recently, the MUSIC-like processing approach was proposed in [10] (termed standard MUSIC-like algorithm in this paper) and was shown to produce high-resolution performance without subspace decomposition. However it assumed that the noise is spatially uncorrelated, which is not the case for underwater environments [1] [2] [3]. It was also shown in [5] and [6] that super-resolution processing such as the MUSIC [9] and Minimum-Norm [7] algorithms require prior knowledge of noise spatial correlation properties to yield optimal results.

The main motivation of this work is to explore the feasibility of using the MUSIC-like approach to mitigate the effects of reverberation in active sonar because this approach does not require knowledge of number of sources. However, a major weakness of the standard MUSIC-like algorithm of [10] is its inability to cope with the complex and dynamic correlated noise encountered in underwater active sonar applications. A new MUSIC-like processing approach, which combine the benefits of MUSIC-like and Capon algorithm and is termed the MUSIC-Capon algorithm, is presented in this paper. The analysis presented in this paper shows that the proposed algorithm uses the estimated noise correlation matrix that is presently inherently in the Capon's weight vector to improve the performance of MUSIC-like processing. The capability of the new algorithm is validated by results obtained from real data processing of pulsed continuous wave signals. The results from our proposed algorithm are also compared to Capon algorithm and the outcome is also very encouraging.

The organization of this paper is as follows. Section II presents the signal model used in this paper follow by the description of our proposed MUSIC-Capon algorithm in Section III. Results from proof-of-concept data processing are presented in Section IV and a short discussion is presented in Section V. This paper is concluded in Section VI.

II. Signal Model

Consider an L-sensor array situated in the farfield of K narrowband sources along directions $\Theta = [\theta_1, \theta_2, ..., \theta_K]^T$. The sensors' snapshot vector $\mathbf{x}(n)$ can be modelled as

$$\mathbf{x}(n) = \mathbf{A}(\Theta)\,\mathbf{s}(n) + \mathbf{v}(n), \qquad (1)$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_K)]$ is the $L \times K$ matrix of array manifold formed by the K source position vectors along their respective source directions, $\mathbf{s}(n)$ is a $K \times 1$ source signal vector at nth snapshot and $\mathbf{v}(n)$ denotes the noise vector. The symbol $\mathbf{a}(\theta)$ denotes the source position vector at direction θ and can be written as

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{L}} \left[\exp\left(j\mathbf{k}_\theta^T \mathbf{r}_1\right), ..., \exp\left(j\mathbf{k}_\theta^T \mathbf{r}_L\right)\right]^T, \qquad (2)$$

where $\mathbf{r}_i$ is the i^{th} sensor location vector, $\mathbf{k}_\theta = \frac{2\pi f}{\upsilon}\mathbf{u}_\theta$ is the wavenumber vector and $\mathbf{u}_\theta$ is the unit vector along the direction of wave propagation. The signal frequency and propagation speed are denoted by f and υ respectively. The cross-correlation matrices of source signals and noise are respectively given by

$$E\left\{\mathbf{s}\mathbf{s}^H\right\} = \mathbf{R}_{ss}$$

and

$$E\left\{\mathbf{v}\mathbf{v}^H\right\} = \sigma_v^2 \mathbf{R}_{vv},$$

where $E\{.\}$ denotes the expectation operation and σ_v^2 denotes the noise power. When signals and noise are uncorrelated, then $E\{\mathbf{v}^H\mathbf{s}\} = 0$ and the covariance matrix can also be written as

$$\begin{aligned}\mathbf{R} &= E\{\mathbf{x}\mathbf{x}^H\} \\ &= \mathbf{A}(\Theta)\mathbf{R}_{ss}\mathbf{A}^H(\Theta) + \sigma_v^2\mathbf{R}_{vv}.\end{aligned} \quad (3)$$

The sources are assumed to be uncorrelated and of rank one and hence the signal covariance matrix of K sources can be written as

$$\mathbf{A}(\Theta)\mathbf{R}_{ss}\mathbf{A}^H(\Theta) = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k), \quad (4)$$

where θ_k and σ_k^2 denote the bearing and power of the kth source respectively.

III. The Proposed Algorithm

Denoting $\mathbf{R}$ as the covariance matrix and $\mathbf{e}$ as the conventional steering vector given by,

$$\mathbf{e}(\theta) = \frac{1}{\sqrt{L}}\left[\exp\left(j\mathbf{k}_\theta^T\mathbf{r}_1\right), ..., \exp\left(j\mathbf{k}_\theta^T\mathbf{r}_L\right)\right]^T, \quad (5)$$

The proposed approach is as follows,

$$\begin{aligned}&\underset{\mathbf{w}}{\text{minimize}} \;\; \mathbf{w}^H\mathbf{R}\mathbf{w} \\ &\text{subject to} \;\; \mathbf{w}^H\mathbf{d}\mathbf{d}^H\mathbf{w} + \beta\mathbf{w}^H\mathbf{w} = c,\end{aligned} \quad (6)$$

where

$$\mathbf{d} = \frac{\mathbf{R}^{-1}\mathbf{e}}{\mathbf{e}^H\mathbf{R}^{-1}\mathbf{e}}, \quad (7)$$

$\mathbf{w}$ is the solution vector, β is the controlling parameter and c is an arbitrary constant which has no consequence in the solution. It is noted that (7) is the Capon's weight vector, hence the algorithm is termed the MUSIC-Capon algorithm in this paper. Using the Lagrange multiplier method, the objective function to (6) can be written as

$$L(\lambda, \mathbf{w}) = \mathbf{w}^H\mathbf{R}\mathbf{w} - \lambda\mathbf{w}^H\left(\mathbf{d}\mathbf{d}^H + \beta\mathbf{I}\right)\mathbf{w}, \quad (8)$$

where λ denotes the Lagrange multiplier. Differentiating (8) with respect to $\mathbf{w}$ and equate to zero yields

$$\left(\mathbf{d}\mathbf{d}^H + \beta\mathbf{I}\right)^{-1}\mathbf{R}\mathbf{w} = \lambda\mathbf{w}, \quad (9)$$

which is a generalized eigenvalue problem. To minimize the objective function in (6), the weight solution of the proposed MUSIC-Capon algorithm is chosen to be

$$p_{min}\left\{\mathbf{B} = \left(\mathbf{d}\mathbf{d}^H + \beta\mathbf{I}\right)^{-1}\mathbf{R}\right\}, \quad (10)$$

where $p_{min}\{.\}$ denotes the eigenvector associated to the minimum eigenvalue of the matrix in the curly brackets. The MUSIC-like spatial spectrum is then computed by

$$P = |\mathbf{w}^H\mathbf{e}|^{-2}. \quad (11)$$

Using the Sherman-Morrison's formula, (10) can be rewritten as

$$\mathbf{B} = \left(\beta^{-1}\mathbf{I} - \frac{\beta^{-2}}{1+\beta^{-1}\mathbf{d}^H\mathbf{d}}\mathbf{d}\mathbf{d}^H\right)\mathbf{R}. \quad (12)$$

Denoting $\gamma = \left(\mathbf{e}^H\mathbf{R}^{-1}\mathbf{e}\right)^{-1}$ and $\mathbf{d} = \gamma\mathbf{R}^{-1}\mathbf{e}$, (12) is simplified to

$$\mathbf{B} = \left(\beta^{-1}\mathbf{I} - \alpha\gamma^2\mathbf{R}^{-1}\mathbf{e}\mathbf{e}^H\mathbf{R}^{-1}\right)\mathbf{R}, \quad (13)$$

where $\alpha = \frac{\beta^{-2}}{1+\beta^{-1}}$. Since the eigenvalue has no consequence to the outcome of $\mathbf{w}$, then the matrix $\mathbf{B}$ of the proposed algorithm can be rewritten to

$$\begin{aligned}\tilde{\mathbf{B}} &= \mathbf{R} - \alpha\beta\gamma^2\mathbf{R}^{-1}\mathbf{e}\mathbf{e}^H \\ &= \mathbf{R} - \eta\gamma^2\mathbf{R}^{-1}\mathbf{e}\mathbf{e}^H,\end{aligned} \quad (14)$$

where $\eta = \alpha\beta = \frac{1}{\beta+1}$ and $\gamma^2 = \left(\mathbf{e}^H\mathbf{R}^{-2}\mathbf{e}\right)^{-1}$. The advantage of the proposed algorithm as observed from (14) is that the in-situ estimated noise correlation matrix $\hat{\mathbf{R}}_{vv}$ is included in our proposed MUSIC-Capon processor implicitly with the covariance matrix $\mathbf{R}$ which will help to ensure the robustness of MUSIC-Capon algorithm for active sonar processing against dynamic noise.

IV. Proof of Concept

The proposed MUSIC-Capon algorithm is used to process data collected from active sonar experiments in shallow-water environment. During the experiment, an echo-repeater is used to emulate as target and its settings were set to return five echoes to emulate targets at range of 3km, 4km, 5km, 6km and 7km and with velocity of -5 knots. Pulsed continuous wave signals with duration of 0.5sec were used in the experiment and the bearing of the echo-repeater is at approximately 220°.

A. Range-Bearing Results

The range-bearing spectra of the standard MUSIC-like and our proposed MUSIC-Capon algorithms are plotted in Figure 1 and Figure 2 respectively.

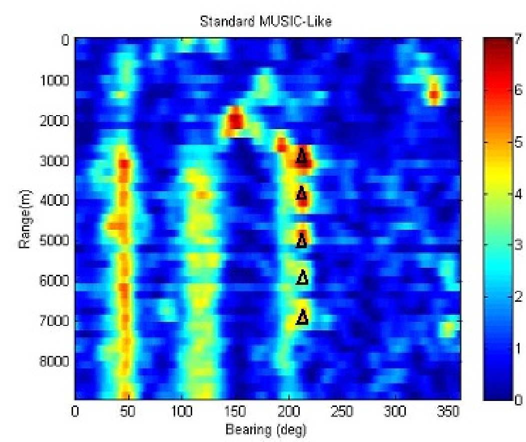

Fig. 1. Power Spectrum of Range-Bearing Processing Using Standard MUSIC-like Method

The positions of the targets are annotated by red triangles in the spectral results of the standard MUSIC-like processing in Figure 1 for reference but there is no need to do so in Figure 2 as the results are very clear. Visual inspection shows that the spectral results from the MUSIC-Capon algorithm is much

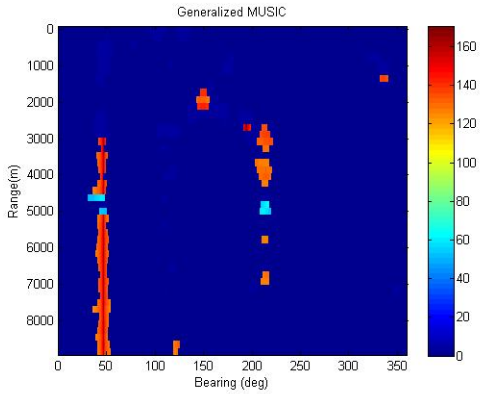

Fig. 2. Power Spectrum of Range-Bearing Processing Using Proposed MUSIC-Capon Method

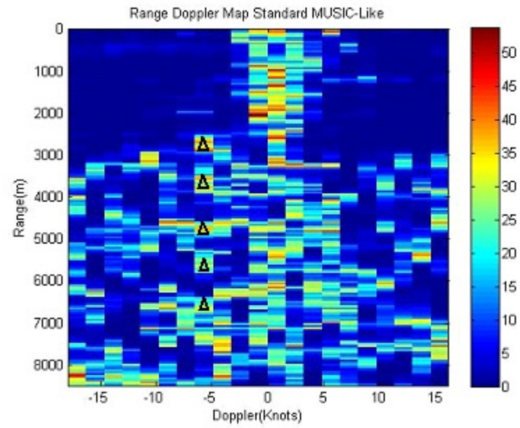

Fig. 3. Power Spectrum of Single Sensor Range-Doppler Processing Using Standard MUSIC-like Processing Method

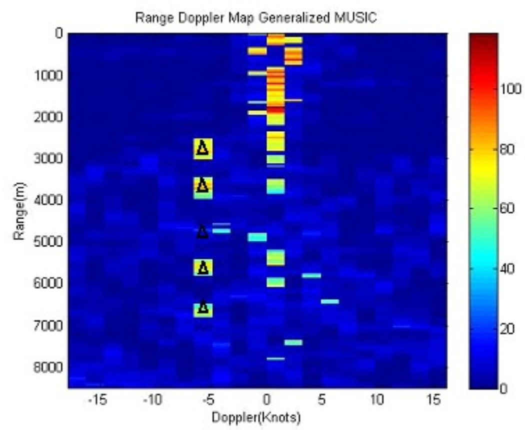

Fig. 4. Power Spectrum of Single Sensor Range-Doppler Processing Using MUSIC-Capon Processing Method

cleaner with fewer cells containing enough spectral power to intefere with visual detection. Both the standard MUSIC-like and MUSIC-Capon processing detect five targets at bearing 220° and the estimated ranges are at 3km to 7km with 1km interval. These estimates are consistent with the experiment parameters. Comparing the two results, it is obvious that the performance of our proposed MUSIC-Capon algorithm is much better than the standard MUSIC-like processing as follows. The peak-to-spectral-floor ratio at the target cells shown in Figure 2 are quite high at >80dB and its spectral floor at non-target cells are consistently flat. Using (11), the spectral results show that the weight vector generated by the MUSIC-Capon algorithm is orthogonal to the source position vector at target cells while is almost parallel to the source position vector at non-target cells. However, the peak-to-spectral-floor ratio of the standard MUSIC-like algorithm is <7dB which reflects poor orthogonality of between $\mathbf{w}$ and source position vector; this leads to relatively high energy at non-target cells compared to the spectral results from MUSIC-Capon algorithm.

B. Range-Doppler Results with Single Sensor

The Range-Doppler processing is applied to single chosen sensor. The range-Doppler spectra of the standard MUSIC-like and our proposed MUSIC-Capon algorithms are plotted in Figure 3 and Figure 4 respectively; the correct target cells are annotated with triangles in both figures. The estimates from the MUSIC-Capon algorithm are quite consistent with four targets estimated to have Doppler of -5 knots and 1 target at -4 knots. The results of the standard MUSIC-like processing are however not conclusive as there are many cells with high power level.

The spectral results from the figures show that the performance of our proposed MUSIC-Capon algorithm is better than the standard MUSIC-like algorithm from the following observations. Firstly, the spectral results observed from the MUSIC-Capon algorithm in Figure 4 is much cleaner where target detection is easier and more obvious compared to standard MUSIC-like algorithm. This is because the orthogonality of the weight vector $\mathbf{w}$ obtained from the MUSIC-Capon algorithm is (i) good in target cells and (ii) consistently poor at non-target cells. These properties will result in high resolution peak in target cells and flat spectral in non-target cells which will help in target detection. The range-Doppler results of our proposed MUSIC-Capon algorithm also show no signs of spectral leakage resulting in very little false alarms near zero-doppler cells.

V. Discussion

Since the results presented in this paper are the first ones for MUSIC-like processing in active sonar, it would be interesting to compare their performances with other well-known methods.

A. Range-Bearing

The conventional beamforming and Capon spectral estimation are used in range-bearing processing for benchmarking purpose and their results are shown in Figure 5 and Figure 6 respectively.

The first observation from the figures are that both spectra shows high energy clutter at near ranges <2km while the

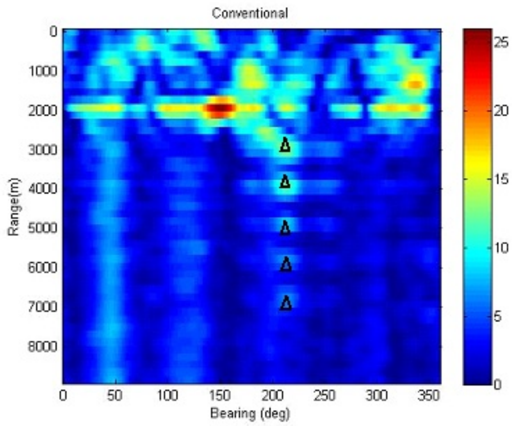

Fig. 5. Power Spectrum of Range-Bearing Processing Using Conventional Method

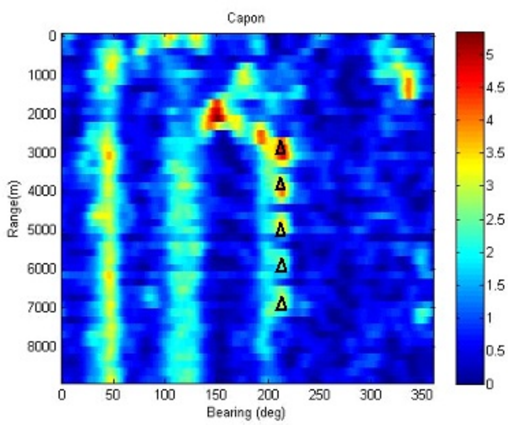

Fig. 6. Power Spectrum of Range-Bearing Processing Using Capon's Method

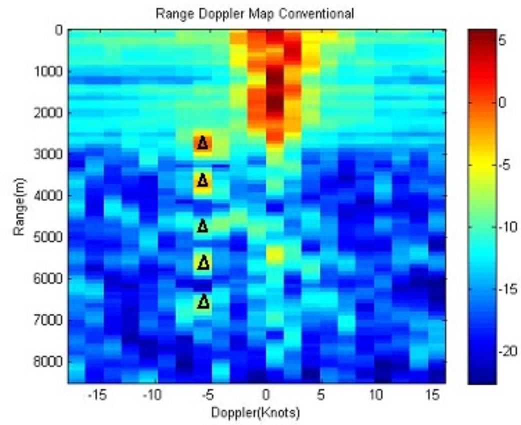

Fig. 7. Power Spectrum of Single Sensor Range-Doppler Processing Using FFT Processing Method

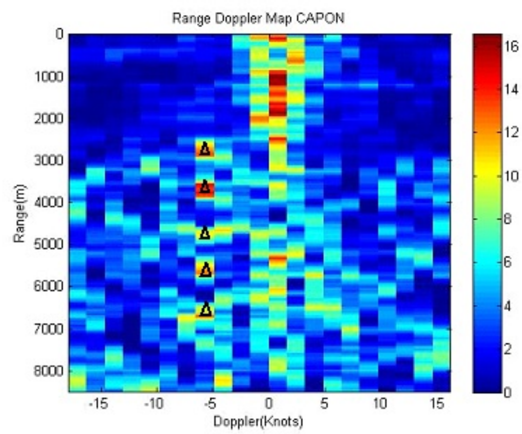

Fig. 8. Power Spectrum of Single Sensor Range-Doppler Processing Using Capon Processing Method

results of our proposed MUSIC-Capon algorithm is able to suppress them. The results also demonstrate that the MUSIC-Capon algorithm offers much better resolution. While Capon method is able to show all 5 targets at the correct bearing and range clearly, the results of conventional beamforming can only show two targets at 3km and 4 km clearly. It is also noted that the results of standard MUSIC-like algorithm is very similar to the Capon spectral estimator.

B. Range-Doppler

In this work the Fast Fourier Transform (FFT) and Capon methods are used. The range-Doppler results of FFT and Capon processing are plotted in Figure 7 and Figure 8 respectively.

Comparing between Figures 4, 7 and 8, the performance of the proposed MUSIC-Capon algorithm is still much better than the FFT and Capon methods. It is very much easier to distinguish the targets in the spectral results of MUSIC-Capon algorithm than the other methods. It can also be observed from Figure 7 and Figure 8 that spectral leakage across the low Doppler cells adjacent to zero-Doppler are high for the conventional and Capon method while energy are confined to zero-Doppler cells for MUSIC-Capon algorithm. This could help in reducing false alarms.

VI. Conclusion

This paper presents the MUSIC-Capon algorithm for MUSIC-like processing of continuous wave active sonar signals. Encouraging results from real data processing shows that it is feasible to use the MUSIC-Capon processing in active sonar signals and its performance is better than the standard MUSIC-like, conventional and Capon processing methods. The results of the MUSIC-Capon algorithm also show that super-resolution approach have advantages in active sonar processing with its non-spectral-leakage property. Future potential works include extending the processing method to other waveforms and development of detection framework.

References

[1] B.F.Cron and C.H.Sherman, "Spatial Correlation Functions for Various Noise Models", *The Journal of Acoustic Society of America*, Vol.34, No.11, pp 1732-1737, November 1962.

[2] B.F.Cron, B.C.Hassell and F.J.Keltonic, "Comparison of Theoretical and Experimental Values of Spatial Correlation", *The Journal of Acoustic Society of America*, Vol.37, No.3, pp 523-529, March 1965.

[3] E.M.Arase and T.Arase, "Correlation of Ambient Sea Noise", *The Journal of Acoustic Society of America*, Vol.40, No.1, pp 205-20, July 1966.

[4] J.Capon, "High Resolution Frequency-Wavenumber Specrum Analysis", *Proceedings IEEE*, Vol.57, No.8, pp 1408-1418, August 1969.

[5] G.Bienvenu and L.Kopp, "Influence of the Spatial Coherence of the Background Noise on High Resolution Passive Methods", *IEEE International Conference on Acoustic Speech and Signal Processing*, Vol.4, pp 306-309, 1979.

[6] G.Bienvenu and L.Kopp, "Adaptivity to Backgroung Noise Spatial Coherence for High Resolution Passive Methods", *IEEE Internation Conference on Acoustic Speech and Signal Processing*, Vol.5, pp 307-310, 1980.

[7] R.Kumaresan and D.W.Tuft, "Estimating the Angles of Arrival of Multiple Plane Waves", *IEEE Transactions on Aerospace and Electronic Systems*, Vol.19, No.1, pp 134-139, January 1983.

[8] G.Bienvenu and L.Kopp, "Decreasing High Resolution Method Sensitivity by Conventional Beamforming Processing", *IEEE Internation Conference on Acoustic Speech and Signal Processing*, Vol.9, No.1, pp 714-717, 1984.

[9] R.O.Schmidt, "Multiple Emitter Location and Signal Parameter Estimation", *IEEE Transactions on Antennas and Propagation*, Vol.34, No.3, pp 276-280, 1986.

[10] Y.Zhang and B.P.Ng, "MUSIC-like DOA Estimation Without Estimating the Number of Sources", *IEEE Transactions on Signal Processing*, Vol.58, No.3, pp 1668-1676, March 2010.

[11] V.V.Reddy, B.P.Ng and A.W.H.Khong, "Insights Into MUSIC-like Algorithm", *IEEE Transactions on Signal Processing*, Vol.61, No.10, pp 2551-2556, May 2013.