ONLINE PATH LOSS ESTIMATION FOR LOCALIZATION USING LARGE APERTURE ARRAY SIGNAL PROCESSING

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ABSTRACT

In this paper, a novel array-based method to estimate the path loss exponent (PLE) is developed. The method is designed as a part of an automatic calibration step, prior to localization of a source transmitting in the near-far field of the array. The method only requires the knowledge of the ranges between the array elements. By making the antenna elements transmit in turn, the array response model in the near-far field is exploited to estimate the current environment PLE. Simulation results show that this method can achieve good performance with one transmission round. The performance of the PLE estimation is investigated in the context of source localization with a sensitivity analysis to the PLE estimation.

Index Terms— Array processing, localization, path loss estimation, spherical wave propagation.

Notations

a, A	Scalar
$\underline{a}, \underline{A}$	Column Vector
$\mathbb{A}, \mathbf{A}$	Matrix
$(\cdot)^T, (\cdot)^H$	Transpose, Hermitian transpose
$\odot, \oslash$	Element-wise matrix product, division
$\exp(\underline{A})$	Element-wise exponential of the vector $\underline{A}$
$\underline{A}^b$	Element-wise power of vector $\underline{A}$
$\Vert\underline{A}\Vert$	Euclidean norm of the vector $\underline{A}$
$\Vert\mathbf{A}\Vert$	Norm (largest singular value) of matrix $\mathbf{A}$
$\mathbb{A}^\#$	Pseudo-inverse of the matrix $\mathbb{A}$
$\mathbb{I}_N$	Identity matrix of $N \times N$ dimensions
$\underline{1}_N$	Column vector of N ones
$\mathcal{R}, \mathcal{C}$	Set of real, complex numbers

1. INTRODUCTION

In various signal processing and communications applications, locating transmitting sources enables a myriad location-based services at both application and network level [1–3]. In the most common case, position related measurements are first extracted from the received signals, and then used in a second step for estimating the position of the source node by means of a specific algorithm: fingerprinting, geometric or statistical methods [2]. The first step is also called "Association" phase, and the second one "Metric-Fusion" phase [4].

Recently, a novel large aperture array (LAA) processing technique was proposed in [4, 5] to provide a range-based source localization solution. The LAA procedure is carried in two steps as depicted in Fig. 1. In the association phase, a set

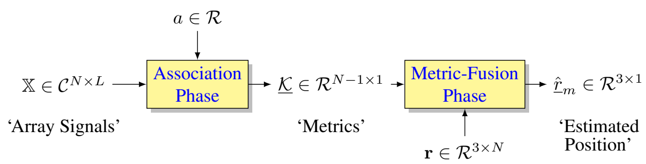

Fig. 1. The LAA localization procedure ($PLE \triangleq a$) from [4].

of metrics is estimated from the impinging signal on the array elements. Those metrics associate the range of the source to the array elements with the signal eigenvalues under both narrow and wide band assumptions. Those metrics are then fed to the metric-fusion process that estimates the source location by using the knowledge of the array geometry $\mathbf{r}$ and the path loss exponent (PLE) denoted as a. In the association phase, the source is assumed to lie in the near-far field of the array, and thus undergoes spherical wave propagation. Under such assumption, the array manifold vector depends on range and direction of the source, in contrast with the far-field region where the manifold vector only depends on the direction of arrival [6,7]. This particularity also led to the development of novel array pilot and self calibration techniques that exploit such form of the array manifold vector [7, 8] where the PLE is assumed to be known.

In this paper, we relax the assumption of the LAA localization on the prior knowledge of the PLE. We propose an array-based technique to automatically estimate the PLE by the array itself. This is achieved by making its elements act as pilot sources in turn, whilst the rest of the array receiving elements form a sub-array. Since the array geometry is known, the PLE can be estimated by inverting the localization procedure in Fig. 1 by estimating the PLE when the source posi-

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tion is known. The advantage of this approach is to use the same formalism for calibration and LAA localization, in contrast with PLE estimation in the framework of received signal strength localization [9, 10].

2. PROBLEM FORMULATION

Consider a fully calibrated large aperture sparse array of N omnidirectional antennas (sensors, transceivers...etc), with a common reference point (zero-phase reference point taken to be the origin of the coordinate system). The array antenna locations are known and defined by the matrix $r \in \mathcal{R}^{3\times N}$ with respect to the system origin. That is

$$\mathbf{r} = \left[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N\right] = \left[\underline{r}_x, \underline{r}_y, \underline{r}_z\right]^T \tag{1}$$

where $\underline{r}_i \in \mathcal{R}^{3\times 1}$ for $i = 1 \ldots N$ is the location of i^{th} antenna in the array and $\underline{r}_x, \underline{r}_y, \underline{r}_z \in \mathcal{R}^{N\times 1}$ denote the x, y and z coordinates of the N antennas. The array aperture is therefore given by

$$D = \max_{\forall i,j} \left\|\underline{r}_i - \underline{r}_j\right\| \tag{2}$$

Assume the array operates in the presence of a single source node located at an unknown position[1] with respect to the array reference point

$$\underline{r}_m = [x, y, z]^T = \rho \cdot \underline{u}(\theta, \phi). \tag{3}$$

The vector $\underline{u}(\theta, \phi) = [\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi]^T$ denotes the unity norm vector pointing in the direction of the source, and ρ the source distance to the origin. The source transmits an unknown narrowband message signal $m(t)$ with a carrier frequency F_c. The message can also be represented as a complex signal, defined as the complex envelope (baseband signal). The wavenumber, also pointing in the direction of the source is then given by

$$\underline{k}(\theta, \phi) = \frac{2\pi F_c}{c} \cdot \underline{u}(\theta, \phi) \tag{4}$$

where c is the speed of light. It is clear from (3) and (4) that $\underline{k}(\theta, \phi)$ is collinear to the source location vector

$$\underline{r}_m = \frac{\rho c}{2\pi F_c} \cdot \underline{k}(\theta, \phi) \tag{5}$$

The source is assumed to lie in the near-far field of the array, which is given by

$$\rho < \frac{2D^2 F_c}{c}. \tag{6}$$

In the case of a single path (LOS condition[2]), the signal $\underline{x}(t) \in \mathcal{C}^{N\times 1}$ received by the array (downconverted to baseband) can be modeled as

$$\underline{x}(t) = \underline{q} \odot \underline{S} m(t) + \underline{n}(t), \tag{7}$$

[1]The azimuth angle θ is measured anticlockwise with respect to the positive x-axis, and the elevation angle ϕ is measured anticlockwise from the x-y plane.

[2]In presence of multipath, the signal LOS component can be extracted from the received signal by prepossessing the signal prior to calibration [11].

where $\underline{q} \in \mathcal{C}^{N\times 1}$ models the unknown channel's slow fading effect. The vector $\underline{S} \triangleq \underline{S}(\theta, \phi, \rho, \mathbf{r}, F_c)$ represents the array manifold vector (array response/steering vector). The vector $\underline{n}(t) \in \mathcal{C}^{N\times 1}$ denotes the baseband additive white Gaussian noise of power σ^2 and covariance matrix (isotropic noise)

$$\mathbb{R}_{nn} = \sigma_n^2 \mathbb{I}_N \in \mathcal{R}^{N\times N}. \tag{8}$$

For a signal arriving from position (θ, ϕ, ρ), the modeling of the array manifold vector in the array's *near-far* region can be expressed as follows.

$$\underline{S} = (\rho \cdot \underline{1}_N \oslash \underline{\rho})^a \odot \exp(-j\frac{2\pi F_c}{c} \cdot (\rho \cdot \underline{1}_N - \underline{\rho})) \tag{9}$$

where the parameter $\underline{\rho}$ denotes the unknown vector of ranges from the source to each of the array elements

$$\underline{\rho} = [\rho_1, \rho_2, \ldots, \rho_N]^T \in \mathcal{R}^{N\times 1}. \tag{10}$$

Consider an observation of L data snapshots received by the $N - elements$ array in the presence of a single source located at position $\underline{r}_m$. The signals received over this interval can be expressed as a matrix in accordance with the model in (7)

$$\mathbb{X} = [\underline{x}(t_1), \underline{x}(t_2), \ldots, \underline{x}(t_L)] \in \mathcal{C}^{N\times L} \tag{11}$$

with the second order statistics, the covariance matrix

$$\mathbb{R}_{xx} = \mathcal{E}\left\{\underline{x}(t)\underline{x}(t)^H\right\} \approx \mathbb{R} = \frac{1}{L}\mathbb{X}\mathbb{X}^H. \tag{12}$$

It is proved in [4] that the signal eigenvalue λ can be estimated from the channel noise power and the principal eigenvalue $\gamma = \max(\text{eig}\,\mathbb{R})$ as

$$\lambda = \gamma - \hat{\sigma}_n^2, \tag{13}$$

where the noise power in the channel $\hat{\sigma}_n^2$, is estimated by the average of the $N - 1$ smallest eigenvalues of the matrix $\mathbb{R}$.

Now consider that the i^{th} sensor of the array is the array reference point, that is all measurements $\mathbb{X}$ are taken with respect to this new reference point. The corresponding model becomes :

- Manifold vector $\underline{S}_i$ with array geometry $\mathbf{r_i} = \mathbf{r} - \underline{r}_i \underline{1}_N^T$ and source position $(\theta_i, \phi_i, \rho_i)$ relative to this new reference point.
- Observation $\mathbb{X}_i$ with the sample covariance matrix $\mathbb{R}_i$ and eigenvalue λ_i.

Without any loss of generality, assume that the primary reference point is placed at the first sensor $\underline{r}_1$. By changing the reference point to be the i^{th} sensor, for all array elements in turn, it is proved in [4] that the following metrics can be constructed to relate the ratio of ranges between the source and the array elements:

$$\underline{\mathcal{K}} = \begin{bmatrix} \frac{\lambda_2}{\lambda_1} \\ \frac{\lambda_3}{\lambda_1} \\ \vdots \\ \frac{\lambda_N}{\lambda_1} \end{bmatrix}^{\frac{1}{2a}} = \begin{bmatrix} \frac{\rho_2}{\rho_1} \\ \frac{\rho_3}{\rho_1} \\ \vdots \\ \frac{\rho_N}{\rho_1} \end{bmatrix}. \tag{14}$$

3. ARRAY ONLINE PATH-LOSS ESTIMATION

The PLE estimation method is presented under narrowband assumption for both single pilot calibration and array self calibration. The same methods remain valid under wideband assumption by using the corresponding association metrics in [4].

3.1. Single Pilot PLE Estimation

Consider a single pilot source, with known ranges to the array elements $\underline{\rho}$, that is transmitting in the near-far field of the array. Under narrowband assumption, in accordance with (14), the following metrics can be constructed

$$\underline{\mathcal{K}} = \left(\underline{\lambda} \oslash \left(\underline{1}_N \cdot \lambda_1\right)\right)^{\frac{1}{2a}} = \underline{\rho} \oslash \left(\underline{1}_N \cdot \rho_1\right) \tag{15}$$

where

$$\underline{\lambda} = \left[\lambda_1, \quad \lambda_2, \dots, \lambda_N\right]^T . \tag{16}$$

Therefore the path loss exponent can be estimated as

$$a = \frac{1}{2} \cdot \frac{\| \log(\underline{\lambda} \oslash (\underline{1}_N \cdot \lambda_1))\|}{\| \log(\underline{\rho} \oslash (\underline{1}_N \cdot \rho_1))\|}. \tag{17}$$

This equation is the key for online path loss estimation using the antenna array itself, by assuming that its elements operate as transceivers.

3.2. Array self PLE Estimation

In order to achieve online path loss estimation, all N array elements transmit in turn to the other $N-1$ elements operating as a receivers thus forming a sub-array. One can build the following matrix of eigenvalues $\mathbf{\Lambda} \in \mathcal{R}^{(N-1)\times(N-1)}$ as,

$$\mathbf{\Lambda} = \begin{array}{c} \begin{array}{cccc} \underline{\lambda}_2 & \underline{\lambda}_3 & \cdots & \underline{\lambda}_N \end{array} \\ \left[\begin{array}{c|c|c|c} \lambda_{12} & \lambda_{13} & \cdots & \lambda_{1N} \\ \lambda_{32} & \lambda_{23} & & \lambda_{2N} \\ \lambda_{42} & \lambda_{43} & & \lambda_{3N} \\ \vdots & & \ddots & \vdots \\ \lambda_{N2} & \lambda_{N3} & \cdots & \lambda_{(N-1)N} \end{array}\right] \end{array} \tag{18}$$

where $\underline{\lambda}_j \in \mathcal{R}^{(N-1)\times 1}$ is the vector of eigenvalues λ_{ij} obtained when the transmitting element is $\underline{r}_j$, and the reference point is rotated on elements $\underline{r}_i$ $(i = 1 \dots N, i \neq j)$. By introducing the matrix $\mathbb{K} \in \mathcal{R}^{(N-1)\times(N-1)}$ such that

$$\mathbb{K} = \left(\mathbf{\Lambda} \oslash \left(\underline{1}_{N-1} \cdot \left(\underline{\delta}_1^T \cdot \mathbf{\Lambda}\right)\right)\right)^{\frac{1}{2a}} \tag{19}$$

where $\underline{\delta}_i \in \mathcal{R}^{(N-1)\times 1}$ denotes a zero vector having its i^{th} element equal to 1. By taking the first antenna element as the primary reference point, in accordance with (14), the matrix $\mathbb{K}$ corresponds to

$$\mathbb{K} = \begin{array}{c} \begin{array}{ccccc} \underline{\mathcal{K}}_2 & \underline{\mathcal{K}}_3 & \underline{\mathcal{K}}_4 & \cdots & \underline{\mathcal{K}}_N \end{array} \\ \left[\begin{array}{c|c|c|c|c} 1 & 1 & 1 & \cdots & 1 \\ \frac{\rho_{23}}{\rho_{21}} & \frac{\rho_{32}}{\rho_{31}} & \frac{\rho_{42}}{\rho_{41}} & \cdots & \frac{\rho_{N2}}{\rho_{N1}} \\ \frac{\rho_{24}}{\rho_{21}} & \frac{\rho_{34}}{\rho_{31}} & \frac{\rho_{43}}{\rho_{41}} & & \frac{\rho_{N3}}{\rho_{N1}} \\ \vdots & & & \ddots & \vdots \\ \frac{\rho_{2N}}{\rho_{21}} & \frac{\rho_{3N}}{\rho_{31}} & \frac{\rho_{4N}}{\rho_{41}} & \cdots & \frac{\rho_{N(N-1)}}{\rho_{N1}} \end{array}\right] \end{array}, \tag{20}$$

where $\rho_{ij} = \|\underline{r}_j - \underline{r}_i\|$. Let us introduce the distance matrix $\boldsymbol{\varrho} \in \mathcal{R}^{(N-1)\times(N-1)}$ defined as

$$\boldsymbol{\varrho} = \left[\underline{\rho}_2, \underline{\rho}_3, \dots, \underline{\rho}_N\right], \tag{21}$$

where $\underline{\rho}_i \in \mathcal{R}^{(N-1)\times 1}$ is defined for $i = 2 \dots N$ as

$$\underline{\rho}_i = \left[\rho_{i1}, \rho_{i2}, \dots, \rho_{i(i-1)}, \rho_{i(i+1)}, \dots, \rho_{iN}\right]^T . \tag{22}$$

Under the above notation, the vector $\underline{\rho}_i$ now corresponds to the range of pilot antenna $\underline{r}_i$ to the rest of the array elements. Equation (20) becomes

$$\mathbb{K} = \boldsymbol{\varrho} \oslash \left(\underline{1}_{N-1} \cdot \left(\underline{\delta}_1^T \cdot \boldsymbol{\varrho}\right)\right) \tag{23}$$

Finally, the path loss exponent can be estimated as

$$a = \frac{1}{2} \cdot \frac{\left\|\log\left(\mathbf{\Lambda} \oslash \left(\underline{1}_{N-1} \cdot \left(\underline{\delta}_1^T \cdot \mathbf{\Lambda}\right)\right)\right)\right\|}{\left\|\log\left(\boldsymbol{\varrho} \oslash \left(\underline{1}_{N-1} \cdot \left(\underline{\delta}_1^T \cdot \boldsymbol{\varrho}\right)\right)\right)\right\|}, \tag{24}$$

where $\| \cdot \|$ is the matrix spectral norm i.e its maximum singular value.

Based on the above discussion, the proposed path loss estimation algorithm can be summarized as follows:

- STEP-1 Take the 1st element $\underline{r}_1$ as the primary array reference point, and make the remaining $N-1$ elements transmit in turn.
- STEP-2 When the j^{th} element transmits, the other $N-1$ remaining elements form a receiving sub-array, and the array reference point is rotated from $\underline{r}_1$ to be at each element of the sub-array.
- STEP-3 When the reference point is at a given array element, the signal eigenvalue is extracted from the received covariance matrix in accordance to (13). By repeating this step, construct the matrix $\mathbf{\Lambda}$ in (18).
- STEP-4 Construct the matrix $\boldsymbol{\varrho}$ from the known array inter-element spacing using (21)
- STEP-5 Estimate the path-loss exponent a using (24).

Note that the above process can also be repeated several rounds and the estimation results can be averaged to refine the PLE estimation. For an extra degree of freedom, the process can also be repeated by changing the primary reference point from $\underline{r}_1$ to other antenna elements in turn.

4. NUMERICAL SIMULATION RESULTS

In this section, the performance of the proposed PLE estimation approach is evaluated versus the SNR and the number of snapshots L. In addition, the LAA localization sensitivity to PLE inaccuracies is evaluated and its performance with PLE being estimated by the proposal. For consistency with reference LAA source localization, we consider a planar array of $N = 4$ elements as follows:

$$\mathbf{r} = \begin{bmatrix} 50 & -40 & 5.05 & 9.45 \\ 0 & 0 & 30 & -20 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

We suppose that the wireless channel has a fading, denoted by the vector $\underline{q}$, with its magnitude modeled by log-normal distribution with mean 0 dB and standard deviation 8 dB and a phase modeled by a uniform distribution over $0 - 2\pi$ radians. For transmission, the source baseband message $m(t)$ is modeled by complex circular normal distribution of zero mean and unity power.

4.1. Performance of Path Loss Estimation

In practice, the observation interval is limited to the number of snapshots L, and the signal covariance matrix is approximated by the sample covariance matrix (in (12)). Thus the estimated signal eigenvalues will cause errors in the estimation of the matrix $\boldsymbol{\Lambda}$, the latter having its elements affected by the noise power (in (13)). Consequently, we investigate the RMS of the PLE estimation over 200 realizations under increasing $SNR \times L$. The PLE true value is set to $a = 2$. Fig. 2 shows that estimation quality is good, and it increases with the SNR and the number of snapshots L. Another interesting view of the estimator performance is shown in In Fig 3, where the distribution of the estimated values is plotted under different SNR and L values. The results show that low SNR leads to higher dispersion around the average value, and vice versa. Finally, the number of snapshots has limited effect on the estimation dispersion, and rather affects the average value. This suggests that, under low SNR, it is more efficient to perform several estimations and average them, whereas under good SNR conditions the PLE estimation can be performed with a single round with a high number of snapshots.

4.2. Sensitivity of LAA Localization

Since the targeted application is LAA localization [4, 5], it is key to study the quality of the PLE estimation by analyzing its overhead on the positioning error. We use the metric-fusion in the section III of [4]. The latter estimates the source position by solving a set of linear equations $\mathbb{H}\underline{r}'_m = \underline{b}$.

By setting the PLE to $a = 3$, 200 independent simulations are performed for different SNR and L values. Fig. 4 shows the reference LAA localization is sensitive to the PLE parameter. Clearly, inaccuracies in the PLE introduce a bias in the

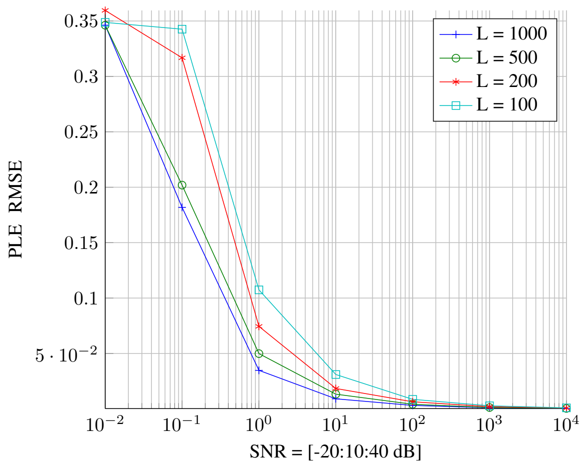

Fig. 2. PLE estimation performance versus SNR over 200 realizations ($PLE = 2$).

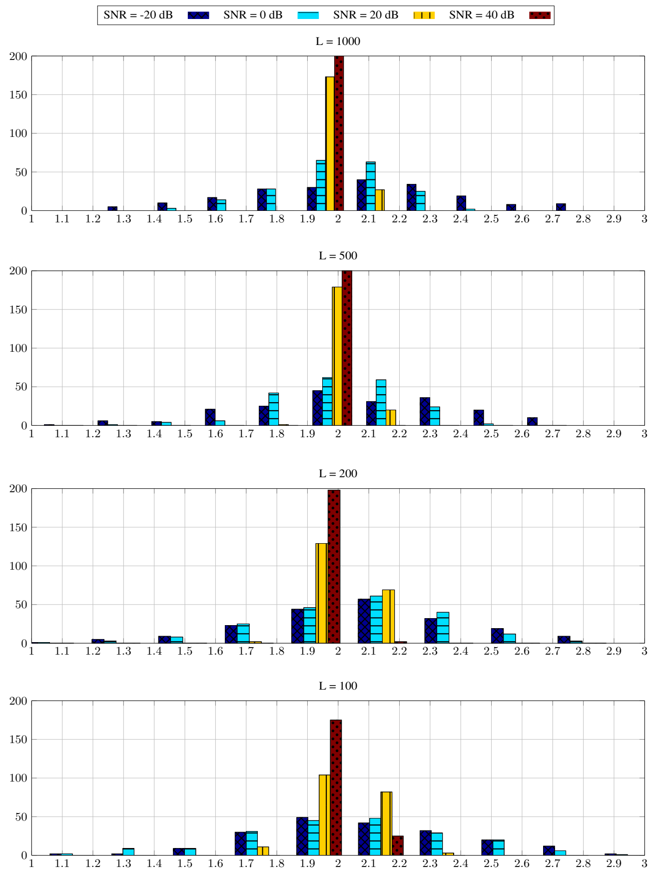

Fig. 3. PLE estimation performance distribution over 200 realizations ($PLE = 2$).

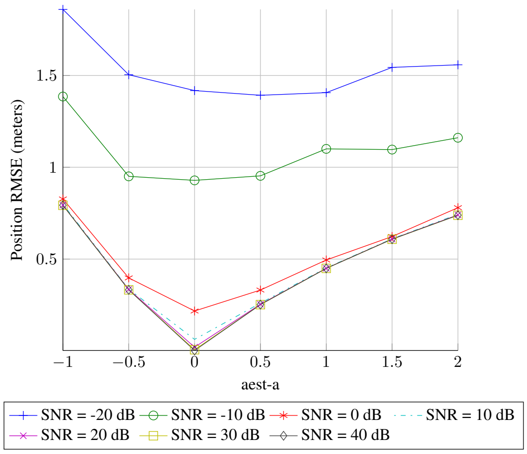

Fig. 4. LAA localization sensitivity over 200 realizations ($PLE = 3, L = 500$ snapshots).

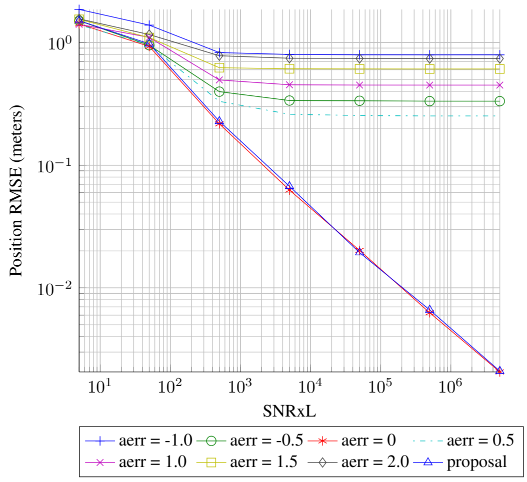

Fig. 5. LAA localization sensitivity and proposal performance over 200 realizations ($PLE = 3, L = 500$ snapshots).

source location estimation. Results in Fig. 5 show that when using the proposed PLE estimation procedure as self calibration step prior to localization, the localization performance is practicality the same as with the true PLE.

5. CONCLUSION

In this paper, an array based method is proposed for estimating the path loss exponent of the array environment. The method can be used in a calibration phase prior to source localization and can be automatically performed by the system itself at any time. Simulation results illustrate the performance of the proposal and its adequacy with the targeted localization process. Future work will concern full array self calibration where the PLE and the array shape are estimated jointly, using a prediction-correction loop that is trained by the measured signals.

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