BAYESIAN TRACK-BEFORE-DETECT FOR CLOSELY SPACED TARGETS

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ABSTRACT

Track-Before-Detect (TBD) is an effective approach to multi-target tracking problems with low signal-to-noise (SNR) ratio. In this paper we propose a novel Labeled Random Finite Set (RFS) solution to the multi-target TBD problem for a generic pixel based measurement model. In particular, we discuss the applicability of the Generalized Labeled Multi-Bernoulli (GLMB) distribution to the TBD problem for low SNR and closely spaced targets. In such case, the commonly used separable targets assumption does not hold and a more sophisticated algorithm is required. The proposed GLMB recursion is effective in the sense that it matches the cardinality distribution and Probability Hypothesis Density (PHD) function of the true joint posterior density. The approach is validated through simulation results in challenging scenarios.

1. INTRODUCTION

Multi-target tracking is often performed using data that have been preprocessed into point measurements or detections [1]. Popular algorithms in multi-target tracking using detections include the Probability Hypothesis Density (PHD) filter [2], Cardinalized PHD (CPHD) filter [3], multi-Bernoulli filter [4], the Multiple Hypotheses Tracking (MHT) algorithm [5], the Joint Probabilistic Data Association (JPDA) [6], and the recently introduced Generalized Labeled Multi-Bernoulli (GLMB) and Labeled Multi-Bernoulli (LMB) trackers [7, 8]. Applications of these multi-target tracking solutions are discussed in [5, 9, 10] for radar/sonar measurements, in [11] for computer vision surveillance, in [12] for autonomous vehicle control, and in [13] for automotive safety problems.

In multi-target tracking problems with low signal-to-noise (SNR) ratio, preprocessing the collected data might lead to poor tracking performance. In this case, an effective solution is to make use of all the information contained in the data before the detection step. This approach is known as track-before-detect (TBD) and was first investigated in [14–16]. Since then a number of TBD techniques have been proposed for various applications [17–22]. The multi-Bernoulli filter proposed in [20] is a Bayes optimal approach to multi-target filtering using image data under separable assumption, which has been successfully demonstrated in TBD using radar images [23, 24], and visual tracking [11, 25]. However, this approach is not a multi-target tracker because it rests on the premise that targets are indistinguishable [7].

In this paper, we present a novel solution to the multi-target TBD tracking problem based on Labeled Random Finite Sets (RFSs) [7, 8]. Compared to the approach in [20], using Labeled RFSs enables sequential estimation of target trajectories. The proposed approach is based on a principled GLMB approximation of multi-target densities which captures target dependencies [26] and does not require the separable likelihood assumption commonly used in TBD problems [19, 20, 23–25]. This allows the use of a more general/precise measurement model which correctly describes the sensor measurements in scenarios with closely spaced targets. In this work, we consider a generic pixel-based measurement model of interest for TBD applications in ground-tracking [23]. This model has also been used in forward-backward smoothing [24] and computer vision [11, 25]. Simulation results for low SNR and closely spaced targets confirm the applicability of the proposed approach.

The paper is organized as follows: in Section II we briefly recall the Bayesian multi-target tracking problem using Labeled Random Finite Sets (RFS); in Section III we review the pixel based measurement model for TBD and detail the GLMB recursion; simulation results are discussed in Section IV, while our conclusions and future research directions are collected in Section V.

2. BAYESIAN TRACKING AND LABELING

Suppose that at time k, there are N_k objects with their states denoted by $x_{k,1}, \ldots, x_{k,N_k}$, each taking values in a state space $\mathcal{X}$. An RFS is a random variable $X_k = \{x_{k,1}, \ldots, x_{k,N_k}\}$ that takes values in $\mathcal{F}(\mathcal{X})$, the space of all finite subsets of $\mathcal{X}$. Mahler's Finite Set Statistics (FISST) provides powerful mathematical tools for dealing with RFSs [1, 27]. In this work we are interested in the multi-object filtering density, which can be propagated in time by

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the *multi-object Bayes filter* as detailed in [1].

2.1. Labeled RFS

The labeled RFS model incorporates a unique label in the object's state vector to identify its trajectory [1]. The single-object state space $\mathcal{X}$ is a Cartesian product $\mathbb{X}\times\mathbb{L}$, where $\mathbb{X}$ is the feature/kinematic space and $\mathbb{L}$ is the (discrete) label space. A finite subset $\mathbf{X}$ of $\mathbb{X}\times\mathbb{L}$ has distinct labels if and only if $\mathbf{X}$ and its labels $\{\ell : (x,\ell) \in \mathbf{X}\}$ have the same cardinality [7]. A recently developed class of labeled RFS is the generalized labeled multi-Bernoulli (GLMB) family [7,8]. Under the standard multi-object model, the GLMB is a conjugate prior that is also closed under the Chapman-Kolmogorov equation. Let $\mathcal{L} : \mathbb{X}\times\mathbb{L} \to \mathbb{L}$ be the projection $\mathcal{L}((x,\ell)) = \ell$, and $\Delta(\mathbf{X}) \triangleq \delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|)$ denote the *distinct label indicator*. A GLMB is a RFS on $\mathbb{X}\times\mathbb{L}$ distributed according to [7,8]:

$$\boldsymbol{\pi}(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{c\in\mathbb{C}} w^{(c)}(\mathcal{L}(\mathbf{X})) \left[p^{(c)}\right]^{\mathbf{X}}, \qquad (2.1)$$

where $\mathbb{C}$ is a discrete index set, $\left[p^{(c)}\right]^{\mathbf{X}}$ is a density on $\mathbb{X}$, and $w^{(c)}$ is such that $\sum_{L\subseteq\mathbb{L}} \sum_{c\in\mathbb{C}} w^{(c)}(L) = 1$. The GLMB distribution is a mixture of multi-object exponentials, i.e. it uses single-target densities to build a multi-target density with statistical dependence between the objects and a general cardinality distribution. The cardinality distribution and PHD of a GLMB are given by [7]

$$\rho(n) = \sum_{c\in\mathbb{C}} \sum_{L\subseteq\mathbb{L}} \delta_n(|L|) w^{(c)}(L), \qquad (2.2)$$

$$v(x,\ell) = \sum_{c\in\mathbb{C}} p^{(c)}(x,\ell) \sum_{L\subseteq\mathbb{L}} 1_L(\ell) w^{(c)}(L). \qquad (2.3)$$

More details GLMB densities can be found in [7,8].

2.2. Prediction

To ensure distinct labels we assign each target an ordered pair of integers $\ell = (k,i)$, where k is the time of birth and i is a unique index to distinguish targets born at the same time. The label space for targets born at time $k+1$ is denoted as $\mathbb{L}_{k+1}$, and the label space for targets at time $k+1$ (including those born prior to $k+1$), denoted as $\mathbb{L}_{0:k+1}$, is constructed recursively by $\mathbb{L}_{0:k+1} = \mathbb{L}_{0:k} \cup \mathbb{L}_{k+1}$ [7, 8]. The set of newborn objects is distributed according to $\mathbf{f}_B(\mathbf{Y}) = \Delta(\mathbf{Y}) w_B(\mathcal{L}(\mathbf{Y})) \left[p_B\right]^{\mathbf{Y}}$, where the birth density $\mathbf{f}_B$ is defined on $\mathbb{X}\times\mathbb{L}_{k+1}$. The multi-object state at the next time $\mathbf{X}$ is the superposition of surviving objects and new born objects and the multi-object transition kernel is given by [7]:

$$\mathbf{f}_{k+1|k}(\mathbf{X}|\mathbf{X}') = \mathbf{f}_S(\mathbf{X}\cap(\mathbb{X}\times\mathbb{L}_{0:k})|\mathbf{X}')\mathbf{f}_B(\mathbf{X}-(\mathbb{X}\times\mathbb{L}_{0:k})).$$

The predicted multi-object density is a GLMB given by [7]

$$\boldsymbol{\pi}_{k+1|k}(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{I\in\mathcal{F}(\mathbb{L}_{0:k+1})} w_{k+1|k}^{(I)}(\mathcal{L}(\mathbf{X})) \left[p_{k+1|k}^{(I)}\right]^{\mathbf{X}}, \qquad (2.4)$$

where,

$$w_{k+1|k}^{(I)} = w_S^{(I)}(I\cap\mathbb{L}_{0:k}) w_B(I\cap\mathbb{L}_{k+1}), \qquad (2.5)$$

$$w_S^{(I)}(L) = [\eta_S^{(I)}]^L \sum_{J\subseteq\mathbb{L}_{0:k}} 1_J(L)[1-\eta_S^{(I)}]^{J-L} w_k^{(I)}(J), \qquad (2.6)$$

$$p_{k+1|k}^{(I)}(x,\ell) = 1_{\mathbb{L}_{0:k}}(\ell) p_S^{(I)}(x,\ell) + (1-1_{\mathbb{L}_{0:k}}(\ell)) p_B(x,\ell), \qquad (2.7)$$

$$p_S^{(I)}(x,\ell) = \frac{\left\langle p_S(\cdot,\ell) f_{k+1|k}(x|\cdot,\ell), p_k^{(I)}(\cdot,\ell)\right\rangle}{\eta_S^{(I)}(\ell)}, \qquad (2.8)$$

$$\eta_S^{(I)}(\ell) = \left\langle p_S(\cdot,\ell), p_k^{(I)}(\cdot,\ell)\right\rangle. \qquad (2.9)$$

Eqs. (2.4)-(2.9) explicitly describes how to calculate the parameters of the predicted multi-object density from the parameters of the previous multi-object density [8].

2.3. Separable Likelihood Update

In multi-target TBD problems the measurement collected at time k usually consists of an array of cells with a scalar intensity, i.e. $z_k = [z_{1,k}, \dots, z_{C,k}]$ where C is the number of cells. A common assumption in TBD solutions [19, 20, 23–25] is that the templates $T(\cdot)$ of single targets do not overlap, i.e. $T(\mathbf{x}_{i,k}) \cap T(\mathbf{x}_{j,k}) = \emptyset$ for $i \neq j$, where $T(\mathbf{x}_{i,k})$ is the set of cells illuminated by the target i. A separable multi-target likelihood function $g^{\text{sep}}(z_k|\mathbf{X})$ is such that:

$$g^{\text{sep}}(z_k|\mathbf{X}) \quad \propto \quad \prod_{\mathbf{x}\in\mathbf{X}} \gamma_z(\mathbf{x}). \qquad (2.10)$$

If the multi-object prediction density $\boldsymbol{\pi}_{k|k-1}$ is a GLMB and the multi-target likelihood is separable, then the multi-target posterior $\boldsymbol{\pi}_{k|k}$ is a GLMB [26]. The separable likelihood assumption/approximation leads to closed form equations which in turns yields an efficient implementation of the GLMB recursion. However, the obtained GLMB tracker can only guarantee satisfactory tracking performance in multi-target scenarios with non-overlapping target templates.

3. GLMB UPDATE FOR TBD OF CLOSE TARGETS

In this section we describe the proposed GLMB update step that uses a general multi-target likelihood function $g(z_k|\mathbf{X})$ to describe the collected measurement at time k. In applications involving multiple closely-spaced targets, each pixel intensity in the image observation can be affected by more than one target. Thus, the likelihood is not separable. The basic idea of our approach is to: (*1*) perform Bayes update using the general likelihood to obtain a joint posterior density; (*2*) approximate the posterior density using the GLMB form; (*3*) proceed to the next time step using the GLMB prediction.

3.1. TBD Measurement Model

At each time step k, the observation consists of an array of cells with a scalar intensity, i.e. $z_k = [z_{1,k}, \dots, z_{C,k}]$. The measurement $z_{i,k}$ in cell (i) is given by:

$$z_{i,k} = h_{i,k}(\mathbf{X}_k) + n_i \tag{3.1}$$

$$h_{i,k}(\mathbf{X}_k) = \sum_{\mathbf{x}_k \in \mathbf{X}_k : i \in T(\mathbf{x}_k)} \frac{\Delta_x \Delta_y I_k}{2\pi\sigma_h^2} \tilde{h}_{i,k}(\mathbf{x}_k) \tag{3.2}$$

$$\tilde{h}_{i,k}(\mathbf{x}_k) = \exp\left(-\frac{(\Delta_x a_i - p_{x,k})^2 + (\Delta_y b_i - p_{y,k})^2}{2\sigma_h^2}\right) \tag{3.3}$$

where $h_{i,k}(\mathbf{X}_k)$ is the target-generated intensity in cell (i); I_k is the known source intensity, (Δ_x, Δ_y) are the cell side lengths in (x, y) coordinates; (a_i, b_i) are discrete indices; σ_h^2 is a blurring coefficient; and $n_i \sim \mathcal{N}(0, \sigma^2)$ is zero-mean white Gaussian noise. The sum over targets in eq. (3.2) models the fact that the intensity of each pixel can be affected by the contribution of multiple closely-spaced targets. The multi-object likelihood function $g(z_k|\mathbf{X}_k)$ is given by:

$$g(z_k|\mathbf{X}_k) = \prod_{i=1}^{C} g(z_{i,k}|\mathbf{X}_k) \tag{3.4}$$

$$g(z_{i,k}|\mathbf{X}_k) = \begin{cases} \mathcal{N}(z_{i,k}; 0, \sigma^2), & \mathbf{X}_k = \{\emptyset\} \\ \mathcal{N}(z_{i,k}; h_{i,k}(\mathbf{X}_k), \sigma^2), & \text{otherwise} \end{cases} \tag{3.5}$$

where the product in eq. (3.4) comes from the assumption that the intensities $z_{i,k}$ are conditionally independent given the multi-target state $\mathbf{X}_k$. Notice that in practice we implement the tracker using the likelihood ratio $\ell(z_{i,k}|\mathbf{X}_k) = g(z_{i,k}|\mathbf{X}_k)/g(z_{i,k}|\emptyset)$ so that the product in (3.4) is taken over the cells in the union of the target templates, i.e.

$$\ell(z_k|\mathbf{X}_k) = \prod_{i \in T(\mathbf{X}_k)} \frac{g(z_{i,k}|\mathbf{X}_k)}{g(z_{i,k}|\emptyset)} \tag{3.6}$$

$$T(\mathbf{X}_k) = T(\mathbf{x}_{1,k}) \cup T(\mathbf{x}_{2,k}) \cup \ldots \cup T(\mathbf{x}_{|\mathbf{X}_k|,k})$$

3.2. Non-Separable GLMB Update

We now detail the GLMB update step for a likelihood of the form (3.2)-(3.5). Given the density $\boldsymbol{\pi}_{k|k-1}$ in GLMB form and a non-separable likelihood $g(z_k|\mathbf{X})$, the Bayes update step [1] yields the following joint posterior:

$$\boldsymbol{\pi}_k(\mathbf{X}|z_k) = \Delta(\mathbf{X}) \sum_{I \in \mathcal{F}(\mathbb{L}_{0:k})} \delta_I(\mathcal{L}(\mathbf{X})) w_k^{(I)}(z_k) p_k^{(I)}(\mathbf{X}|z_k), \tag{3.7}$$

where

$$p_k^{(I)}(\mathbf{X}|z) = g(z_k|\mathbf{X}) \left[p_{k|k-1}^{(I)}\right]^{\mathbf{X}} / \eta_{z_k}(I), \tag{3.8}$$

$$w_k^{(I)}(z_k) \propto \eta_{z_k}(I) w_{k|k-1}^{(I)}, \tag{3.9}$$

$$\eta_{z_k}(\{\ell_1, \ldots, \ell_n\}) = \int g(z_k|\{(x_1, \ell_1), \ldots, (x_n, \ell_n)\}) \times \prod_{i=1}^{n} p_{k|k-1}^{\{\ell_1, \ldots, \ell_n\}}(x_i, \ell_i) d(x_1, \ldots, x_n). \tag{3.10}$$

From eq. (3.8), we see that each component $\left[p_{k|k-1}^{(I)}\right]^{\mathbf{X}}$ from the prior GLMB is updated into a joint density $p^{(I)}(\mathbf{X}|z_k)$ and not into a multi-object exponential. From [26, Prop. 5], the approximate GLMB $\hat{\boldsymbol{\pi}}_k(\mathbf{X}|z_k)$ is given by:

$$\hat{\boldsymbol{\pi}}_k(\mathbf{X}|z_k) = \Delta(\mathbf{X}) \sum_{I \in \mathcal{F}(\mathbb{L}_{0:k})} \delta_I(\mathcal{L}(\mathbf{X})) \hat{w}_k^{(I)} \left[\hat{p}_k^{(I)}(\cdot|z_k)\right]^{\mathbf{X}}, \tag{3.11}$$

$$\hat{p}_k^{(\{\ell, \ell_1, \ldots, \ell_n\})}((x, \ell)|z_k) = \tag{3.12}$$

$$\int p_k^{(\{\ell, \ell_1, \ldots, \ell_n\})}(\{(x, \ell), (x_1, \ell_1), \ldots, (x_n, \ell_n)\}|z_k) d(x_1, \ldots, x_n),$$

$$\hat{w}_k^{(I)} = w_k^{(I)}. \tag{3.13}$$

Notice that we only approximate the spatial distribution in each component of the GLMB and retain the weighting terms $w_k^{(I)}(z_k)$ from the joint update. In particular, retaining the weights $w_k^{(I)}(z_k)$ in eq. (3.13) leads to matching the cardinality distribution of the posterior density in (3.7) [26]. Furthermore, the approximation in (3.12) amounts to representing each track $\ell \in I$ by the marginal density $p^{(I)}((x, \ell))$, which is obtained performing a standard marginalization of the joint density $p^{(I)}(\{(x, \ell), (x_1, \ell_1), \ldots, (x_n, \ell_n)\})$. Additional details and derivations of eqs.(3.11)-(3.13) can be found in [26]. In practice we use the likelihood ratio (3.6) in place of the multi-object likelihood in eqs. (3.8)- (3.10), and approximate the spatial densities using particles [8].

4. SIMULATION RESULTS

In this section we report the simulation results using the newly proposed GLMB update step for non-separable likelihood function in TBD problems. Synthetic measurements are generated using eqs. (3.1)-(3.3) and the new algorithm is tested against the Cardinality Balanced MeMBer (CB-MeMBer) filter presented in [20] and the GLMB tracker using the separable likelihood approximation. Notice that the expressions for the separable likelihood function are obtained from eq. (3.2) by assuming that the single target templates do not overlap, i.e. $T(\mathbf{x}_{i,k}) \cap T(\mathbf{x}_{j,k}) = \emptyset$ for $i \neq j$, so that the sum in (3.2) has at most one term.

Parameter	Symbol	Value
Cell Side Length (x)	(Δ_x, Δ_y)	$(1, 1)$
Blurring Coefficient	σ_h	1
Sampling Time	T_s	1 sec
Birth Uncertainty	P_0	diag $([1, 1, 1, 1])$
Number of Particles	N_p	10000

Table 1. Parameters used in simulation.

The kinematic part of the single-target labeled state vector $\mathbf{x}_k = (x_k, \ell_k)$ at time k comprises the planar position and velocity vectors $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]^T$ in 2D Cartesian coordinates. The single-target motion model is given by,

$$x_{k+1} = F\, x_k + v_k, \;\; v_k \sim \mathcal{N}(0; Q)$$

where v_k is a zero-mean Gaussian noise, T_s is the radar sampling time, and the matrices F and Q define the Nearly Constant Velocity (NCV) model [6]. We consider the scenario depicted in Fig. 4.1 where a maximum of 4 targets enter/leave the surveillance area at different time instants. The scenario parameters are reported in Tab. 1, and a snapshot of the TBD measurement and true targets positions is depicted in Fig. 4.2 for the time instant $k = 16$.

Simulation results are reported in Fig. 4.3 in terms of the Optimal Sub-Pattern Assignment (OSPA) [28] distance and estimated number of targets. From the results we verify that the proposed approach can solve the multi-target tracking problem with satisfactory performance in relatively low SNR and closely spaced targets. Notice that when target are closely spaced in the measurement space, i.e. the targets templates overlap, the CB-MeMBer and GLMB with separable likelihood filters will generally lose targets due to merging of tracks. Removing the merging procedure could solve this problem, but it would also lead to an overestimated number of targets. In fact, the merging procedure is needed since the separable likelihood model is not consistent with the birth model. Specifically, if the SNR is low and/or the target speed is not high compared to the sensor sampling time, the templates of two target tracks from the same birth location at successive time instant will always overlap. In this case the separable likelihood lead to double counting of the measurement, thus leading to an overestimated number of targets. The proposed approximate GLMB recursion solves these problems since by allowing the use of a non-separable likelihood, it avoids overestimating the number of targets at birth, and prevents underestimating closely spaced targets since it does not require a merging procedure.

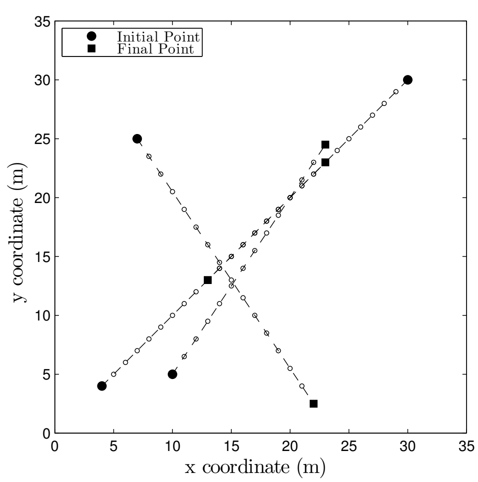

Fig. 4.1. Scenario with closely spaced targets and SNR=4 dB.

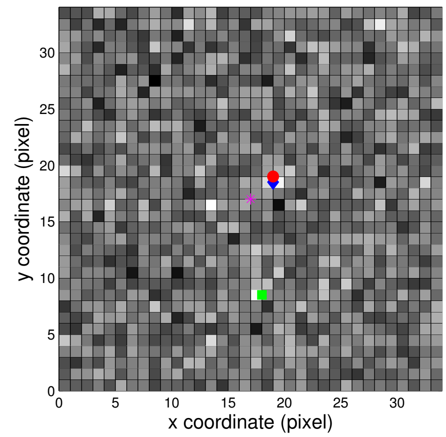

Fig. 4.2. Snapshot of the TBD measurement and true targets positions at time instant $k = 16$.

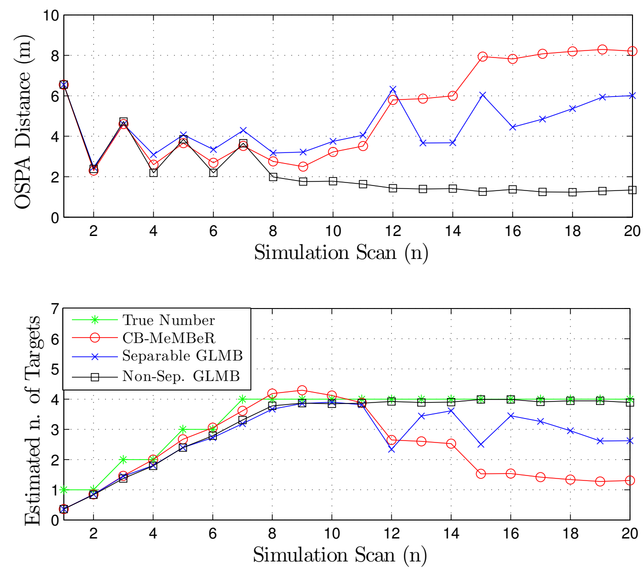

Fig. 4.3. OSPA distance (top) for the position estimates and (bottom) Estimated number of targets.

5. CONCLUSIONS

In this paper, a novel approach to multi-target TBD with closely spaced targets was presented. The solution is based on GLMB distribution which is a principled approximation to a general joint distribution with target dependencies. The approach is of great interest in TBD problems with closely spaced targets where the sensor returns cannot be correctly described using a separable likelihood function. Simulation results in challenging scenarios were presented to validate the applicability of the proposed approach.

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