NORMALIZED RECURSIVE LEAST ADAPTIVE THRESHOLD NONLINEAR ERRORS ALGORITHM

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ABSTRACT

This paper proposes a new adaptation algorithm named Normalized Recursive Least Adaptive Threshold Nonlinear Errors (NRLATNE) algorithm for complex-domain adaptive filters which makes the filters fast convergent for correlated filter inputs and robust against two types of impulse noise: one is found in additive observation noise and another at filter input. Analysis of the proposed NRLATNE algorithm is fully developed to theoretically calculate filter convergence behavior. Through experiments with some examples, we demonstrate the effectiveness of the proposed algorithm in improving the filter performance. Good agreement is observed between simulated and theoretically calculated filter convergence that shows the validity of the analysis.

Index Terms—Adaptive filter, recursive least squares, impulse noise, adaptive threshold, nonlinear error, normalization

1. INTRODUCTION

Among many adaptation algorithms for adaptive filters, the LMS or NLMS algorithm is most widely applied to practical communication systems and most intensively studied [1, 2]. Although the LMS or NLMS algorithm attracts many implementers for its excellent performance, a serious drawback is its vulnerability to impulse noise [3, 4].

Two types of impulse noise are found in adaptive filtering systems: one in additive observation noise and another at filter input. The latter type of impulse noise is often found in active noise cancellers. One of the solutions for robust filtering in the presence of impulsive observation noise is use of the Sign Algorithm (SA) in the real-number domain [5] or the Least Mean Modulus (LMM) algorithm in the complex-number domain [6]. However, the filter convergence with these algorithms is much slower than the LMS algorithm.

The author proposed Adaptive Threshold Nonlinear Algorithm [7] to preserve the fast convergence speed of the LMS algorithm while improving the robustness against the impulsive observation noise. To make the filter robust against large variations of the filter input, we introduce a normalizing factor as in the NLMS algorithm [8].

When the filter input is correlated (or colored), the filter convergence becomes considerably slower. To solve this problem, recursive least squares estimation of the inverse covariance matrix of the filter input is combined. A typical example is the well-known Recursive Least Squares (or Square Errors) (RLS) algorithm [2]. If this recursive estimation is combined with the LMM algorithm, we derive Recursive Least Moduli (RLM) algorithm which successfully makes the filter convergence significantly faster for a strongly correlated filter input and, at the same time, realizes high robustness against both types of impulsive noise [9].

In this paper, with a different approach, combining the above stated methods, we derive an adaptation algorithm named *Normalized Recursive Least Adaptive Threshold Nonlinear Errors* (NRLATNE) algorithm. Theoretical analysis of the proposed NRLATNE algorithm is developed, and experiments with some examples are carried out to examine the performance of the NRLATNE algorithm and to compare simulated and theoretically calculated filter convergence behavior.

2. IMPULSE NOISE MODELS

2.1. Impulsive Observation Noise

Impulse noise found in the additive observation noise is often modeled as Contaminated Gaussian Noise (CGN) that is mathematically a combination of two independent Gaussian noise sources [10], i.e., Gaussian noise $\nu^{(0)}(n)$ with variance $\sigma^2_{\nu^{(0)}}$ and probability of occurrence $p_{\nu^{(0)}}$, and $\nu^{(1)}(n)$ with $\sigma^2_{\nu^{(1)}}$ and $p_{\nu^{(1)}}$, where n is the time instant. Note that $p_{\nu^{(0)}} + p_{\nu^{(1)}} = 1$ holds. Usually, $\sigma^2_{\nu^{(1)}} >> \sigma^2_{\nu^{(0)}}$ and $p_{\nu^{(1)}} < p_{\nu^{(0)}}$. The variance of CGN is given by $\sigma^2_{\nu} = p_{\nu^{(0)}}\sigma^2_{\nu^{(0)}} + p_{\nu^{(1)}}\sigma^2_{\nu^{(1)}}$. For "pure" Gaussian noise, $\sigma^2_{\nu} = \sigma^2_{\nu^{(0)}}$ and $p_{\nu^{(1)}} = 0$.

2.2. Impulse Noise at Filter Input [11]

A "noisy" filter input $b(n)$ with impulse noise added to the reference input $a(n)$ is given by $b(n) = a(n) + \tau(n)\, \nu_a(n)$, where $\tau(n)$ is an independent Bernoulli random variable taking 1 with probability p_{ν_a} and 0 with $1 - p_{\nu_a}$. The impulse noise $\nu_a(n)$ itself is assumed to be a White & Gaussian process with variance $\sigma^2_{\nu_a}$ independent of $a(n)$.

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3. NORMALIZED RECURSIVE LEAST ADAPTIVE THRESHOLD NONLINEAR ERRORS ALGORITHM

3.1. Least Mean Adaptive Threshold Nonlinear Error Algorithm

In a complex-domain FIR-type adaptive filter, let the cost function of the error $e(n)$ be defined by

$$L_e(n) = F[|\, e(n)\, |; A(n)],$$

where $|\, e(n)\, |$ is the modulus of the complex-valued error and $A(n)$ is a threshold. The function $F(\,\cdot\,;\,\cdot\,)$ is given by

$$F(|\, e\, |; A) = \begin{cases} |\, e\, |^2/2 & \text{for } |\, e\, | \le A \\ A^2/2 & \text{for } |\, e\, | > A. \end{cases}$$

Defining a *nonlinear* function of *error* by

$$\phi_F(|\, e\, |; A) = |\, e\, |^{-1}\, dF(|\, e\, |; A) \,/\, d|\, e\, | = \begin{cases} 1 & \text{for } |\, e\, | \le A \\ 0 & \text{for } |\, e\, | > A, \end{cases} \quad (1)$$

we derive an update equation for the tap weight vector $\mathbf{c}(n)$:

$$\mathbf{c}(n+1) = \mathbf{c}(n) + \alpha_c\, f[e^*(n); A(n)]\, \mathbf{a}(n), \quad (2)$$

where $f(e; A) = \phi_F(|\, e\, |; A)\, e$, $\mathbf{a}(n) = [a(n) \cdots a(n-k) \cdots a(n-N+1)]^T$ is the filter reference input vector, N is the number of tap weights, α_c is the step size and $(\,\cdot\,)^*$ denotes complex conjugate. The *adaptive threshold* $A(n)$ is calculated as

$$A(n+1) = (\, 1 - \rho_A\,)\, A(n) + \rho_A\, M_A\, |\, e(n)\, | \quad (3)$$

with M_A being a multiplier and ρ_A a leakage factor.

The above adaptation algorithm is named Least Mean Adaptive Threshold Nonlinear Error (LMATNE) algorithm. The LMATNE algorithm makes adaptive filters converge as fast as the LMS algorithm and robust against impulsive observation noise [7]. Note that the threshold is not fixed but adapted according to the average error magnitude.

3.2. Normalized Recursive Least Adaptive Threshold Nonlinear Errors Algorithm

First, we combine the LMATNE algorithm with a normalizing factor $\|\, \mathbf{a}(n)\, \|^2$ as used in the NLMS algorithm [8]. Next, combining the NLMATNE algorithm with the recursive least squares estimation of the inverse covariance matrix as used in the RLS algorithm, we derive an adaptation algorithm named *Normalized Recursive Least Adaptive Threshold Nonlinear Errors* (NRLATNE) algorithm whose tap weight update equation is given by

$$\mathbf{c}(n+1) = \mathbf{c}(n) + \alpha_c\, \mathbf{P}(n)\, f[e^*(n); A(n)] \cdot \mathbf{a}(n) \,/\, \|\, \mathbf{a}(n)\, \|^2, \quad (4)$$

in which usually $\alpha_c = 1$ and the estimate of the inverse covariance matrix $\mathbf{P}(n)$ is calculated in two ways.

Method <A>: $\mathbf{P}(n+1) = \mathbf{Q}^{-1}(n+1)$ (5)

$$\mathbf{Q}(n+1) = \lambda\, \mathbf{Q}(n) + \phi_F[|\, e(n)\, |; A(n)]\, \mathbf{a}(n)\, \mathbf{a}^H(n) \,/\, \|\, \mathbf{a}(n)\, \|^2 \quad (6)$$

or

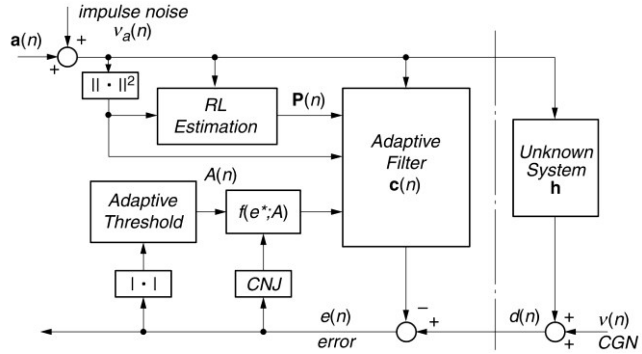

Fig. 1. Schematic diagram for NRLATNE algorithm.

Method <B>: $\mathbf{P}(n+1) = \lambda^{-1}\, \{\, \mathbf{P}(n) - \phi_F[|\, e(n)\, |; A(n)]\, \mathbf{P}(n)\, \mathbf{a}(n)\, \mathbf{a}^H(n)\, \mathbf{P}(n) \,/\, \|\, \mathbf{a}(n)\, \|^2 \,/\, \{\lambda + \phi_F[|\, e(n)\, |; A(n)]\, \mathbf{a}^H(n)\, \mathbf{P}(n)\, \mathbf{a}(n) \,/\, \|\, \mathbf{a}(n)\, \|^2\}\, \}$, (7)

where λ is the forgetting factor. Method <B> is derived from Method <A> by applying the famous Matrix Inversion Lemma. Although the update equation for Method <A> is simpler, the computational complexity is much lower for Method <B>. Thus, we use Method <B> for simulations.

The NRLATNE algorithm is expected to make adaptive filters fast convergent for correlated filter inputs and robust against *both* types of impulse noise stated in Section 2. Fig. 1 is a schematic diagram for the NRLATNE algorithm.

4. ANALYSIS

In this section, for ease of analysis, we assume *absence* of impulse noise at the filter input. In the experiments in the next section, both types of impulse noise are considered.

4.1. Assumptions

For the analysis in this section, we make the following assumptions.

A1: The number of tap weights N is large, say $N > 20$.

A2: The filter reference input $\mathbf{a}(n)$ is a colored Gaussian process with a covariance matrix $\mathbf{R_a} = E[\mathbf{a}(n)\mathbf{a}^H(n)]/2$ and a variance $\sigma^2_a = E[|\, a(n)\, |^2]/2$.

A3: Two types of impulse noise are modeled in Section 2.

A4: The filter input $\mathbf{a}(n)$ and the tap weights $\mathbf{c}(n)$ are mutually independent (*Independence Assumption*).

A5: The estimate $\mathbf{P}(n)$ is independent of $e(n)$ and $\mathbf{a}(n)$.

A6: The error $e(n)$ given $\mathbf{a}(n)$ is Gaussian distributed [5].

4.2. Difference Equations for Tap Weight Misalignment

Define a tap weight misalignment vector $\boldsymbol{\theta}(n) = \mathbf{h} - \mathbf{c}(n)$ where $\mathbf{h}$ is the response vector of the unknown stationary system. For $\boldsymbol{\theta}(n)$, we have an update equation

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) - \alpha_c\, \mathbf{P}(n)\, \phi_F[|\, e(n)\, |; A(n)]\, e^*(n) \cdot \mathbf{a}(n) \,/\, \|\, \mathbf{a}(n)\, \|^2. \quad (8)$$

From (8), a set of difference equations for the mean vector $\mathbf{m}(n) = E[\boldsymbol{\theta}(n)]$ and the second-order moment matrix $\mathbf{K}(n) = E[\boldsymbol{\theta}(n)\, \boldsymbol{\theta}^H(n)]$ is derived as

$$\mathbf{m}(n+1) = \mathbf{m}(n) - \alpha_c\, \mathbf{p}(n) \quad (9)$$

and $$\mathbf{K}(n+1) = \mathbf{K}(n) - \alpha_c\, [\mathbf{V}(n) + \mathbf{V}^H(n)] + \alpha^2_c\, \mathbf{T}(n), \quad (10)$$

where $\mathbf{p}(n) = E[\mathbf{P}(n)]\ \mathbf{W}(n)\ \mathbf{m}(n)$, $\mathbf{V}(n) = E[\mathbf{P}(n)]\ \mathbf{W}(n)\ \mathbf{K}(n)$ and $\mathbf{T}(n) \cong E[\mathbf{P}(n)]\ \mathbf{S}(n)\ E[\mathbf{P}(n)]$.

First, let us calculate $\mathbf{W}(n)$. Recognizing that $|\, e(n)\, |$ is subject to the Rayleigh distribution [12], we calculate

$$E\{\phi_F[|\, e(n)\, |;\, A(n)]\ e^*(n)\ \mathbf{a}(n) / \|\, \mathbf{a}(n)\, \|^2 \,\big|\, \boldsymbol{\theta}(n)\}$$
$$\cong \int_0^{r(n)} (t^2/2)\exp(-t^2/2)tdt\ E[\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^2]\ \boldsymbol{\theta}(n)$$
$$\cong H[r(n)]\ E[\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^2]\ \boldsymbol{\theta}(n),$$

whence

$$\mathbf{W}(n) = H[r(n)]\ \mathbf{W_a} \quad (11)$$

with $\mathbf{W_a} = E[\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^2]$ and $H(r) = 1 - (1+r^2/2) \exp(-r^2/2)$. The error $e(n) = \in(n) + \nu(n)$, and $\in(n) = \boldsymbol{\theta}^H(n)\, \mathbf{a}(n)$ is the excess error, $\nu(n)$ is the additive observation noise, $r(n) = E[A(n)] / \sigma_e(n)$ is the normalized threshold, $\sigma^2_e(n) = \varepsilon(n) + \sigma^2_\nu$ is the error variance, and we define Excess Mean Square Error (EMSE) by $\varepsilon(n) = E[|\in(n)|^2]/2 = \mathrm{tr}[\mathbf{R_a K}(n)]$.

Next, we calculate for N>>1

$$\mathbf{S}(n) = E\{\phi_F[|\, e(n)\, |;\, A(n)]\ |\, e(n)\, |^2\ \mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^4\}$$
$$\cong \int_0^{r(n)} t^2\exp(-t^2/2)tdt\ \sigma^2_e(n)\ E[\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^4]$$
$$\cong 2H[r(n)]\ \sigma^2_e(n)\ \mathbf{S_a} \quad (12)$$

with $\mathbf{S_a} = E[\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^4]$.

For N>>1, we can approximate

$$\mathbf{W_a} \cong E[\mathbf{a}(n)\mathbf{a}^H(n)]/E[\|\, \mathbf{a}(n)\, \|^2] \cong \mathbf{R_a} / (\sigma^2_a N) \quad (13)$$

and $$\mathbf{S_a} \cong \mathbf{R_a} / [2(\sigma^2_a N)^2]. \quad (14)$$

For the CGN, we calculate an average for $\sigma^2_\nu{}^{(i)}$, $i = 0, 1$.

4.3. Difference Equation for Adaptive Threshold

For the adaptive threshold, the difference equation is:

$$E[A(n+1)] = (\, 1 - \rho_A\,)\ E[A(n)] + \rho_A\, M_A\, (\pi/2)^{1/2}\, \sigma_e(n). \quad (15)$$

4.4. Analysis of Method <A> for Calculation of $E[\mathbf{P}(n+1)]$

For Method <A>, we derive, from (5) and (6)

$$E[\mathbf{P}(n+1)] = E[\mathbf{Q}^{-1}(n+1)] \cong E[\mathbf{Q}(n+1)]^{-1} \quad (16)$$

and $$E[\mathbf{Q}(n+1)] = \lambda E[\mathbf{Q}(n)] + \boldsymbol{\Xi}_\mathbf{Q}(n), \quad (17)$$

where

$$\boldsymbol{\Xi}_\mathbf{Q}(n) \cong E\{\phi_F[|\, e(n)\, |;\, A(n)]\}\ E[\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^2]$$
$$\cong G[r(n)]\ \mathbf{W_a} \quad (18)$$

with $G(r) = \int_0^r \exp(-t^2/2)tdt = 1 - \exp(-r^2/2)$.

4.5. Analysis of Method <B> for Calculation of $E[\mathbf{P}(n+1)]$

For Method <B>, we derive, from (7), a difference equation

$$E[\mathbf{P}(n+1)] = \lambda^{-1}E[\mathbf{P}(n)]\{\mathbf{I} - \boldsymbol{\Phi}_\mathbf{P}(n)\ E[\mathbf{P}(n)]\}, \quad (19)$$

where

$$\boldsymbol{\Phi}_\mathbf{P}(n) = E\{\ \mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^2$$
$$/\ \{\lambda / \phi_F[|\, e(n)\, |;\, A(n)] + \mathbf{a}^H(n)\mathbf{P}(n)\mathbf{a}(n)/\|\, \mathbf{a}(n)\, \|^2\}\ \}$$
$$\cong \int_0^\infty d\beta\ E\{\exp\{-\beta\lambda/\phi_F[|\, e(n)\, |;\, A(n)]\}\}$$
$$\cdot\ E\{\mathbf{a}(n)\mathbf{a}^H(n)/\|\, \mathbf{a}(n)\, \|^2 \exp[-\beta\mathbf{a}^H(n)\mathbf{P}(n)\mathbf{a}(n)/\|\, \mathbf{a}(n)\, \|^2]\}.$$

Since $\exp\{-\beta\lambda/\phi_F[|\, e(n)\, |;\, A(n)]\} = \exp(-\beta\lambda)$ for $|\, e(n)\, | \le A(n)$ and $= 0$ for $|\, e(n)\, | > A(n)$, we obtain for N>>1

$$\boldsymbol{\Phi}_\mathbf{P}(n) \cong \int_0^\infty d\beta\ \exp(-\beta\lambda) \int_0^{r(n)}\exp(-t^2/2)tdt$$
$$\cdot\ \mathbf{W_a}\ \exp\{-\beta\mathrm{tr}\{\mathbf{W_a}\, E[\mathbf{P}(n)]\}\}$$
$$\cong G[r(n)]\ \mathbf{W_a} / \{\ \lambda + \mathrm{tr}\{\mathbf{W_a}\, E[\mathbf{P}(n)]\}\}$$
$$\cong \boldsymbol{\Xi}_\mathbf{Q}(n) / \{\ \lambda + \mathrm{tr}\{\mathbf{W_a}\, E[\mathbf{P}(n)]\}\}. \quad (20)$$

4.6. Initial Conditions

For $\mathbf{c}(0) = \mathbf{0}$, we have $\mathbf{m}(0) = \mathbf{h}$ and $\mathbf{K}(0) = \mathbf{h}\ \mathbf{h}^H$. $A(0) = (\pi/2)^{1/2}\, M_A\, \sigma_e(0)$ and $\mathbf{P}(0) = P_0\, \mathbf{I}$ with

$$P_0 = \mathrm{tr}[\mathbf{W}(0)\mathbf{K}(0)] / \{\alpha_c\, \mathrm{tr}[\mathbf{S}(0)]\}$$
$$\cong 1 / \alpha_c$$

that minimizes $\varepsilon(1)$.

4.7. Steady-State Solution

As $n \to \infty$, $E[A(\infty)] = M_A\, (\pi/2)^{1/2}\sigma_e(\infty)$ and $r(\infty) = (\pi/2)^{1/2}\, M_A$. Then, $\mathbf{W}(\infty) \cong H_\infty\, \mathbf{W_a}$ and $\mathbf{S}(\infty) \cong 2\, H_\infty\, \sigma^2_e(\infty)\, \mathbf{S_a}$ with $H_\infty = H[(\pi/2)^{1/2}\, M_A]$.

For Method <A>,

$$E[\mathbf{P}(\infty)] = \lambda_c\, \boldsymbol{\Xi}_\mathbf{Q}^{-1}(\infty) \cong \lambda_c\, G_\infty^{-1}\mathbf{W_a}^{-1} \quad (21)$$

with $G_\infty = G[(\pi/2)^{1/2}\, M_A]$ and $\lambda_c = 1 - \lambda$.

For Method <B>, we find

$$E[\mathbf{P}(\infty)] = \lambda_c\, \boldsymbol{\Phi}_\mathbf{P}^{-1}(\infty) \cong \lambda_c\, \{\lambda + \mathrm{tr}\{\mathbf{W_a} E[\mathbf{P}(\infty)]\}\, G_\infty^{-1}\mathbf{W_a}^{-1}$$

from which we solve $\mathrm{tr}\{\mathbf{W_a}E[\mathbf{P}(\infty)]\} \cong \lambda_c / (1-\lambda_c\, N\, G_\infty^{-1}) \cdot \lambda_c\, N\, G_\infty^{-1}$, hence

$$E[\mathbf{P}(\infty)] \cong \lambda_c\, \rho_P\, G_\infty^{-1}\mathbf{W_a}^{-1} \quad (22)$$

with $\rho_P = \lambda_c/(1-\lambda_c N G_\infty^{-1}) > 1$. For Method <A>, clearly $\rho_P = 1$.

Then we derive, with $\rho_P \ge 1$,

$$\mathbf{K}(\infty) = (\alpha_c/2)\ \mathbf{W}^{-1}(\infty)\ \mathbf{S}(\infty)\ E[\mathbf{P}(\infty)]$$
$$\cong \alpha_c\, \lambda_c\, \rho_P\, G_\infty^{-1}\sigma^2_e(\infty)\mathbf{W_a}^{-1}\, \mathbf{S_a}\, \mathbf{W_a}^{-1}$$

and the steady-state EMSE for N>>1

$$\varepsilon(n) \cong \delta / (1 - \delta) \cdot \sigma^2_\nu \quad (23)$$

with

$$\delta = \alpha_c\, \lambda_c\, N\, \rho_P\, G_\infty^{-1} / 2. \quad (24)$$

5. EXPERIMENTS

In this section, experiments are carried out for the proposed NRLATNE algorithm. In the experiments, the simulation result is plotted as an ensemble average of the squared excess error $<|\in(n)|^2>/2$ over 1000 Monte Carlo simulations of filter convergence.

For the experiments, two examples are prepared as given below, where $N = 32$, the filter input is an AR1 Gaussian process with $\sigma^2_a = 1$ (0 dB) and the regression coefficient $\eta = 0.9$. For the unknown system, $\mathbf{h} = [0.01\text{–j}0.05\ \ 0.758\text{–j}0.02\ \ 0.05\text{+j}0.05\ \ –0.5\text{+j}0.1\ \ –0.25\text{+j}0.05\ \ h_5 \cdots h_{31}]^T$ with $h_k = 0.8\, h_{k-1}$ for $k = 5$ to 31 ($\|\, \mathbf{h}\, \|^2 \cong 1$). For the adaptive threshold, $M_A = 1.5$ and $\rho_A = 2^{-11}$.

Example #1 "pure" Gaussian noise: $\sigma^2_\nu = 0.01$ (–20 dB)
no impulse noise at filter input

for NRLATNE: $\alpha_c = 1$, $\lambda_c = 2^{-11}$
for NLMATNE: $\alpha_c = 2^{-6}$
analysis of Methods <A> and <B>

Example #2 Case 1: "pure" Gaussian noise $\sigma^2_v = 0.01$
no impulse noise at filter input
Case 2: CGN $\sigma^{2}_{v}{}^{(0)} = 0.01$; $p_v^{(0)} = 0.9$
$\sigma^{2}_{v}{}^{(1)} = 10$; $p_v^{(1)} = 0.1$
no impulse noise at filter input
Case 3: "pure" Gaussian noise $\sigma^2_v = 0.01$
impulse noise at filter input
$\sigma^2_{va} = 1000$; $p_{va} = 0.1$
Case 4: CGN as in Case 2
impulse noise at filter input
as in Case 3
for NRLATNE: $\alpha_c = 1$, $\lambda_c = 2^{-11}$
analysis of Method <B>

Fig. 2 shows results for *Example #1*, where simulated and theoretically calculated filter convergence curves for the NRLATNE algorithm in the absence of impulse noise are plotted. For the estimate $\mathbf{P}(n)$, analyses of Methods <A> and <B> are compared. We see slightly better accuracy of Method <B> than Method <A>. In the figure, filter convergence for the NLMATNE algorithm is also shown. We observe much faster convergence for the NRLATNE algorithm than for the NLMATNE algorithm, which demonstrates the effectiveness of the recursive estimation of the inverse covariance matrix.

For *Example #2*, filter convergence for the NRLATNE algorithm in the presence of two types of impulse noise is shown in Fig. 3. Case 2 is for impulsive observation noise, Case 3 for impulse noise at filter input, and Case 4 for both types of impulse noise. Only simulation results are given for Cases 3 and 4. We observe high robustness of the algorithm against both types of impulse noise.

6. CONCLUSION

In this paper, we have proposed an adaptation algorithm for complex-domain adaptive filters named Normalized Recursive Least Adaptive Threshold Nonlinear Errors (NRLATNE) algorithm, combining the LMATNE algorithm with a normalizing factor and a recursively estimated inverse covariance matrix of the filter input.

Through analysis and experiments, we have demonstrated that the algorithm makes adaptive filters fast convergent for correlated filter inputs and highly robust against the two types of impulse noise found in adaptive filtering systems.

Simulated and theoretically calculated filter convergence curves are in good agreement that shows the validity and accuracy of the analysis for practical use.

Analysis of the NRLATNE algorithm in the presence of both types of impulse noise is left as a future work.

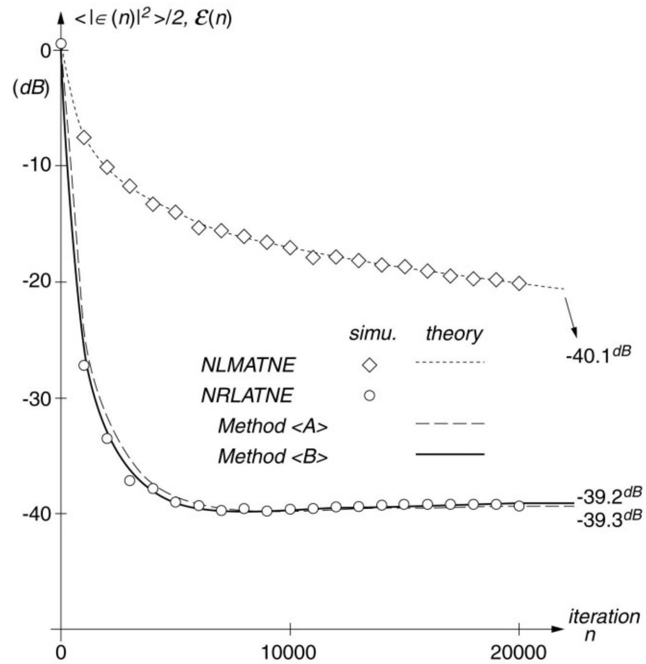

Fig. 2. Adaptive filter convergence curves. (*Example #1*, NRLATNE and NLMATNE, Analysis of Methods <A> and <B>)

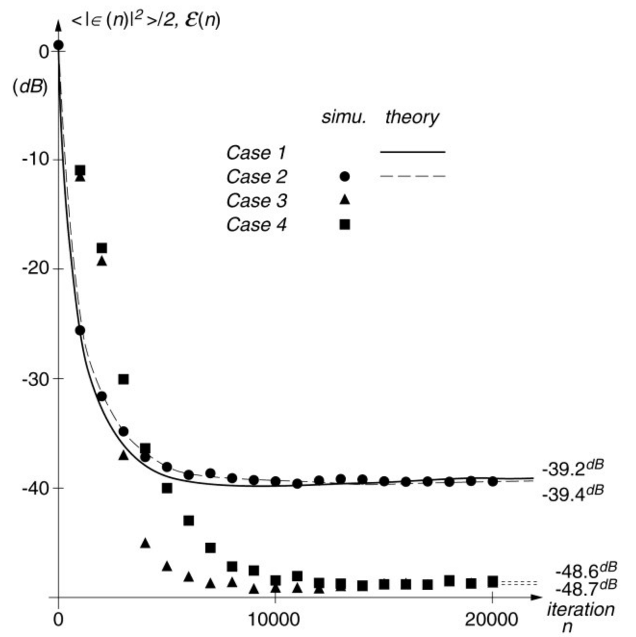

Fig. 3. Adaptive filter convergence curves. (*Example #2*, NRLATNE, Cases 1 to 4).

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