OPTIMAL ADAPTIVE TRANSMIT BEAMFORMING FOR COGNITIVE MIMO SONAR IN A SHALLOW WATER WAVEGUIDE

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ABSTRACT

This paper addresses the problem of adaptive beamforming for target localization by active cognitive multiple-input multiple-output (MIMO) sonar in a shallow water waveguide. Recently, a sequential waveform design approach for estimation of parameters of a linear system was proposed. In this approach, at each step, the transmit beampattern is determined based on previous observations. The criterion used for waveform design is the Bayesian Cramér-Rao bound (BCRB) for estimation of the unknown system parameters. In this paper, this method is used for target localization in a shallow water waveguide, and it is extended to account for environmental uncertainties which are typical to underwater acoustic environments. The simulations show the sensitivity of the localization performance of the method at different environmental prior uncertainties.

Index Terms— MIMO sonar, cognitive sonar, sequential waveform design, adaptive beamforming, underwater acoustics

1. INTRODUCTION

Optimal beamforming for active arrays has been extensively studied in the past three decades. The optimization is usually performed in order to achieve better performance for detection or localization under some constraints, such as transmitted power constraint. The optimization criterion may be statistical bounds for localization performance, probability of detection, output signal-to-noise ratio (SNR), or information theoretic criteria.

Since the introduction of colocated multiple-input multiple-output (MIMO) radar in [1, 2], several works have been devoted for transmit beamform design [3–7]. In a colocated constellation, both the transmitting and receiving arrays are assumed to be close to each other in space so that they observe targets at same directions. In [1,2] it was shown that transmitting spatially orthogonal signals provides higher estimation accuracy performance over traditional spatially coherent signals transmission. A cognitive approach for transmit beamforming was studied in [5, 6], where the effectiveness of the adaptive transmit beamforming over non-adaptive transmit beamforming was demonstrated.

The concept of cognitive radar was introduced in [8]. A cognitive radar system adaptively interrogates a propagation channel using all available information. Then, it facilitates the newly acquired knowledge through feedback from the receiver to the transmitter. The whole cognitive radar system constitutes a dynamic closed feedback loop encompassing the transmitter, environment and receiver. In [7] a new adaptive transmit beamforming approach was proposed for target parameters estimation with cognitive MIMO array, where the beampattern in each pulse is adaptively determined based on previous observations. The algorithm was implemented in the case of free-space environment. This approach suggests a transmit beamforming scheme, which adaptively minimizes the Bayesian Cramér-Rao bound (BCRB) or the Reuven-Messer bound (RMB) on the system parameter estimation based on historical observations. At each pulse step, the system parameters were estimated by using the minimum mean squared error (MMSE) estimator that was implemented using the posterior distribution from the previous step.

Underwater localization of a point source has been studied in several works (see e.g. [9–15]) and various underwater target localization approaches, such as matched-field processing (MFP) [11, 12] and maximum likelihood (ML) localization [9], have been introduced. Several performance bounds, such as the Cramér-Rao bound [9,10,12] or a Ziv–Zakai-type bound [12] for source localization in underwater waveguides were derived and studied.

Source localization in a shallow water waveguide in the presence of environmental uncertainties has been studied in several works (see e.g. [10,11,14]). It was shown in [10] that uncertainty in sensors location severely decreases the estimation accuracy. In [11] it was shown that MFP is sensitive to environmental mismatch. In [14] a robust ML source localization method, was proposed based on nulling the modes that are sensitive to environmental uncertainties.

In this paper, we apply the adaptive transmit beamforming technique proposed in [7] for the case of shallow water waveguide environment. Additionally, the adaptive beamforming algorithm will be extended to address environmental uncertainties. The effect of the environmental uncertainties on the performance of the adaptive beamforming algorithm is studied via simulations.

The rest of this paper is organized as follows. In Section 2, the cognitive MIMO signal model is described and the underwater channel model is presented. In Section 3, we review the BCRB-based sequential beamforming. In Section 4, the performance of the adaptive algorithm is evaluated via simulations in the presence of environmental uncertainties. Our conclusion appears in Section 5.

2. THE SIGNAL MODEL AND SHALLOW UNDERWATER CHANNEL MODEL FORMULATION

2.1. Cognitive MIMO signal model

Consider a narrowband signal transmission and a static target scenario. The following general data model describes a colocated MIMO system of N_T transmitters and N_R receivers:

$$\mathbf{x}_k[l] = \mathbf{H}(\boldsymbol{\Theta})\,\mathbf{s}_k[l] + \mathbf{n}_k[l] \quad l = 1,\dots,L,\ \ k = 1,2,\dots \tag{1}$$

where $\mathbf{x}_k[l] \in \mathbb{C}^{N_R}$, $\mathbf{s}_k[l] \in \mathbb{C}^{N_T}$, and $\mathbf{n}_k[l] \in \mathbb{C}^{N_R}$ denote the lth snapshot of the observation, the transmit signal, and the noise vectors, respectively, at the kth pulse step. L is the number of total snapshots in each pulse step. $\mathbf{H}(\boldsymbol{\Theta}) \in \mathbb{C}^{N_R \times N_T}$ is the MIMO channel matrix, dependent of the unknown parameter vector $\boldsymbol{\Theta}$, which may consist of target location parameters, target complex attenuation, and unknown environmental parameters. $\boldsymbol{\Theta} \in \mathbb{R}^{Q_T}$ is assumed to be a random vector, with *a-priori* probability density function (pdf), $f_{\boldsymbol{\Theta}}(\cdot)$.

Equation (1) can be rewritten in a matrix form as follows:

$$\mathbf{X}_k = \mathbf{H}(\boldsymbol{\Theta})\,\mathbf{S}_k + \mathbf{N}_k, \ \ k = 1,2,\dots \tag{2}$$

where $\mathbf{X}_k = [\mathbf{x}_k[1],\dots,\mathbf{x}_k[L]]$, $\mathbf{S}_k = [\mathbf{s}_k[1],\dots,\mathbf{s}_k[L]]$, and $\mathbf{N}_k = [\mathbf{n}_k[1],\dots,\mathbf{n}_k[L]]$. We assume that the columns of $\mathbf{N}_k$ are independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian random vectors with zero mean and known covariance matrix, $\mathbf{R}$.

We are interested in the optimal beamforming of the transmit signal matrix at the kth step, $\mathbf{S}_k$, given observations from previous steps, denoted by $\mathbf{X}^{(k-1)} \triangleq [\mathbf{X}_1,\dots,\mathbf{X}_{k-1}]$. The optimization criterion is the BCRB on the estimation performance of the target unknown parameters, in the presence of environmental uncertainties. Fig. 1 describes the cognitive system for sequential beamforming.

2.2. Shallow underwater channel model formulation

In this work, we describe an active MIMO sonar system with colocated transmit and receive arrays. Consider a shallow underwater waveguide channel, in which the propagation model can be described by normal-modes [9, 11, 13, 14]. A point target is located in the waveguide at depth z_0 and range r_0 from vertical arrays of omnidirectional transmit and receive elements. The target is assumed to be in the far-field of the arrays. The array radiates a narrowband signal. Denote the target location vector by $[z_0, r_0]^T$ and its complex attenuation factor by α. Assume the signal model in (2), with the channel matrix given by $\mathbf{H}(\boldsymbol{\Theta}) \triangleq \alpha \mathbf{a}_R \mathbf{a}_T^T$. The transmit and receive steering vectors are given by $\mathbf{a}_T = \mathbf{T}_T \mathbf{q}(z_0, r_0)$ and $\mathbf{a}_R = \mathbf{T}_R \mathbf{q}(z_0, r_0)$, respectively. The elements of the matrices $\mathbf{T}_T \in \mathbb{C}^{N_T \times M}$ and $\mathbf{T}_R \in \mathbb{C}^{N_R \times M}$ are given by $[\mathbf{T}_T]_{im} = \phi_m(z_{T_i})$ and $[\mathbf{T}_R]_{im} = \phi_m(z_{R_i})$, respectively, and M denotes the number of propagating modes. The function $\phi_m(\cdot)$ is the mth modal depth eigenfunction and the terms z_{T_i} and z_{R_i} are the depths of the ith element of the transmit and receive arrays, respectively. The mth element of the vector $\mathbf{q}(z_0, r_0) \in \mathbb{C}^{M \times 1}$ is given by $[\mathbf{q}(z_0, r_0)]_m = \phi_m(z_0) \frac{e^{j\kappa_m r_0}}{\sqrt{\kappa_m r_0}}$, where κ_m is the horizontal wavenumber of mode m.

Define the entire unknown parameter vector as $\boldsymbol{\Theta} \triangleq \left[\boldsymbol{\theta}^T, \boldsymbol{\psi}^T\right]^T$, where $\boldsymbol{\theta} \in \mathbb{R}^{Q_1}$ and $\boldsymbol{\psi} \in \mathbb{R}^{Q_2}$ represent the target and environmental parameter vectors, respectively. The target unknown random vector is defined as $\boldsymbol{\theta} \triangleq [\mathrm{Re}(\alpha), \mathrm{Im}(\alpha), z_0, r_0]^T$. Environmental parameters in an underwater waveguide may consist of the sound velocity c, the sensors locations $\{z_{T_i}\}_{i=1}^{N_T}$ and $\{z_{R_i}\}_{i=1}^{N_R}$, the channel depth D, and other possible parameters. It is implicit that $\mathbf{T}_T$, $\mathbf{T}_R$ and $\mathbf{q}$ are dependent of the environmental parameters, where $\mathbf{q}$ depends also on the target location parameters.

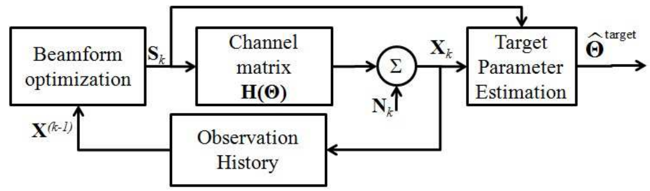

Fig. 1. Cognitive system scheme for sequential beamforming.

3. REVIEW OF THE BCRB-BASED SEQUENTIAL BEAMFORMING

3.1. Derivation of the objective function for optimal sequential beamforming in the presence of environmental uncertainties

In this section, we review the adaptive algorithm for transmit beamforming derived in [7], and extend it to the case of environmental uncertainties. We consider the environmental uncertainties as additional random parameters, as in [10]. The transmitted signal at each step is constrained by the total power, i.e. $\mathrm{tr}(\mathbf{R}_{\mathbf{s}_k}) \leq P$ where P is the average power limit for the transmitted signal vector at each snapshot, tr $(\cdot)$ is the matrix trace operator and $\mathbf{R}_{\mathbf{s}_k}$ is the transmit auto-correlation matrix defined as $\mathbf{R}_{\mathbf{s}_k} \triangleq \frac{1}{L}\mathbf{S}_k\mathbf{S}_k^H$. In this algorithm, at the kth step, $\mathbf{R}_{\mathbf{s}_k}$ is determined based on previous observations $\mathbf{X}^{(k-1)} \triangleq [\mathbf{X}_1, \ldots, \mathbf{X}_{k-1}]$. The BCRB for estimating $\boldsymbol{\theta}$ given $\mathbf{X}^{(k-1)}$ is considered as a criterion for optimization. We will choose the objective function as follows:

$$\hat{\mathbf{R}}_{\mathbf{s}_k} = \text{argmin}_{\mathbf{R}_{\mathbf{s}_k}} \text{tr}\left(\mathbf{W}\mathbf{C}_k^{(\text{BCRB})}(\boldsymbol{\theta})\right) \quad \text{s.t. } \text{tr}(\mathbf{R}_{\mathbf{s}_k}) \leq P, \;\; \mathbf{R}_{\mathbf{s}_k} \succeq 0 \tag{3}$$

where $\mathbf{C}_k^{(\text{BCRB})}(\boldsymbol{\theta})$ is the BCRB for estimating the target parameter vector, $\boldsymbol{\theta}$, at the kth pulse step, and $\mathbf{W} = \text{diag}(w_1, \ldots, w_{Q_1})$ is a weighting matrix. In [17] it was shown that (3) can be transformed into the following SDP optimization problem:

$$\begin{aligned} \hat{\mathbf{R}}_{\mathbf{s}_k} &= \text{argmin}_{\mathbf{R}_{\mathbf{s}_k}, \{t_i\}_{i=1}^{Q_1}} \sum_{i=1}^{Q_1} w_i t_i \\ &\text{s.t.} \\ &\begin{bmatrix} \triangle\mathbf{J}_{D_k} + \mathbf{J}_{P_{k-1}} & \mathbf{e}_i \\ \mathbf{e}_i^T & t_i \end{bmatrix} \succeq 0, \;\; i = 1, \ldots, Q_1 \\ &\text{tr}(\mathbf{R}_{\mathbf{s}_k}) \leq P, \;\; \mathbf{R}_{\mathbf{s}_k} \succeq 0 \end{aligned} \tag{4}$$

where the vector $\mathbf{e}_i$ is the ith column of the identity matrix of size Q_T, $\{t_i\}_{i=1}^{Q_1}$ are auxiliary variables, and $Q_T = Q_1 + Q_2$. Let $\triangle\mathbf{J}_{D_k} \in \mathbb{R}^{Q_T \times Q_T}$ and $\mathbf{J}_{P_{k-1}} \in \mathbb{R}^{Q_T \times Q_T}$ be the Bayesian Fisher information matrices (BFIM) for estimating the entire unknown parameter vector $\boldsymbol{\Theta}$ from the observations $\mathbf{X}^{(k-1)}$. The term $\triangle\mathbf{J}_{D_k}$ represents the incremental BFIM, which is linearly dependent of $\mathbf{R}_{\mathbf{s}_k}$. The term $\mathbf{J}_{P_{k-1}}$ represents the posterior BFIM from previous observations, which is independent of $\mathbf{s}_k$. In [7] it was shown that

$$\triangle\mathbf{J}_{D_k} = 2L\text{Re}\left\{\mathbf{Q}_I\left(\boldsymbol{\Gamma}\left(\mathbf{X}^{(k-1)}\right) \odot \left(\mathbf{1}_{Q_T \times Q_T} \otimes \mathbf{R}_{s_k}^T\right)\right)\mathbf{Q}_I^T\right\} \tag{5}$$

where $\boldsymbol{\Gamma}\left(\mathbf{X}^{(k-1)}\right) = \text{E}\left[\frac{d\mathbf{H}}{d\boldsymbol{\Theta}}^H \mathbf{R}^{-1}\frac{d\mathbf{H}}{d\boldsymbol{\Theta}}\middle|\mathbf{X}^{(k-1)}\right]$, $\mathbf{1}_{Q_T \times Q_T}$ is a $Q_T \times Q_T$ matrix whose entries are equal to one, $\mathbf{Q}_I \triangleq \mathbf{I}_{Q_T} \otimes \mathbf{1}_{1 \times N_T}$, and $\frac{d\mathbf{H}}{d\boldsymbol{\Theta}} \triangleq \left[\frac{\partial\mathbf{H}}{\partial\Theta_1}, \ldots, \frac{\partial\mathbf{H}}{\partial\Theta_{Q_T}}\right]$. The operators $\text{Re}\{\cdot\}$, $\odot$, and $\otimes$ are the real part operator, Hadamard product, and Kronecker product, respectively. The term $\mathbf{J}_{P_{k-1}}$ is given by

$$\mathbf{J}_{P_{k-1}} = \mathbf{J}_{P_0} + \mathbf{J}_{N_{k-1}} + 2L\sum_{m=1}^{k-1}\text{Re}\left\{\mathbf{Q}_I\left(\boldsymbol{\Gamma}\left(\mathbf{X}^{(k-1)}\right) \odot \left(\mathbf{1}_{Q_T \times Q_T} \otimes \mathbf{R}_{\mathbf{s}_m}^T\right)\right)\mathbf{Q}_I^T\right\} \tag{6}$$

where $\mathbf{J}_{P_0}$ and $\mathbf{J}_{N_{k-1}}$ are given by

$$[\mathbf{J}_{P_0}]_{i,j} = -\text{E}\left[\frac{\partial^2 \log f_{\boldsymbol{\Theta}}}{\partial\Theta_i \partial\Theta_j}\right] \tag{7}$$

$$[\mathbf{J}_{N_{k-1}}]_{i,j} = -2\sum_{m=1}^{k-1}\text{Re} \left\{\text{E}\left[\text{tr}\left(\left(\mathbf{X}_m - \mathbf{H}\mathbf{S}_m\right)^H \mathbf{R}^{-1}\frac{\partial^2\mathbf{H}}{\partial\Theta_i\partial\Theta_j}\mathbf{S}_m\right)\middle|\mathbf{X}^{(k-1)}\right]\right\} \tag{8}$$

The appropriate expectations in (5) and (6) are performed w.r.t. the posterior pdf of the entire parameter vector $\boldsymbol{\Theta}$ given previous observations, denoted by $f_{\boldsymbol{\Theta}|\mathbf{X}^{(k-1)}}$. This formulation allows minimizing the trace of the weighted BCRB of target parameter vector $\boldsymbol{\theta}$ only, while using the entire information in the posterior pdf $f_{\boldsymbol{\Theta}|\mathbf{X}^{(k-1)}}$.

3.2. Adaptive algorithm review

The computation of $\triangle\mathbf{J}_{D_k}$ and $\mathbf{J}_{P_{k-1}}$ in (5) and (6) involve performing expectations w.r.t. the posterior pdf $f_{\boldsymbol{\Theta}|\mathbf{X}^{(k-1)}}$, which can be obtained sequentially at each pulse step. The adaptive beamforming algorithm solves the SDP problem in (4) at each iteration and obtains an optimal auto-correlation matrix $\hat{\mathbf{R}}_{\mathbf{s}_k}$ based on the information from $\mathbf{X}^{(k-1)}$. This information is embedded in the posterior pdf $f_{\boldsymbol{\Theta}|\mathbf{X}^{(k-1)}}$. The sequential derivation of $f_{\boldsymbol{\Theta}|\mathbf{X}^{(k-1)}}$ and the adaptive beamforming algorithm are described in [7]. In the simulations, we will apply the adaptive beamforming algorithm in order to solve the extended problem of optimal beamforming for target localization in the presence of environmental uncertainties.

4. RESULTS

In this section, we evaluate the performance of the adaptive beamforming technique described above for different cases of environmental uncertainties. Consider a time-invariant homogeneous waveguide, as considered in [10], with constant sound speed $c = 1500$m/s and depth $D_0 = 105\,$m. We use a uniform linear array (ULA) of $N_T = N_R = N = 7$ transceivers. The elements of the arrays are equally spaced across the channel depth. The transmit array radiates a narrowband signal centered at frequency $f = 50\,$Hz. Consider a single point target that is located at $[z_0, r_0]$, with a complex attenuation factor α. The mth modal eigenfunction is given by $\phi_m(z) = \sqrt{\frac{2}{D}}\sin(\gamma_m z)$ where $\kappa_m = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \gamma_m^2}$ and $\gamma_m = \left(m - \frac{1}{2}\right)\frac{\pi}{D}$.

In the simulations, we consider uniform *a-priori* distribution of the unknown parameters. The regularity conditions of the BCRB are not satisfied for compact support distributions. The problem re-occurs in each pulse step of the adaptive algorithm. Therefore, we artificially assume that $\mathbf{J}_{P_0}$ is constant within the *a-priori* boundaries of the unknown parameters. Assume uniform *a-priori* distribution for the unknown target location parameters as $z_0 \sim U\left[0, 105\,\mathrm{m}\right]$, $r_0 \sim U\left[1150\,\mathrm{m}, \;\; 1350\,\mathrm{m}\right]$. The unknown environmental parameter is the channel depth D_0, which is uniformly distributed $D_0 \sim U\left[105\,\mathrm{m} - \triangle, \;\; 105\,\mathrm{m} + \triangle\right]$, where $\triangle$ represents the maximum deviation from the true value of the channel depth.

In the simulations, assume a mismatch between the true value of D_0 and its *a-priori* uncertainty. The entire unknown vector parameter is $\mathbf{\Theta} = \left[z_0, r_0, D_0\right]^T$.

Consider the signal model in (2), where the additive Gaussian noise matrix $\mathbf{N}_k$ is randomly generated at each step of each trial. The noise covariance matrix is denoted by $\mathbf{R} = \sigma^2 \mathbf{I}_N$. The number of snapshots in each pulse step is $L = 10$. The total SNR is defined as $SNR \triangleq \frac{P|\alpha|^2}{\sigma^2} \cdot \frac{\|\mathbf{Tq}(z_0, r_0)\|^4}{N^2}$. In the simulations the number of propagating modes M remains constant for all cases of channel depth uncertainty. Consider the following definition of the 2-dimensional transmit beampattern, which describes the transmitted energy distribution over the $[z, r]$ plane at the true channel depth D_0 :

$$Q\left(z, r\right) \triangleq \frac{\mathbf{q}^H\left(z, r\right) \mathbf{T}^H \mathbf{R}_{s_k} \mathbf{T} \mathbf{q}\left(z, r\right)}{\left\|\mathbf{T} \mathbf{q}\left(z, r\right)\right\|^2} \qquad (9)$$

In Fig. 2 two cases of channel depth uncertainties are tested: the case of high uncertainty $\triangle = 5$ m and the case of low uncertainty $\triangle = 0.1\,\mathrm{m}$. The simulation was performed for 300 trials. In each trial the true value of the target parameter vector $\left[z_0, r_0\right]^T$ was independently and uniformly generated according to boundaries mentioned above. We compare the root mean squared error (RMSE) of the optimal beamforming MMSE estimator for target localization and the square root of the BCRB vs. pulse step, for $SNR = -15\,\mathrm{dB}$ and $SNR = -10\,\mathrm{dB}$. The MMSE estimator of $[z_0, r_0]$ at the $(k-1)$th step was obtained via the posterior pdf of the entire parameter vector $f_{\mathbf{\Theta}|\mathbf{X}^{(k-1)}}$, which is available at step k. It is evident that the performance of the estimator is considerably better for smaller uncertainty in the channel depth. However, little difference is apparent between the lower bounds, with a slight advantage to the case of smaller uncertainty. Additionally, the BCRB poorly describes the estimation performance even in the case of $SNR = -10\,\mathrm{dB}$. The difference of the RMSE performance between the two cases of channel depth uncertainty can be explained by the high level of the sidelobes in the ambiguity function in an underwater environment, which dominates in the case of high uncertainty of the channel depth as the BCRB criterion ignores large errors with high probability.

Fig. 3 illustrates the sequential beamforming in an underwater waveguide. An example of the posterior pdf of the target parameters and the resulting optimal transmitted beampattern in various pulse steps, was derived for a target located at $[z_0, r_0] = [25\,\mathrm{m}, 1300\,\mathrm{m}]$, $SNR = -15\,\mathrm{dB}$, and channel depth uncertainty of $\triangle = 5\,m$. The posterior pdf has various high peak levels distributed across the $[z, r]$ plane in early steps and after a few more steps the peak converges around the true location of the target. The high sidelobe ambiguity in the transmit beampattern, combined with high peak levels in the posterior pdfs results in large estimation error with high probability. This is compatible to the RMSE performance shown in Fig. 2.

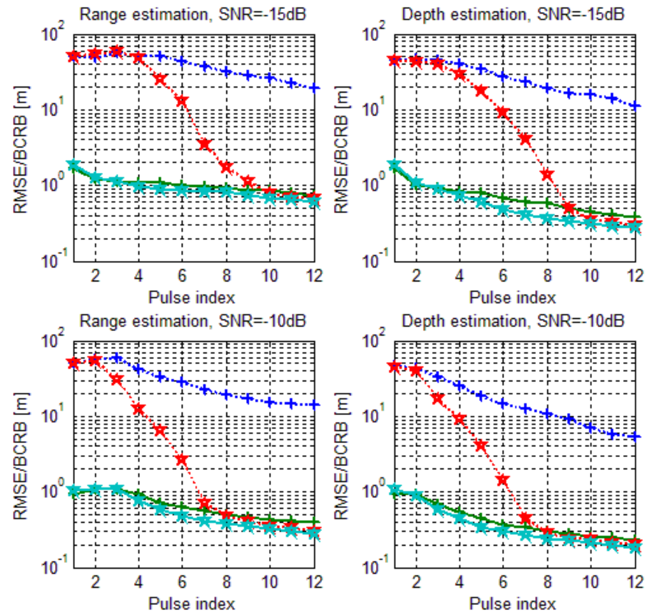

Fig. 2. Performance of RMSE (dotted) and BCRB (solid) vs. pulse step. A comparison between uncertainty of $\triangle = 5\,\mathrm{m}$ ('plus' sign) and uncertainty of $\triangle = 0.1\,\mathrm{m}$ ('star' sign).

5. CONCLUSION

In this paper, we extended the adaptive beamforming approach introduced in [7] for the case of shallow underwater channel model in the presence of environmental uncertainties. The BCRB was chosen as the criterion for optimization. It was shown that the ambiguity dominates the localization performance. The ambiguity is strongly influenced by environmental uncertainties.

Further research can focus on the analysis of high sidelobe environment (as in underwater channels) with minimization of the RMB which accounts for large errors due to high sidelobes. Additionally, possible research can focus on analysis of dynamic targets. In cases where the target dynamics obey the Markovian model, the adaptive algorithm can be extended to accommodate the target dynamics and use tracking algorithms in order to improve the adaptive beamforming algorithm.

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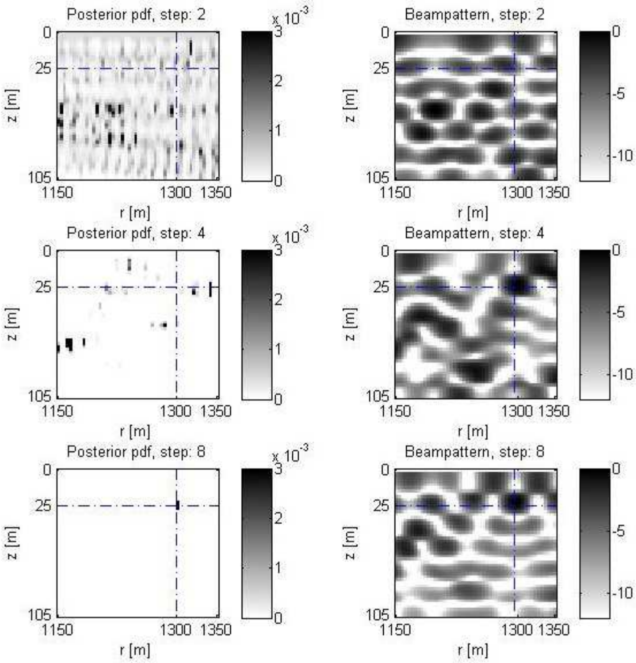

Fig. 3. Posterior pdfs (left column) and the resulting optimal transmit beampattern in [dB] (right column) against the $[z, r]$ plane, for different pulse steps. The target is located at $[z_0, r_0] = [25\,\text{m}, 1300\,\text{m}]$, $SNR = -15\,\text{dB}$, and $\triangle = 5\,\text{m}$.

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