MARKED POISSON POINT PROCESS PHD FILTER FOR DOA TRACKING

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ABSTRACT

In this paper we propose a Track Before Detect (TBD) filter for Direction Of Arrival (DOA) tracking of multiple targets from phased-array observations. The phased-array model poses a new problem since each target emits a signal, called source signal. Existing methods consider the source signal as part of the system state. This is inefficient, especially for particle approximations of posteriors, where samples are drawn from the higher-dimensional posterior of the extended state. To address this problem, we propose a novel Marked Poisson Point Process (MPPP) model and derive the Probability Hypothesis Density (PHD) filter that adaptively estimates target DOAs. The PPP models variations of both the number and the location of points representing targets. The mark of a point represents the source signal, without the need of an extended state. Recursive formulas for the MPPP PHD filter are derived with simulations showcasing improved performance over state-of-the art methods.

Index Terms— DOA tracking, marked Poisson point process, PHD filter, track before detect, DBSCAN.

1. INTRODUCTION AND RELATED WORK

Modern radar and sonar systems employ antenna and hydrophone arrays in order to detect, localize and track various targets. Target localization is achieved by estimating the Direction Of Arrival (DOA) of the source signal. DOA tracking is traditionally achieved through a two stage process referred to as Track While Scan (TWS). Firstly, targets are detected and a set of DOA estimates is formed. The resulting DOA estimates suffer from origin uncertainty, that is, we do not know from which target the DOA estimate originates. Secondly, filtering the DOA estimates with the kinematic target-model and resolving the origin uncertainty problem is dealt with probabilistic data association filters [1]. In [2, 3] we proposed a TWS system for a sonar image reconstruction application.

Track Before Detect (TBD) filters, are aimed at tracking targets directly from the antenna signal, i.e. without performing any pre-detection or estimation procedures. Furthermore, the MPPP model that we employ is capable of taking into account target birth and death, i.e. a variable number of targets. The MPPP is taken to be simple, hence a random finite set formalism is also possible with the PHD as the first moment density [4, Sec. IV.C]. Derivation of the PHD filter within the point process framework is considered in [5]. The PHD filter of [4], with subsequent implementations given in [6], is derived for TWS systems and is not directly applicable for TBD. A PHD-TBD filter for amplitude sensors was derived in [7]. However, in array processing, the DOA information is contained in the phase differences between the signals received by the sensor array and, in general, we are not directly interested in signal amplitudes. Furthermore, array observations result from the superposition of individual source contributions. Unlike [7], the individual source amplitudes are unknown and modeled as randomly fluctuating complex signals. Thus, [7] cannot be directly used for the standard DOA tracking problem, and further developments of the PHD must be addressed to tackle this problem. Augmenting the state of each target with the source signal leads to inefficient particle filter implementations, which approximate the higher-dimensional posterior corresponding to the augmented state. In [8] an augmented state is considered, while target births and deaths are resolved by a reversible jump MCMC algorithm. In [9] a multi-Bernoulli filter for DOA tracking is proposed, by using the MUSIC [10, Ch. 4.5] pseudo-spectrum as the likelihood function. A sparse DOA estimator is proposed in [11], by minimizing a functional defined on a discrete space and under sparse constants. Given the sequential nature of the estimator, DOA tracking of kinematic targets is possible.

The novelty of our approach is the proposal of an MPPP model and the derivation of the first-order moment measure density filter (PHD) for DOA tracking, effectively extending in the case of MPPP the aforementioned multi-target TBD filters. A PPP (denoted as ground PPP) is employed to describe both the target number and target locations in some state space. Each target (or point) generates a source signal, modeled as a stochastic process and representing the mark of the point. The process containing the ground PPP and the target marks is the Marked PPP (MPPP). The PHD of the ground PPP is shown to be a sufficient statistic for target state inference, with the marks being analytically integrated in the

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filter update procedure. Therefore, the proposed filter propagates a particle approximation for the PHD of the ground PPP, instead of a density function defined on the target and mark spaces. The point-mark relationship is naturally modeled by the MPPP, which would be otherwise lost in an extended state formalism. The ability of the proposed filter to yield one estimated set of target DOAs per array observation is necessary for high-resolution image formation algorithms, like sonar bathymetry [12, 13, 14]. In contrast, most TWS methods process several array observations to produce one set of target DOA estimates, and hence, lead to a reduced resolution effect in imaging applications.

The paper is organized as follows: section 2 presents the array signal model and the MPPP state formalism. In section 3 we derive the filtering equations. Results on simulated array data are given in section 4 and we conclude in section 5.

2. ARRAY SIGNAL AND MPPP FORMALISM

We assume a simple and finite PPP $X_t \equiv \{\mathbf{x}_t^1, \cdots, \mathbf{x}_t^{N_t}\}$ at time t that characterizes both the number of points/targets N_t and their locations $\mathbf{x}_t$ as random. N_t is a Poisson random variable and conditional to the number of points, the elements of X_t are independent and identically distributed in the d dimensional state-space $\mathcal{R} \subset \mathbb{R}^d$. The probability distribution of X_t can be specified by the distribution of the number of targets N_t and the joint distribution of the target positions, conditional on the total number of targets N_t. Thus X_t has an associated density [5, eq. 2.4]:

$$p(X_t) = \mathrm{e}^{-\int_{\mathcal{R}} D_t(\mathbf{x})d\mathbf{x}} \prod_{\mathbf{x}\in X_t} D_t(\mathbf{x}), \tag{1}$$

where D_t represents the intensity function defined on the state-space $\mathcal{R}$. For PPP, the first-order moment measure density, that is the PHD coincides with the intensity [4, Prop. 5]. Targets are presented as peaks of D_t and thus target locations can be inferred from D_t. Next we consider that each point $\mathbf{x}_t$ generates a mark s_t in a mark space $\mathcal{K} \subset \mathbb{C}$. The point process $\{(\mathbf{x}_t^i, s_t^i)\}$ on $\mathcal{R} \times \mathcal{K}$ is Poisson and represents the MPPP $\tilde{X}_t$. With X_t usually called ground PPP [15, Ch. 6.4]. In this paper, marks represent the source signals generated by the targets, and are taken to be centered and circular Gaussian random variables with power P: $s_t \sim \mathcal{N}(s_t; 0, P)$. The MPPP $\tilde{X}_t$ is a PPP with points $\tilde{\mathbf{x}} = (\mathbf{x}, s)$ and intensity function $\tilde{D}_t(\tilde{\mathbf{x}}) = D_t(\mathbf{x})\mathcal{N}(s; 0, P)$, as given by the Marking Theorem [16, Prop. 3.9] and [17, p. 55].

We assume an array signal $\mathbf{y}_t \in \mathbb{C}^{M\times 1}$ received at time t by an $M-$element array as the random sum

$$\mathbf{y}_t = \sum_{\tilde{\mathbf{x}}\in\tilde{X}_t} \mathbf{g}(\tilde{\mathbf{x}}) + \mathbf{n}_t, \tag{2}$$

where $\mathbf{g}(\tilde{\mathbf{x}}_t) = \mathbf{a}(\mathbf{x}_t)s_t$ and $\mathbf{a}$ is the array manifold vector. The additive noise $\mathbf{n}_t \in \mathbb{C}^{M\times 1}$ is circular Gaussian with $\mathbf{n}_t \sim \mathcal{N}(\mathbf{n}_t; 0, \sigma_t^2\,\mathbf{I}_M)$. The Signal to Noise Ratio (SNR) is given by $SNR = \frac{P}{\sigma_t^2}$.

Generally, target tracking is conducted in the cartesian coordinate system, with several kinematic models such as the nearly constant velocity or the Wiener acceleration model [18]. For distant targets, tracking in polar coordinates may be adequate since pseudo-acceleration is small [19, Ch. 1.5]. In this paper, we consider a simple first order kinematic model, i.e. a nearly constant angular velocity model:

$$\mathbf{x}_t = F_t\mathbf{x}_{t-1} + \mathbf{v}_t, \tag{3}$$

where $\mathbf{x}_t = [\theta_t, \dot{\theta}_t]^T \in \mathcal{R}$ presents the single target state vector. $\mathbf{v}_t \sim \mathcal{N}(\mathbf{v}_t; 0, Q)$ represents the white model noise. F_t is a transition matrix specific for a constant velocity model [18, Ch. 6.3.1].

Assuming a uniform linear array and far-field narrowband sources, the array manifold vector $\mathbf{a}(\mathbf{x}_t)$ is defined as:

$$\mathbf{a}(\mathbf{x}_t) \triangleq \begin{bmatrix}1 & e^{-j\,k\Delta\sin(\theta_t)} & \dots & e^{-j\,k(M-1)\Delta\sin(\theta_t)}\end{bmatrix}^T,$$

where $\{\cdot\}^T$ represents the transpose operator. k represents the wave number and Δ the inter-receiver spacing.

The proposed filter adaptively detects targets and estimates target states $\mathbf{x}_t$ from the MPPP posterior $p_{t|t}(\tilde{X}) \triangleq p(\tilde{X}_t|\mathbf{y}_{0:t})$, given the sequence $\mathbf{y}_{0:t}$ of past and current array observations. To this goal, the filter recursively propagates of the PHD $D_{t|t}(\mathbf{x})$ of the ground PPP, while the PHD of the MPPP intervenes in the observation likelihood function. Hence, by propagating the PHD $D_{t|t}(\mathbf{x})$ on the state-space $\mathcal{R}$, we avoid the propagation of an equivalent PHD function on the augmented space $\mathcal{R} \times \mathcal{K}$ [8]. In the following section we derive approximate propagation formulas for $D_{t|t}(\mathbf{x})$.

3. APPROXIMATE MPPP PHD FILTER

The filter comprises two stages: prediction (sec. 3.1) and update (sec. 3.2). Target kinematics, as given by eq. (3), in conjunction with target birth/death processes are employed to derive an exact formula for the predictive PHD $D_{t+1|t}(\mathbf{x})$. The update stage corrects the predicted intensity with the current observation $\mathbf{y}_{t+1}$. Since closed-form formulas are not available for the updated intensity, an approximation similar to [7] is proposed.

3.1. MPPP PHD Prediction

The prediction starts with the PHD $D_{t|t}$ corresponding to the posterior density $p_{t|t}(X)$ of the ground PPP, eq. (1). The PPP ground process, describing target locations at time t, evolves due to target birth/death and the kinematics of surviving targets. The resulting process is shown to be a PPP, with the PHD function $D_{t+1|t}$ being sufficient for target state inference.

Target death (extinction) is modeled by a process of independent thinning, i.e. targets survive independently of each other with probability $p_s(\mathbf{x})$. The resulting process is a PPP with intensity and PHD function $p_s(\mathbf{x})D_{t|t}(\mathbf{x})$ [16, Prop. 3.7]. Targets that survive the thinning process undergo a kinematic transformation modeled by $f_{t+1|t}(\mathbf{x}|\xi) = \mathcal{N}(\mathbf{x}; F_{t+1}\xi, Q_t)$ of eq. (3). The resulting process is again a PPP [20, Ch. 2.11.1]. Target birth is accounted for by superposing an independent PPP with intensity γ_{t+1} to the transformed PPP. From the superposition theorem [17, Ch. 2.2], the predicted PHD is given by

$$D_{t+1|t}(\mathbf{x}) = \gamma_{t+1}(\mathbf{x}) + \int_{\mathcal{R}} f_{t+1|t}(\mathbf{x}|\xi)\, p_s(\xi)\, D_{t|t}(\xi) d\xi. \quad (4)$$

The associated marked PPP has PHD (or intensity) function:

$$\tilde{D}_{t+1|t}(\tilde{\mathbf{x}}) = D_{t+1|t}(\mathbf{x})\mathcal{N}(s; 0, P) \quad (5)$$

The propagation of the PHD function $D_{t|t}(\mathbf{x})$ is sufficient for inferring the target states, while the MPPP PHD occurs in the array likelihood model. Hence, an efficient particle implementation propagating only the PHD $D_{t|t}(\mathbf{x})$ is possible.

3.2. MPPP PHD Update

By definition [15, Lemma 5.4.III], $D_{t+1|t+1}$ is given by

$$D_{t+1|t+1}(\mathbf{x}) = \sum_{n=0}^{\infty} \frac{1}{n!} \int \cdots \int p_{t+1|t+1}(\{\mathbf{x}, \mathbf{w}_1, \cdots, \mathbf{w}_n\}) d\mathbf{w}_1 \cdots d\mathbf{w}_n.$$

Or more compactly, by using the set integral representation [4, eq. 21]

$$D_{t+1|t+1}(\mathbf{x}) = \int p_{t+1|t+1}(\{\mathbf{x}\} \cup W) \delta W \quad (6)$$

where we denote $W = \{\mathbf{w}_1, \cdots, \mathbf{w}_n\}$. Applying the Bayes rule for the posterior $p_{t+1|t+1}(W)$, and considering a Poisson expression (eq. (1)) for $p_{t+1|t}(W)$ we obtain [7]:

$$D_{t+1|t+1}(\mathbf{x}) = D_{t+1|t}(\mathbf{x})\; L_{\mathbf{y}_{t+1}}(\mathbf{x}). \quad (7)$$

The ratio $L_{\mathbf{y}_{t+1}}(\mathbf{x})$ is given by:

$$L_{\mathbf{y}_{t+1}}(\mathbf{x}) = \frac{\int p_{t+1}(\mathbf{y}_{t+1}|\{\mathbf{x}\} \cup W) p_{t+1|t}(W) \delta W}{\int p_{t+1}(\mathbf{y}_{t+1}|W) p_{t+1|t}(W) \delta W}. \quad (8)$$

The array likelihood involves target positions and associated marks (source signals), and can be further written as:

$$p_{t+1}(\mathbf{y}_{t+1}|W) = \int \cdots \int p_{t+1}(\mathbf{y}_{t+1}|\tilde{W}) \prod_{i=1}^{|\tilde{W}|} \mathcal{N}(s_i; 0, P) ds_i, \quad (9)$$

where $\tilde{W}$ is the MPPP corresponding to the ground PPP W. Both the ground and marked PPP have the same cardinality, denoted by $|\tilde{W}|$. By employing the specific form of the likelihood of eq. (9), the expression of $p_{t+1|t}(W)$ given by eq. (1) and grouping of terms we obtain:

$$L_{\mathbf{y}_{t+1}}(\mathbf{x}) = \frac{\iint p_{t+1}(\mathbf{y}_{t+1} - \mathbf{a}(\mathbf{x})s|\widetilde{W}) \mathcal{N}(s; 0, P) p_{t+1|t}(\widetilde{W}) \delta \widetilde{W} ds}{\int p_{t+1}(\mathbf{y}_{t+1}|\widetilde{W}) p_{t+1|t}(\widetilde{W}) \delta \widetilde{W}}$$

By employing the change of variables formula proposed in [21, Prop 4., p.180], that is, denoting $\mathbf{z} \triangleq \sum_{\tilde{\mathbf{w}} \in \tilde{W}} \mathbf{g}(\tilde{\mathbf{w}})$ we obtain:

$$L_{\mathbf{y}_{t+1}}(\mathbf{x}) = \frac{\iint \mathcal{N}(\mathbf{y}_{t+1} - \mathbf{a}(\mathbf{x})s; \mathbf{z}, \sigma_t^2 \mathbf{I}_M) p(\mathbf{z}) d\mathbf{z}\, \mathcal{N}(s; 0, P) ds}{\int \mathcal{N}(\mathbf{y}_{t+1}; \mathbf{z}, \sigma_t^2 \mathbf{I}_M) p(\mathbf{z}) d\mathbf{z}} \quad (10)$$

Observe that in eq. (10) the set integrals are now reduced to ordinary integrals, where $p(\mathbf{z})$ is the distribution induced by the change of variables. As proposed in [7] if we approximate $p(\mathbf{z}) \approx \mathcal{N}(\mathbf{z}, \widetilde{\mu}, \widetilde{\Sigma})$ with a Gaussian distribution we are able to obtain an analytic formula for the updated PHD. The first and second order moments of $p(\mathbf{z})$ are obtainable from the MPPP predicted distribution $p_{t+1|t}(\tilde{X})$ by means of the Marking Theorem [17, Ch. 5.3]:

$$\begin{aligned} \widetilde{\mu} &= \int \mathbf{g}(\widetilde{\mathbf{x}})\, \widetilde{D}_{t+1|t}(\widetilde{\mathbf{x}}) d\widetilde{\mathbf{x}} \\ &= \iint \mathbf{a}(\mathbf{x})\, D_{t+1|t}(\mathbf{x})\, s\, \mathcal{N}(s; 0, P) d\mathbf{x} ds = 0, \quad (11a) \\ \widetilde{\Sigma} &= \int \mathbf{g}(\widetilde{\mathbf{x}}) \mathbf{g}^H(\widetilde{\mathbf{x}})\, \widetilde{D}_{t+1|t}(\widetilde{\mathbf{x}}) d\widetilde{\mathbf{x}} \\ &= P \int \mathbf{a}(\mathbf{x}) \mathbf{a}^H(\mathbf{x})\, D_{t+1|t}(\mathbf{x}) d\mathbf{x}, \quad (11b) \end{aligned}$$

where $\widetilde{D}_{t+1|t}(\widetilde{\mathbf{x}})$ is the predicted MPPP PHD given by eq. (5), and $\{\cdot\}^H$ represents the transpose conjugate operator. The formula for combination of quadratic terms [22, Appendix 3.8] is employed to solve the denominator and inner integral of the numerator of the ratio given in eq. (10):

$$L_{\mathbf{y}_{t+1}}(\mathbf{x}) \approx \frac{\int \mathcal{N}(\mathbf{y}_{t+1}; \mathbf{a}(\mathbf{x})s, \sigma_t^2 \mathbf{I}_M + \widetilde{\Sigma}) \mathcal{N}(s; 0, P) ds}{\mathcal{N}(\mathbf{y}_{t+1}; 0, \sigma_t^2 \mathbf{I}_M + \widetilde{\Sigma})}.$$

Solving again for the numerator we obtain

$$L_{\mathbf{y}_{t+1}}(\mathbf{x}) \approx \frac{\mathcal{N}(\mathbf{y}_{t+1}; 0, \mathbf{a}(\mathbf{x})\, P\, \mathbf{a}^H(\mathbf{x}) + \sigma_t^2 \mathbf{I}_M + \widetilde{\Sigma})}{\mathcal{N}(\mathbf{y}_{t+1}; 0, \sigma_t^2 \mathbf{I}_M + \widetilde{\Sigma})}. \quad (12)$$

4. SIMULATION RESULTS

The final form of the pseudo-likelihood is given in eq. (12), were $\widetilde{\Sigma}$ is given by eq. (11b). This analytic relationship facilitates the update of the predictive PHD with eq. (7). Due to the

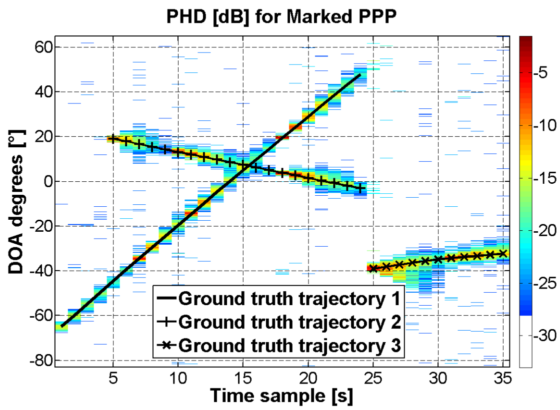

Fig. 1. Logarithm of PHD for proposed filter.

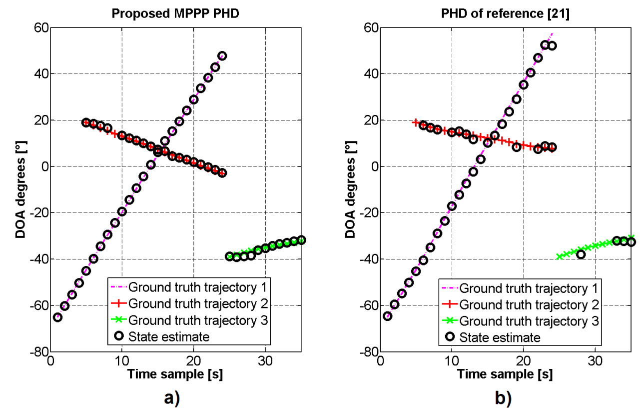

Fig. 2. DBSCAN clustering results: a) with proposed PHD filter b) the method in [25]

approximation of $p(\mathbf{z})$, the integral of the PHD is no longer a reliable estimate for the number of targets, and hence a clustering methodology is proposed in [7] that extracts the number of targets and their state estimates. In [7], this is achieved by iterating the $k-$means clustering algorithm for different numbers of clusters and evaluating the cluster separation with the silhouette method [23]. However, the silhouettes are only computable when there are two or more clusters, hence there isn't any explicit way of distinguishing when there are less than two clusters. In fact, the authors in [7] notice the increased error of the method whenever there is only one target present. Here, we propose to use the DBSCAN clustering algorithm [24], which does not require *apriori* knowledge of the number of clusters and considers the existence of particles not belonging to any cluster, i.e. outliers. This is necessary since a number of particles corresponding to the birth process γ_t, are expected to be dispersed at times when no actual target is born. Note that the $k-$means aims to minimize the within-cluster sum of squares and hence behaves poorly in presence of such outliers. A crossing-target scenario is envisaged, with an auxiliary particle filter implementation of the PHD filter as in [7]. The number of particles per target is fixed at 1000. The kinematic model in eq. (3) is employed with $\mathbf{v}_t \sim \mathcal{N}(0, Q_t)$ and covariance $Q_t = Gq_t^2G^T$ with $q_t = 0.5^o/s^2$ representing an acceleration noise [18, Ch. 6.3.1]. Accordingly, $G = \left[T_s^2/2, T_s\right]^T$ and $T_s = 1$s represents the sampling period. PHD prediction is performed with eq. (4) where the probability of target survival is a constant $p_s = 0.9$. The intensity γ_t for birth location is chosen uniform over $[-\pi/2, \pi/2]$, while for speed we use $\mathcal{N}(0, 3)$. The whole birth intensity integrates to 0.2. The array consists of $M = 30$ receivers and targets are generated with the same

Table 1. Average OSPA error over 1000 Monte Carlo runs.

	OSPA error for SNR $5dB$			OSPA error for SNR $0dB$		
Algorithm	$c = 1.5$	$c = 2.5$	$c = 5$	$c = 1.5$	$c = 2.5$	$c = 5$
Proposed PHD	0.46	0.54	0.67	0.58	0.69	0.87
Method in [25]	1.07	1.43	2.19	1.53	2.44	4.67

source signal power, so that SNR $= 5dB$. PHD update is performed using eq. (7) and the pseudo-likelihood of eq. (12). The resulting point-mass approximation of the PHD is observable in Fig. 1, where the logarithm of particle weights is shown for better visualization. Observe the relatively good fit of the particle clouds for the three target trajectories. From the particle approximation of the PHD, target number and state estimation is performed via DBSCAN clustering. DBSCAN requires two parameters: the minimum number of particles of a cluster (set to 50), and the point-neighborhood distance (set to 1). In Fig. 2-a) clustering of the particle PHD of Fig. 1 is depicted. In Fig. 2-b), results of the method proposed in [25] for the same simulated scenario and parameters are depicted. Observe a better adequacy of the proposed PHD filter, while the method [25] struggles with crossing targets and short tracks. The method in [25] employs a TWS PHD to track the peaks of the array signal spectrogram. Thus, whenever targets cross only one peak is present in the spectrogram (one observation) leading to track loss.

A Monte Carlo analysis is carried out over 1000 runs of the scenario described above at SNR of $5dB$ and $0dB$. The latter representing a very low SNR scenario in order to test the proposed TBD filter. The optimal subpattern assignment (OSPA) [26] error metric is employed to quantify the differences between the estimated target set and ground truth set. Results are synthesized in Table 1 and showcase the superiority of the proposed TBD filter even at very low SNR.

5. CONCLUSIONS

A TBD filter is proposed for DOA tracking based on the superpositional PHD filter. An efficient particle filter implementation is achieved by analytically integrating the source signals in the update step. Tracking of multiple crossing targets is showcased and a Monte Carlo simulation is performed. Results for low SNR scenarios demonstrate the robustness of our proposed method, as compared to state-of-the-art methods.

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