DESIGN OF PR OVERSAMPLED DFT TRANSMULTIPLEXERS WITH MINIMAL DIMENSION

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ABSTRACT

In this paper we propose a new design method for oversampled perfect reconstruction (PR) Discrete Fourier Transform (DFT) transmultiplexers (TMUXs). The resulting multicarrier modulation (MCM) systems are characterized by their minimal dimension, i.e., they involve a minimal number of Givens rotations. Our design method is applicable for system parameters that have never been reached before and also provides improved results in terms of out-of-band energy.

Index Terms— DFT; FMT; OFDM; Oversampled; Transmultiplexer.

1. INTRODUCTION

A new generation of mobile communication is now actively being prepared. This fifth generation (5G) will impact all communication layers from the application layer to the physical one. This strong trend is illustrated by the research on new waveforms which is actively carried out in several European projects [1], [2], [3]. If many communication standards, including 3GPP LTE [4] for the 4G, have adopted the Orthogonal Frequency Division Multiplexing (OFDM) modulation in the past, currently various MCM alternatives are being proposed. The main idea is to preserve the great advantages of OFDM, i.e., fast and simple implementation algorithms and robustness with respect to frequency selectivity, while avoiding its main drawback which is its poor $\sin(x)/x$ frequency behavior. This feature is indeed annoying in the presence of any type of frequency impairment and could definitively prevent the use of OFDM in asynchronous application scenarios due to the bad frequency containment of each user/service.

In this paper, we focus on an alternative known as oversampled OFDM [5] or Filtered MultiTone (FMT) [6]. As it uses the duality with exponentially modulated filter banks, it is also named Discrete Fourier Transform (DFT) transmultiplexer (TMUX). Being an oversampled system, the DFT TMUX can provide in the meantime a PR property and a frequency spectrum which is far better than the OFDM one. However, the design of such a PR TMUX is difficult if one wants to get a modulation system with

- A high spectral efficiency (SE), i.e. an oversampling factor (OSF) close to 1, e.g., 33/32, otherwise said a SE equivalent to the one of an OFDM system having a Cyclic Prefix (CP) equal to 1/32. Then, the underlaying problem is that the number of orthogonality constraints grows when the OSF decreases;
- A high number of subcarriers, knowing this number can attain the maximum size of the (Inverse) Fast Fourier Transform ((I)FFT) used in OFDM systems, e.g., 2048 for 3GPP [4] and 32768 for DVB-NGH [7]. Then, as the number of taps of the prototype filter is proportional to the number of subcarriers, the resulting optimization problem may become huge.

At the exception of TMUX systems equipped with prototype filters having, for given OSF and number of carriers, the shortest possible length [8], the state of the art does not provide solutions fully satisfying the two above objectives. In the early works the authors have exhibited design examples going up first to 32 and then 128 carriers with OSF going down from 3/2 to 5/4 in [5] and [9], respectively. The concept of dimension of a solution is introduced at first in [10] where the authors also provide designs for TMUX systems with 1024 carriers and an OSF equal to 5/4. This result is also made possible by the use of the compact representation method [11]. Then, in [12], for a large set of DFT TMUX parameters, a classification of the corresponding algebraic solutions is provided according to their dimension. It appeared that minimal dimension systems where appropriate to get prototype filters with low out-of band energy. A recent publication [13] has shown that using a more conventional approach, i.e., a Singular Value Decomposition (SVD) or a Cosine-Sine Decomposition (CSD) of the polyphase matrix it was possible to design a 128-subband DFT TMUX system with an OSF equal to 33/32. With our method, we can go up to 32768 subbands while providing in the meantime a better frequency selectivity.

Our paper is organized as follows. Section 2 presents the PR DFT TMUX. In Section 3, we define the set of matrices providing PR for various OSFs and prototype filter lengths. In

Section 4, we provide our notations and definitions leading to a minimal dimension PR Givens set together with the key theorem allowing us to design minimal dimension DFT TMUX. In Section 5, we present some design examples that can be used for practical TMUXs. Section 6 gives our conclusions.

2. THE PR DFT TRANSMULTIPLEXER

Let us denote by M and N two integers such that $M \leq N$. The TMUX scheme is depicted in Fig. 1. Its expansion and decimation factors are equal to N. $F_m(z)$ and $H_m(z)$, $0 \leq m \leq M-1$ denote the filters of the synthesis (SFB) and analysis filter bank (AFB), respectively. In a general setting, they correspond to the z-transform of their complex impulse response $f_m[k]$ and $h_m[k]$, respectively

$$F_m(z) = \sum_{k \in \mathbb{Z}} f_m[k] z^{-k}, \; H_m(z) = \sum_{k \in \mathbb{Z}} h_m[k] z^{-k}. \quad (1)$$

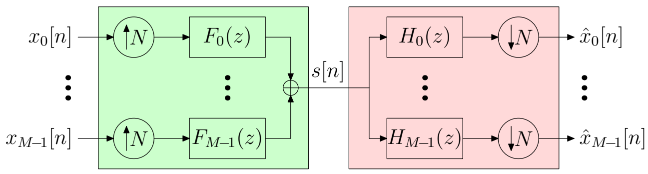

Fig. 1. Transmutiplexer with M subcarriers and expansion/decimation factor equal to N.

This TMUX with OSF equal to $\rho = \frac{N}{M}$ is said to be PR if for any discrete-time input symbols $x_m[n]$ the output symbols $\hat{x}_m[n]$ are such that $\hat{x}_m[n] = x_m[n], \; n \in \mathbb{Z}, \; 0 \leq m \leq M-1$ implying that, for $0 \leq m, m' \leq M-1, \; n, k \in \mathbb{Z}$,

$$\sum_{l \in \mathbb{Z}} f_{m'}[l - kN] h_m[nN - l] = \delta_{m,m'} \delta_{n,k}, \quad (2)$$

where $\delta_{i,j} = 1$ if $i = j$ and 0 otherwise.

In the following, we only focus on *orthogonal* TMUX, i.e., the AFB corresponds to a set of filters that are matched to the ones of the SFB. Otherwise said the coefficients of the $F_m(z)$ and $H_m(z)$ filters are linked together by the following equation $h_m[n] = \overline{f_m[-n]}, \; 0 \leq m \leq M-1, \; n \in \mathbb{Z}$.

The oversampled OFDM ($N > M$) system, we are now considering, is a TMUX with *exponential modulation*, meaning that the $F_m(z)$ filters are given by

$$F_m(z) = P(\omega_M^m z), \; 0 \leq m \leq M-1, \quad (3)$$

where ω_M denotes the Mth root of unity $\omega_M = e^{-2j\pi/M}$ and $P(z) = \sum_{n \in \mathbb{Z}} p[n] z^{-n}$ is a filter, called the *prototype* filter, with real-valued coefficients, $p[n], n \in \mathbb{Z}$.

After some computations, we arrive at a PR condition already reported in [9, (10)]

$$\sum_{k \in \mathbb{Z}} p[s+kM] p[s+kM+nN] = \frac{1}{M} \delta_{n,0}, 0 \leq s < M, n \in \mathbb{Z}. \quad (4)$$

Then, setting $\Delta = \gcd(M, N)$ and defining M_0 and N_0 such that $M = \Delta M_0, N = \Delta N_0$, we get the type 1 Δ-polyphase decomposition of $P(z)$ as follows

$$P(z) = \sum_{i=0}^{\Delta-1} P_i(z^{\Delta}) \; z^{-i}. \quad (5)$$

with $P_i(z)$ the i-th polyphase component. Doing so we can provide a simple reformulation of the decomposition theorem initially proved in [10].

Theorem 2.1. *$P(z)$ is PR for the parameters M and N if and only if the $P_i(z)$ are PR for the parameters M_0 et N_0 for every $i, 0 \leq i \leq \Delta - 1$.*

3. THE $\mathcal{S}(M_0, N_0, L_0)$ SET

Definition 3.1. *Let $N_0 \geq 2$, $M_0 > 0$ with $M_0 \leq N_0$, and $\boldsymbol{A}(X)$ a $N_0 \times M_0$ matrix for which entries are polynomial in X with real valued coefficients. Such a matrix $\boldsymbol{A}(X)$ is* paraunitary *if*

$$\boldsymbol{A}(1/X)^T \; \boldsymbol{A}(X) = \boldsymbol{I}_{M_0}, \quad (6)$$

where $\boldsymbol{I}_{M_0}$ denotes the unit matrix of dimension M_0 and $(.)^T$ is the transpose operator.

If the entries of a paraunitary matrix $\boldsymbol{A}(X)$ are constant then the matrix $\boldsymbol{A} = \boldsymbol{A}(X)$ is constant : it is a$N_0 \times M_0$ orthogonal matrix, such that $\boldsymbol{A}^T \; \boldsymbol{A} = \boldsymbol{I}_{M_0}$.

In the rest of this paper the row and column indexes of matrices will be denoted starting with the index 0 and matrices appear in bold characters while set of matrices are written with standard characters.

Let the functions $a(r,c), p(r,c), q(r,c)$ defined for $(r,c) \in [0, \ldots, N_0-1] \times [0, \ldots, M_0-1]$ be defined as in [12] by

$$a(r,c) = r + p(r,c) N_0 = c + q(r,c) M_0, \quad (7)$$

with $0 \leq p(r,c) < M_0$, $0 \leq q(r,c) < N_0$, $0 \leq a(r,c) < M_0 N_0$. It is proven in [12] that the function $\varepsilon(r,c)$, defined by

$$\varepsilon(r,c) = \alpha(r) - (q(0,c) + q(r,0) - q(r,c))/N_0, \quad (8)$$

where a/b denotes the integer division and where $\alpha(r) = 0$ if r is divisible by M_0 and 1 elsewhere, has values in $\{0, 1\}$.

For M_0, N_0 relatively prime numbers, with $M_0 < N_0$, and $P(z)$ a prototype filter for parameters M_0 and N_0, let us define the $N_0 \times M_0$ matrix $\boldsymbol{U}(X)$, called the U-matrix of $P(z)$, by

$$[\boldsymbol{U}(X)]_{r,c} = X^{\varepsilon(r,c)} V_{a(r,c)}(X), \quad (9)$$

where the $V_a(z)$ are the $M_0 N_0$-polyphase components of $P(z)$.

Theorem 3.2. *[12, Theorem II.2] $P(z)$ has the PR property if and only if its U-matrix $\boldsymbol{U}(X)$ is paraunitary.*

For $L_0 \geq N_0$, we denote by $\mathcal{S}(M_0, N_0, L_0)$ the set of all U-matrices of PR prototype filters for parameters M_0, N_0 and length L_0.

4. A MINIMAL DIMENSION PR GIVENS SET

4.1. Notations and definitions

Definition 4.1. . *Let $s : [0, 1, \dots, M_0 - 1] \to [0, 1, \dots, N_0 - 1]$ be an injective application. We denote by $\boldsymbol{E}(s)$ the $N_0 \times M_0$ constant orthogonal matrix such that $\boldsymbol{E}_{s(c),c} = 1,\ 0 \leq c \leq M_0 - 1$ and $\boldsymbol{E}_{r,c} = 0$, otherwise. Such a matrix is said to be an* elementary paraunitary matrix.

$E(s)$ denotes the one element set $\{\boldsymbol{E}(s)\}$.

In the following s_0 denotes the application defined by $s_0(c) = c, 0 \leq c \leq M_0 - 1$.

Definition 4.2. *Let $N_0 \geq 2$ and $i, j, 0 \leq i, j \leq N_0 - 1;\ i \neq j$ and $\theta \in \mathbb{R}$. The square orthogonal matrix $\boldsymbol{R}_{i,j}(\theta)$ of dimension N_0 defined by $[\boldsymbol{R}_{i,j}(\theta)]_{i,i} = [\boldsymbol{R}_{i,j}(\theta)]_{j,j} = \cos\theta$, $[\boldsymbol{R}_{i,j}(\theta)]_{j,i} = -[\boldsymbol{R}_{i,j}(\theta)]_{i,j} = \sin\theta$, $[\boldsymbol{R}_{i,j}(\theta)]_{r,r} = 1,\ r \neq i,\ r \neq j$ and $[\boldsymbol{R}_{i,j}(\theta)]_{r,c} = 0$ otherwise, is called an* elementary rotation matrix*, or a* Givens matrix*, of indexes i, j.*

$R_{i,j}$ denotes the set $\{\boldsymbol{R}_{i,j}(\theta),\ \theta \in \mathbb{R}\}$.

Definition 4.3. *Let $N_0 \geq 2$ and $0 \leq r \leq N_0 - 1$. The paraunitary diagonal square matrix $\boldsymbol{Z}_r$ of dimension N_0, such that $[\boldsymbol{Z}_r]_{r,r} = X$ and $[\boldsymbol{Z}_r]_{k,k} = 1$ if $k \neq r$ is called the* shift matrix *of row r.*

Z_r denotes the one element set $\{\boldsymbol{Z}_r\}$.

If A and B designate two sets of matrices with compatible dimensions, we denote by AB the set of matrix products $\boldsymbol{AB}$ when $\boldsymbol{A} \in A$ and $\boldsymbol{B} \in B$.

Let us now give the fundamental following definition.

Definition 4.4. *Let M_0 and N_0 be two integers with $N_0 \geq 2$ and $1 \leq M_0 \leq N_0$. A* Givens set *of parameters M_0, N_0 is a product of shape $T_1 T_2 \dots T_{n_T} E$ where E is the set composed of an elementary matrix and where for, each $k,\ 1 \leq k \leq n_T$, T_k is a set of rotations $R_{i,j}$ or a set Z_r.*

The matrices of a Givens set are obviously paraunitary matrices of dimensions $N_0 \times M_0$.

If there exist n_R sets of rotations in the sequence $\{T_i,\ 1 \leq i \leq n_T\}$ of a Givens set G, let $i_1 < i_2 < \dots < i_{n_R}$ denote their indexes in this sequence. If $\theta_l,\ 1 \leq l \leq n_R$ are n_R given real numbers, then we can choose the rotation of angle θ_l in the set of rotations T_{i_l} for $1 \leq l \leq n_R$.

Thus, we get an application ϕ_G from $\mathbb{R}^{n_R}$ to G, which is called the *parametric representation* of G.

In another hand, the set $\mathcal{M}(N_0, M_0)^{(d)}$ of $N_0 \times M_0$ matrices the entries of which are polynomial in X of degree less or equal to $d,\ d \geq 0$ can be identified to $\mathbb{R}^{(d+1)M_0N_0}$ as follows : the $\boldsymbol{E}_{k,r,c}$ matrices with $0 \leq k \leq d$, $0 \leq r \leq N_0 - 1$, $0 \leq c \leq M_0 - 1$, defined by $[\boldsymbol{E}_{k,r,c}]_{r,c} = X^k$ and 0 otherwise, constitute a basis of this set of matrices and the linear application such that $\psi(\boldsymbol{E}_{k,r,c}) = e_n$ with $n = kM_0N_0 + mM_0 + c$, where $e_n, 0 \leq n < (d+1)M_0N_0$ is the canonical basis $\mathbb{R}^{(d+1)M_0N_0}$, is a bijection from $\mathcal{M}(N_0, M_0)^{(d)}$ to $\mathbb{R}^{(d+1)M_0N_0}$. Starting from now, we will identify a matrix $\boldsymbol{A}$ of $\mathcal{M}(N_0, M_0)^{(d)}$ and its image $\psi(\boldsymbol{A})$ in $\mathbb{R}^{(d+1)M_0N_0}$ and we will also consider than $\psi \circ \phi_G$ is the parametric representation of G.

Theorem 4.5. *For a Givens set G containing n_R rotations, the rank of the Jacobian matrix of its parametric representation ϕ_G reaches its maximum value except on a set of null measure in $\mathbb{R}^{n_R}$. This maximum value is called the* dimension *of G.*

If the dimension of G is equal to n_R then, for any point θ of $\mathbb{R}^{n_R}$ where the rank of the Jacobian matrix is exactly equal to n_R, there exists a neighborhood V of θ where the rank is still equal to n_R and the restriction of ϕ_G of V is injective being more precisely a diffeomorphism of V onto $\phi_G(V)$. We will say that G is a *locally injective Givens set*.

The use of a compact representation [11] assumes that for each Δ-polyphase component (PC) the behavior of the d angular parameters is very regular which has been checked for critically decimated [11] and oversampled systems [10, 12]. Otherwise said, for the PC of index i and a number of d rotations, corresponding to the solution dimension, the angle behavior may for instance be accurately represented by the following smoothing function [10]

$$\theta_p^{(i)} = \sum_{k=0}^{K-1} x_{p,k} \left(\frac{2i+1}{2\Delta} \right)^k,\ 0 \leq i < \Delta,\ 0 \leq p \leq d-1. \quad (10)$$

where the $x_{p,k}$ designate the coefficients of this expansion. Then, instead of having $d\Delta$ parameters to optimize the compact representation only involves dK variables. This can ne interesting as soon as $K < \Delta$. Most often a value of K equal to 5 or 6 can be enough and the number of subcarriers can attain thousands, so, in general, $K << \Delta$.

4.2. The basic theorem

When the length L_0 of the solution is a multiple of N_0, $L_0 = mN_0$, and when the set $\mathcal{S}_{M_0, M_0+1, L_0}$ of all solutions has been explicitly computed, the solution of minimal dimension is of dimension m and can be written in a relatively simple form. In the following theorem, we show that this simple form gives rise to a solution, i.e., for each value of $L_0 = mN_0$, we get a set of paraunitary matrices of size $N_0 \times M_0$ providing PR prototype filters for the parameters M_0, $N_0 = M_0 + 1$ and with length L_0.

Theorem 4.6. *Let $M_0 \geq 2$ and $N_0 = M_0 + 1$. For all $k \geq 0$, we define the sets of matrices T_k by*

$$T_k = \begin{cases} Z_{M_0} R_{0,M_0} & \text{if } k > 0 \text{ and } k \mod M_0 = 0, \\ R_{k \bmod M_0, M_0} & \text{otherwise,} \end{cases} \tag{11}$$

Let us define the Givens set S_{M_0,L_0} for $L_0 = mN_0$ with $m \geq 1$ by

$$S_{M_0,L_0} = \prod_{k=0}^{m-1} T_k \; E[s_0]. \tag{12}$$

Then $S_{M_0,L_0} \subset \mathcal{S}_{M_0,M_0+1,L_0}$ and S_{M_0,L_0} has dimension m.

Conjecture 4.7. *For $M_0 \geq 2, N_0 = M_0 + 1$ and $L_0 = mN_0$ with $m \geq 1$, S_{M_0,L_0} is a minimal dimension solution of $\mathcal{S}_{M_0,M_0+1,L_0}$ in its decomposition as an union of locally injective Givens sets, each being maximal for the inclusion and not included in the union of the others.*

Each set T_k depends on an angle and we denote $\boldsymbol{T}_k(\theta_k)$ the matrix obtained when giving the value of θ_k to this angle. Then, we set, for $L_0 = mN_0$,

$$\boldsymbol{S}_{M_0,L_0}(\theta_0, \ldots, \theta_{m-1}) = \prod_{k=0}^{m-1} \boldsymbol{T}_k(\theta_k) \; \boldsymbol{E}(s_0). \tag{13}$$

Since a rotation matrix with a null angle is the identity matrix of dimension N_0 and as $\boldsymbol{Z}_{M_0}\boldsymbol{E}(s_0) = \boldsymbol{E}(s_0)$, we get

$$\boldsymbol{S}_{M_0,L_0}(\theta_0, \ldots, \theta_{m-1}) = \boldsymbol{S}_{M_0,L_0+N_0}(\theta_0, \ldots, \theta_{m-1}, 0), \tag{14}$$

so that $S_{M_0,L_0} \subset S_{M_0,L_0+N_0}$.

Theorem 4.6 is a strict generalization of [12, Theorem VI.4]. Its main interest lies on the fact that the optimization for a given criterion of a PR prototype RP filter of length ΔL_0, with $L_0 = mN_0$, built using Δ Δ-polyphase components corresponding to U-matrices $\boldsymbol{S}_{M_0,L_0}(\theta_0^{(i)}, \theta_1^{(i)}, \ldots, \theta_{m-1}^{(i)})$, $i = 0, \ldots, \Delta - 1$ can be done for $m > 1$ using as initial point the optimum obtained for $m - 1$ and initial angles $\theta_{m-1}^{(i)}$, $i = 0, \ldots, \Delta - 1$ being set to zero.

Then, using the compact representation method, we do not need to add Δ parameters at each step but only K, with K the degree of the compact representation. Furthermore, as the optimization is carried out step by step, it appears that for high values of M_0, a degree $K = 2$ is enough. Note also that compared to [10, 12], where the prototype filter length is constrained to be equal to $m\Delta N_0 M$, we can now explore a far wider set of lengths.

In the next section, our design method is used to get prototype filters with minimal out-of-band energy. It is also shown that it can produce prototype filters up to $\rho = 33/32$ for a 32768-subband PR DFT TMUX, i.e., something comparable to what is proposed in DVB-NGH [7] for OFDM.

5. DESIGN RESULTS

As in [12] we noticed that minimal dimension solutions could provide good attenuation properties, we focus here on the minimization of the out-of-band energy criterion. The objective function to minimize then writes as

$$E = \frac{J(f_c)}{J(0)} \text{ with } J(x) = \int_x^{1/2} |P(e^{j2\pi\nu}|^2 \, d\nu. \tag{15}$$

with $f_c = \frac{1}{2M}$. Then the goal is to find the $x_{p,k}$ variables of the compact representation minimizing E.

For the first targeted example, the parameters are as follows: $M_0 = 32, N_0 = 33, L_0 = 128N_0, \Delta = 2^{10}$ thus leading to a prototype filter of length $L = 33 \times 2^{17}$. According to theorem 2.1, the angular representation corresponds to Δ PR polyphase components described by an element belonging to S_{32,L_0} with $L_0 = 128N_0$ depending on 128 angles, i.e., a total of $128\Delta = 2^{17}$ angles.

Setting $K = 2$, the resulting optimization problem involves $Kd = 2 \times 128$ independent parameters which is too much to get a satisfactory result using a global optimization software.

Using (14), the optimization can be conducted with m increasing from 1 to 128, using as a starting point, for any $m > 1$, the optimal filter obtained for $m - 1$. Only K parameters need to be added at each step, with $K = 2$ for high values of M_0.

In this way, we obtain the prototype filter displayed in Figure 2. Its out-of-band energy is equal to 2.5214×10^{-5}, i.e., -45.97 dB. Only the first 10^6 filter's coefficients are represented in the left part of Figure 2.

Naturally our method also works for other values of M_0. E.g., for $M_0 = 8, N_0 = 9, L_0 = mN_0$ with $1 \leq m \leq 32$ and $\Delta = 8$, we get the best out-of-band energy curve depicted in Figure 3.

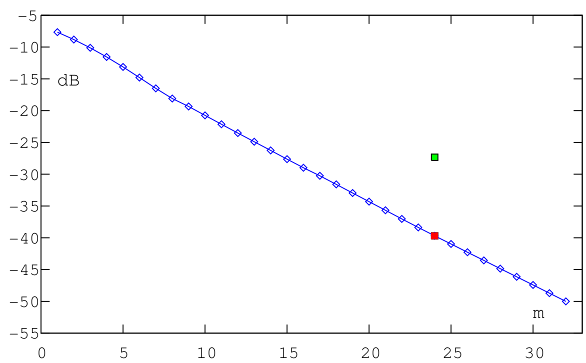

Fig. 3. Variations (in dB) for $1 \leq m \leq 32$ of the best out-of-band energy for PR prototype filter optimized setting: $M_0 = 8, N_0 = 9, L_0 = mN_0$ and $\Delta = 8$.

For $m = 24$, we obtain a filter of length $L = mN_0\Delta = 24 \times 9 \times 8 = 1728$ having a out-of-band energy equal to

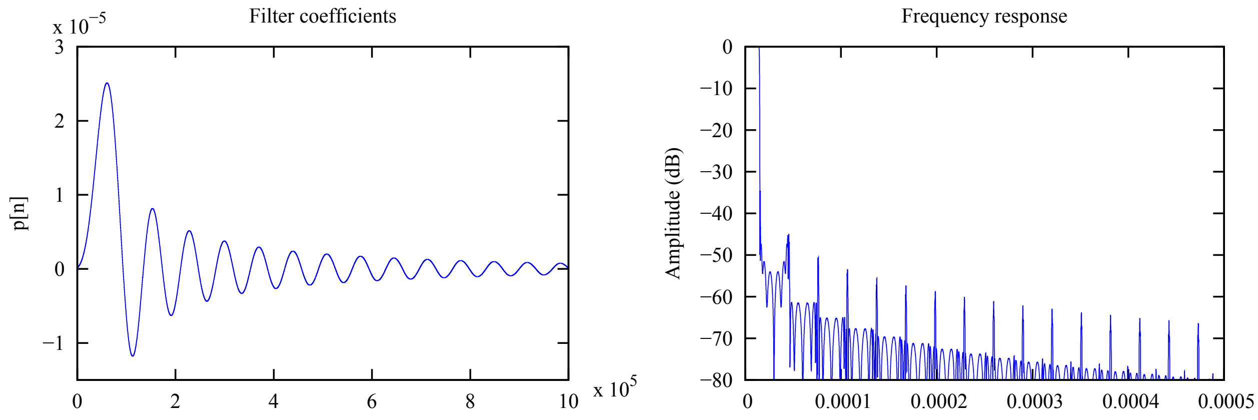

Fig. 2. Prototype PR filter with parameters $M_0 = 32, N_0 = 33, L_0 = 128N_0, \Delta = 2^{10}$ and out-of-band energy -45.97 dB.

1.0736×10^{-4}, i.e., -39.69 dB. It corresponds to the red square in Figure 3. The number of angles for its angular representation is equal to $\Delta m = 192$ while its optimization has been carried out with the $2m = 48$ parameters of a compact representation of degree $K = 2$.

Let us compare with the result provided by Rahimi and Champagne in [13] for a PR prototype filter having similar parameters. According to their own parametric representation, the number of parameters allowing to describe a paraunitary matrix corresponding to the prototype filter is equal to 576 or 448 [13, Table 1]. Their result corresponds to the green square in Figure 3. This clearly shows that our method requires a smaller number of parameters, which is of interest for optimization but also for the implementation point of view, and furthermore we are able to provide a design result which is approximately 13.5 dB better for the out-of-band energy.

6. CONCLUSION

We have presented a design method for oversampled PR DFT TMUX. We considered a class of PR solutions leading to MCM systems of minimal dimension, i.e. involving a minimum number of Givens rotations. We have theoretically proved that the angles can be derived step by step when considering increasing lengths of the prototype filters. This leads to an efficient design algorithm in which the filter's coefficients can be iteratively computed. For a minimization of the out-of-band energy criterion, we can provide nearly optimal designs without practically no length limitation and for very high spectral efficiency, i.e., with oversampling factors very close to 1 (33/32).

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