MULTIPLE STAGE ANT COLONY OPTIMIZATION ALGORITHM FOR NEAR-OPTIMAL LARGE-MIMO DETECTION

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ABSTRACT

In this paper, we propose a multiple stage ant colony optimization (MSACO) algorithm for symbol vector detection in large multiple-input multiple-output (MIMO) systems. The proposed algorithm uses minimum mean squared error (MMSE) solution as an initial solution in every stage, and produces a set of solutions by using the ant colony optimization (ACO) based MIMO detection. Finally, a best solution from the generated solution set is selected using the maximum likelihood (ML) metric. Simulation results show that the proposed algorithm significantly outperforms the existing ACO algorithm and some of the other MIMO detection algorithms in terms of bit error rate (BER) performance and achieves near ML performance. Furthermore, the BER performance of the proposed algorithm shifts towards single input single output (SISO) additive white Gaussian noise (AWGN) performance with increase in number of antennas which adds to the importance of MSACO algorithm for detection in large MIMO systems.

Index Terms— Ant colony optimization, multiple-input multiple-output, minimum mean squared error.

1. INTRODUCTION

Significant increase in wireless channel capacity can be achieved by employing multiple-input multiple-output (MIMO) systems in wireless communication [1]. Multiple data streams can be transmitted simultaneously by using multiple antennas at the transmitter through spatial multiplexing. The challenge lies in reliable detection of these data streams at the receiver. MIMO systems with large number of antennas are said to be the large-MIMO systems [2]. The main bottleneck with implementation of large-MIMO systems is the computational complexity associated with the detection algorithms [3]. Minimum bit error rate (BER) performance can be achieved by using maximum likelihood (ML) detection. But, the number of computations required to achieve ML solution increases exponentially with number of antennas and hence becomes computationally infeasible. Sphere decoder (SD) is a well known ML detector, but is practical only up to 32 real dimensions [4]. Hence, the need for efficient detection techniques is of great interest today. Sub-optimum detection techniques include linear detectors like zero forcing (ZF) detector and MMSE detector, and non-linear detector such as vertical Bell laboratories layered architecture (VBLAST) detector [5]. These detectors offers less complexity but are inferior in terms of BER performance when compared to the ML performance.

Several algorithms based on ordered interference cancellation [6], message passing [7], tabu search [8], likelihood ascent search (LAS) [9] and lattice reduction (LR) aided detection [10, 11] have been proposed in the literature for large MIMO detection. Recently, bio inspired optimization algorithms are getting increased attention by signal processing researchers as a low-complexity alternative over the existing algorithms. Moreover, bio-inspired algorithms involve less mathematics and provide a near optimum solution. Ant colony optimization (ACO) is one of the bio inspired algorithms which mimics the foraging behavior of natural ants. ACO based MIMO detection algorithms include modified ACO (MACO) algorithm [12], congestion control based ACO (CC-ACO) algorithm [13] and LR aided ACO algorithm [14]. MACO algorithm proposed in [12] achieves near ML solution but requires a large number of ants to converge, and also, it does not consider ordering in the detection sequence. The CC-ACO algorithm proposed in [13] uses the concept of negative pheromones for pheromone update. But the BER performance of CC-ACO algorithm degrades for higher order QAM. The algorithm in [14] uses lattice reduction based techniques with ACO for MIMO detection which results in a higher computational complexity.

In this paper, we propose a multiple stage ant colony optimization (MSACO) algorithm which uses partial solution from the MMSE estimate of the received symbol vector as an initial guess in every stage. In each stage, we use the MACO based MIMO detection as proposed in [12]. In MACO, the MIMO detection problem is solved as a path finding problem where all the transmit antennas are considered as cities and possible transmit symbols as the available paths. MIMO detection problem then reduces to minimizing the cost of visiting each city exactly once. The proposed work is an extension of the work in [12] with following contributions: a) The concept of multiple stage is used in order to start the

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journey of ants from different cities with partial knowledge of the path. With this partial knowledge, ants start walking on the remaining available paths and find a complete solution. In every stage, N_{ants} solutions are generated and ML metric is used to select the best solution. This results in avoiding the problem of premature convergence to a local optimum solution, and b) Furthermore, we use log likelihood ratio (LLR) based sorted QR decomposition (SQRD) algorithm proposed in [15] for ordering the detection sequence which reduces the effect of error propagation.

Simulation results show that the proposed MSACO algorithm performs better over the existing ACO algorithm and some other large MIMO detection algorithms available in the literature, and achieves the near ML performance.

2. ANT COLONY OPTIMIZATION

In this section, we provide an overview of ACO algorithm which was originally proposed by Dorigo et. al. in [16]. In ACO, we use N_{ants} ants to find a shortest path between their nest N and the food location F. Ants use the phenomenon known as stigmergy, which means communication through environment. When ants walk to and fro between N and F, they deposits on ground a substance called *pheromone* on each path. The pheromone concentration on each path decays with time and the ants always choose the path with higher pheromone concentration. **Algorithm**-1 shows the basic steps followed by ants in ACO.

Algorithm 1 Ant colony optimization algorithm

procedure
  Initialize parameters
  while termination condition not met **do**
    Construct solutions
    Search best solution
    Update pheromone concentration
  end while
end procedure

3. SYSTEM MODEL

We consider a discrete-time MIMO system with N_t transmit antennas, N_r receive antennas and a frequency flat channel. Let $\widetilde{\mathbf{x}}$ be an $N_t \times 1$ transmitted vector in which $\widetilde{x}_i$, the i^{th} element of $\widetilde{\mathbf{x}}$ denotes the M-ary modulated symbol transmitted from i^{th} transmit antenna. The $N_r \times 1$ received signal vector $\widetilde{\mathbf{y}}$ in MIMO system can be represented as

$$\widetilde{\mathbf{y}} = \widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \widetilde{\mathbf{n}}, \tag{1}$$

where $\widetilde{\mathbf{H}}$ denotes the $N_r \times N_t$ channel matrix with its elements $\left\{\widetilde{h}_{j,i}\right\}$ independent and identically distributed (i.i.d.) $\sim \mathcal{CN}(0,1)$. $\left\{\widetilde{h}_{j,i}\right\}$ denotes the channel gain between the i^{th} transmit antenna and j^{th} receive antenna. $\widetilde{\mathbf{n}}$ is $N_r \times 1$ white complex Gaussian noise vector in which the entries are i.i.d. $\sim \mathcal{CN}\left(0,\sigma^2\right)$ where σ^2 is the noise variance. The average SNR is defined as $10\log_{10}\frac{N_t E_x}{\sigma^2}$ dB, where E_x is the average energy per symbol. The complex-valued received vector $\widetilde{\mathbf{y}}$ can be transformed into an equivalent real-valued representation as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = \begin{bmatrix} \Re(\widetilde{\mathbf{y}}) \\ \Im(\widetilde{\mathbf{y}}) \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} \Re(\widetilde{\mathbf{H}}) & -\Im(\widetilde{\mathbf{H}}) \\ \Im(\widetilde{\mathbf{H}}) & \Re(\widetilde{\mathbf{H}}) \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} \Re(\widetilde{\mathbf{x}}) \\ \Im(\widetilde{\mathbf{x}}) \end{bmatrix}$, and $\mathbf{n} = \begin{bmatrix} \Re(\widetilde{\mathbf{n}}) \\ \Im(\widetilde{\mathbf{n}}) \end{bmatrix}$. $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of $(\cdot)$ respectively. When the receiver has the perfect knowledge of the channel state information (CSI) $\mathbf{H}$, the MMSE estimate of the received vector $\mathbf{y}$ can be written as

$$\mathbf{G}_{MMSE} = \arg\min_{\mathbf{G}} E[\|\mathbf{x} - \mathbf{G}\mathbf{y}\|^2] \tag{3}$$

$$\mathbf{x}_{MMSE} = \mathcal{Q}(\mathbf{G}_{MMSE}\mathbf{y}), \tag{4}$$

where $\mathbf{G}$ is linear transformation matrix applied to the received vector, $E[\cdot]$ is the expectation operation and $\mathcal{Q}(\cdot)$ denotes the nearest neighbor quantizer for the input modulation. The ML solution can be written as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{A}^{2N_t}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \tag{5}$$

where $\mathbb{A}$ is the set of real-valued entries in the signal constellation, e.g., $\mathbb{A} = \{-1, 1\}$ in 4-QAM signaling and $\mathbb{A} = \{-3, -1, 1, 3\}$ in $16 - QAM$ signaling. The channel state information matrix $\mathbf{H}$ can further be decomposed using $\mathbf{QR}$ decomposition as $\mathbf{H} = \mathbf{QR}$, where $\mathbf{Q}$ is a $2N_r \times 2N_t$ orthogonal matrix and $\mathbf{R}$ is a $2N_t \times 2N_t$ upper triangular matrix. Equation (2) can then be rewritten as

$$\mathbf{y} = \mathbf{QRx} + \mathbf{n} \tag{6}$$

$$\mathbf{Q}^H\mathbf{y} = \mathbf{Q}^H\mathbf{QRx} + \mathbf{Q}^H\mathbf{n} \tag{7}$$

$$\hat{\mathbf{y}} = \mathbf{Rx} + \hat{\mathbf{n}}, \tag{8}$$

where $\hat{\mathbf{y}} = \mathbf{Q}^H\mathbf{y}$, $\hat{\mathbf{n}} = \mathbf{Q}^H\mathbf{n}$, and $(.)^H$ denote the matrix Hermitian transpose.

4. PROPOSED ALGORITHM

In this section, we propose MSACO algorithm for symbol vector detection in large MIMO systems. In the proposed algorithm, we use multiple stages of ant colonies with the partial solution from the MMSE estimate of the received symbol. Using ACO, the MIMO detection is modeled as a path finding problem which uses the concept of traveling salesman problem (TSP) as in [12]. All the transmit antennas are treated as cities and the possible symbols are the available paths to reach the i^{th} city. MIMO detection problem now reduces to finding the cheapest way of visiting each city exactly once. From (8) we conclude that the vector $\mathbf{x}$ is chosen from the set of all possible transmit vectors such that it minimizes the ML function $f(\mathbf{x})$ given by

$$\mathbf{f}\left(\mathbf{x}\right)=\|\hat{\mathbf{y}}-\mathbf{R}\mathbf{x}\|^{2}. \tag{9}$$

We use N_s parallel stages and for every stage, we consider a set of artificial ants N_{ants} to search the solution independently. There exists M paths to reach a specific city i, where M is the cardinality of the constellation (for eg. M = 4 in 16-QAM) and $i = 1, 2, \cdots, 2N_t$. Each path is denoted as x_{ik}, $k = 1, 2, \cdots, M$ which is nothing but the transmitted symbol from the i^{th} antenna. At a given time, only one ant independently walks on the available paths and finds a solution. The ML metric for MIMO detection is given as

$$\mathbf{x}_{ML}=\arg\min_{\mathbf{x}\in\mathbb{A}^{2N_t}}\|\hat{\mathbf{y}}-\mathbf{R}\mathbf{x}\|^{2}. \tag{10}$$

Expanding (10) results in

$$\mathbf{x}_{ML}=\arg\min_{\mathbf{x}\in\mathbb{A}^{2N_t}}\sum_{i=1}^{2N_t}\mid\hat{y}_i-\sum_{l=i}^{2N_t}R_{il}x_l\mid^{2}. \tag{11}$$

In ACO, the distance of the kth path towards the i^{th} city is formulated as d_{ik}

$$d_{ik}=\mid\hat{y}_i-\sum_{l=i+1}^{2N_t}R_{il}\tilde{x}_l-R_{ii}x_{ik}\mid, k=1,\cdots,M, \tag{12}$$

where i progressively decreases from $2N_t$ to 1, $x_{ik}\in\mathbb{A}$ denotes all possible transmit symbols from the i^{th} antenna and $\tilde{x}_l$ are the hard decisions of x_l for $l = i+1, i+2, \cdots, 2N_t$. A sigmoid function (as in [12]) is then used to convert the distance d_{ik} into heuristic value ϕ_{ik} as

$$\phi_{ik}=\frac{1}{(1+\exp(d_{ik}))}. \tag{13}$$

Based on these heuristic values the probability of selecting the k^{th} path for the i^{th} city is given as

$$p_{ik}=\frac{[\phi_{ik}]^{\alpha}}{\sum_{k=1}^{M}[\phi_{ik}]^{\alpha}} \tag{14}$$

where α is the weighting parameter.

For the s^{th} stage, all N_{ants} ants assumes a partial solution from the MMSE solution as an initial guess i.e. $x_{2N_t}, x_{2N_t-1}, \cdots, x_{2N_t-s}$ are taken from the MMSE solution as

$$x_{2N_t}=x_{2N_t}^{MMSE},\cdots,x_{2N_t-s}=x_{2N_t-s}^{MMSE}, \tag{15}$$

and the remaining symbols x_{2N_t-s+1}, x_{2N_t-s+2}, $\cdots$, x_1 are searched by the ants. Once the solutions in s^{th} stage are generated, the ML metric as in (10) is used to select the best one. Similarly, every stage produces one best solution and finally a set of N_s solution are generated. Again, we use ML metric as in (10) to select one best solution from these N_s solutions which is the final output of the MSACO algorithm.

Algorithm 2 MSACO algorithm for large-MIMO detection

input: $\mathbf{y}$, $\mathbf{H}$, $N_t, N_r, N_{ants}, \alpha, \mathbf{x}_{MMSE}$;
initialize: $\hat{d}_{best}=\infty$;
Compute $\hat{\mathbf{y}}=\mathbf{Q}^H\mathbf{y}$ where $\mathbf{H}=\mathbf{QR}$[1]
for $s = 1 : 1 : N_s$ **do**
  Initialize the partial solution for s^{th} stage using (15)
  while $j = 1 : 1 : N_{ants}$ **do**
    for $i = 2N_t - s : -1 : 1$ **do**
      Compute d_{ik} using (12);
      Compute ϕ_{ik} using (13);
      Compute p_{ik} using (14);
      Choose x_i according to the probability p_{ik} ;
    end for
    Compute
    $\hat{d}_{new}=\|\hat{\mathbf{y}}-\hat{\mathbf{H}}\mathbf{x}^{(j)}\|^2$;
    if $\hat{d}_{new}\leqslant\hat{d}_{best}$ **then**
      $\mathbf{x}^{(sol)}=\mathbf{x}^{(j)}$;
      $\mathbf{x}^{(int)}=\mathbf{x}^{(j)}$;
      $\hat{d}_{best}=\hat{d}_{new}$;
    else
      $\mathbf{x}^{(sol)}=\mathbf{x}^{(int)}$;
    end if
  end while
end for
output: $\mathbf{x}^{(sol)}$ is output solution vector

The concept of multiple stages could be thought of as restarting ants from different locations in the journey with partial knowledge about the path. This helps ants in avoiding the premature convergence to a local optimum solution, and also, it results in fast convergence of the algorithm (as shown in Section 5). Furthermore, by ordered decoding, the effect of error propagation that may occur due to wrong decisions in early stages can be minimized. To order the symbol detection sequence, we use the LLR-SQRD algorithm from [15]. LLR-SQRD algorithm is used for $\mathbf{QR}$ decomposition of channel matrix $\mathbf{H}$ in MSACO algorithm. Along with $\mathbf{QR}$ decomposition, the detection sequence and the received symbol vector are also arranged as per the changes in the corresponding channel link. The proposed MSACO algorithm is given in **Algorithm**-2.

[1] LLR-SQRD algorithm from [15] is used for $\mathbf{QR}$ decomposition.

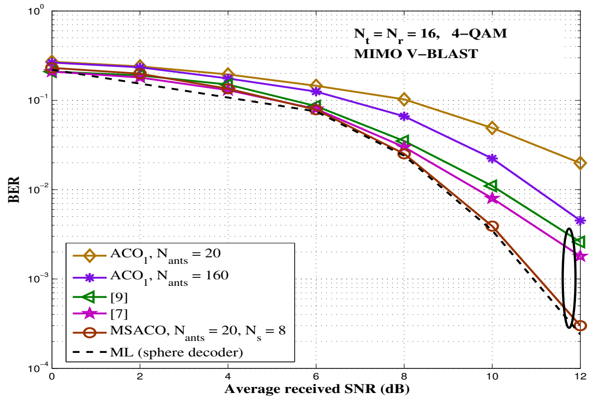

Fig. 1. BER performance comparison of the proposed algorithm for 16×16 MIMO system with 4-QAM.

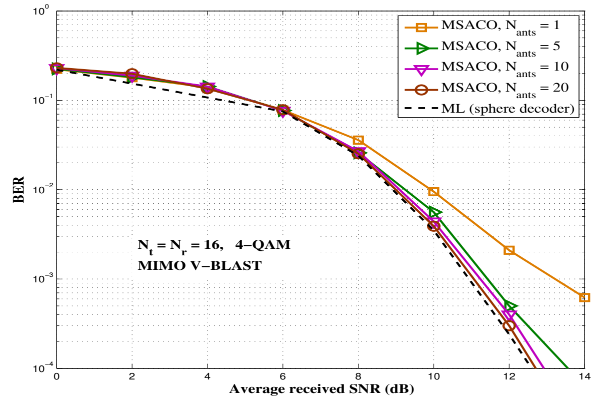

Fig. 2. BER performance of the proposed algorithm with $N_s = 8$ and N_{ants} = 1, 5, 10 and 20 for 16×16 MIMO system with 4-QAM.

5. SIMULATION RESULTS

In this section, we present the simulation results for BER performance and convergence of the proposed algorithm. We compare the performance of the MSACO algorithm with existing ACO algorithm and some of the other large MIMO detection algorithms. We denote the algorithm proposed in [12] by ACO_1. The value used for α is 1 in the proposed algorithm.

In Fig. 1, we compare the BER performance of the proposed algorithm with ACO_1, message passing based algorithm in [7] and LAS algorithm in [9] for 16×16 MIMO systems with 4-QAM. Observation reveals that the proposed algorithm outperform the other algorithms and achieves a near ML performance. The MSACO performs close to within 0.2 dB of the ML performance with just $N_{ants} = 20$ and $N_s = 8$ which is quite attractive. At SNR = 12 dB, MSACO achieves 3×10^{-4} BER whereas other algorithms proposed in [7] and [9] achieves BER of approx. 1.8×10^{-3} and 2.5×10^{-3} respectively.

The BER performance of MSACO algorithm with $N_s = 8$ and N_{ants} = 1, 5, 10 and 20 for 16×16 MIMO systems with 4-QAM is shown in Fig. 2. It is observed that the BER performance of MSACO improves with increase in N_{ants} and achieves a near ML performance. For N_{ants} = 5, 10 and 20 at target BER = 10^{-3}, the MSACO performs close to within 0.5, 0.3 and 0.2 dB of the ML performance respectively. In Fig. 3, the BER performance of MSACO algorithm is plotted for N_t = 8, 16, and 32. Observation reveals that the performance improves with increase in N_t and shifts towards SISO-AWGN performance. This shows the significance of MSACO algorithm for symbol vector detection in large MIMO systems. To check the convergence of the MSACO algorithm, BER performance of MSACO versus N_{ants} with different N_s is plotted in Fig. 4. Results show that with small increase in N_{ants}, the performance of MSACO moves close to ML performance.

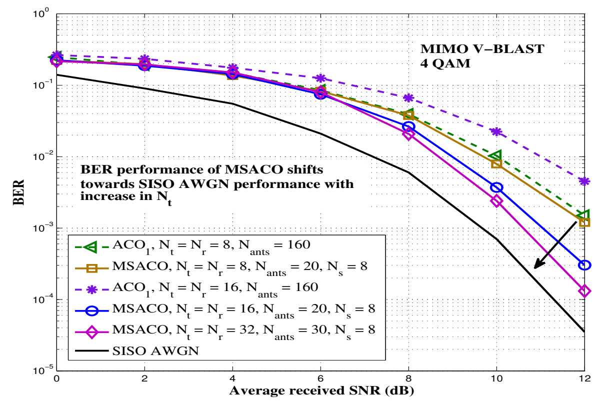

Fig. 3. BER performance of the proposed algorithm for 8×8, 16×16 and 32×32 MIMO system with 4-QAM.

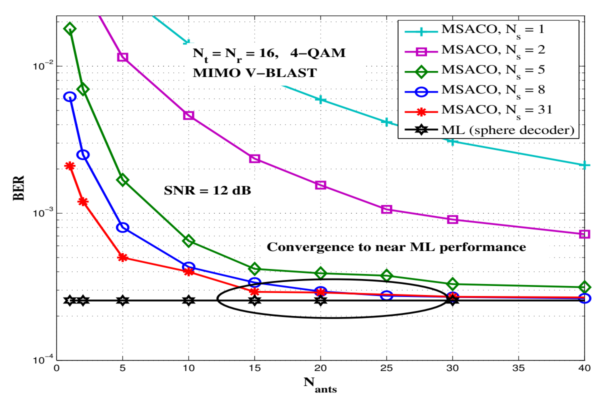

Fig. 4. Convergence curve of the proposed algorithm for 16×16 MIMO system with 4-QAM at SNR = 12 dB.

It can also be seen that further increase in N_s beyond 8 results in the same BER performance for all values of $N_{ants} > 20$. This shows the fast convergence behavior of MSACO algorithm which is very important from the implementation point of view. The additional complexity in MSACO when compared with ACO_1 is due to the MMSE block and ordering in the detection sequence. However, the performance gain of MSACO is superior which results in a better complexity-performance tradeoff.

6. CONCLUSION

We proposed a multiple stage ant colony optimization (MSACO) algorithm for symbol vector detection in large-MIMO systems. The proposed algorithm outperforms the ACO algorithm and some of the other MIMO detection algorithms existing in the literature, and achieves a near ML performance close to within 0.2 dB of the ML performance. Furthermore, the BER performance of MSACO improves with increase in the number of transmit antennas and shifts towards SISO-AWGN performance. This shows the significance of the proposed algorithm for symbol vector detection in large MIMO systems. Also, the proposed algorithm converges very rapidly to the near ML performance with very few ants (i.e $N_{ants} < 30$) and stages ($N_s = 8$) for 16×16 MIMO with 4-QAM. Since we have used hard decision decoding, the errors occurred in early stage of detection propagates and results in wrong decision in later stages of the algorithm. In order to further reduce the effect of error propagation, devising a soft decision decoding based MSACO algorithm is a possible future work.

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