3-D ARRAY CONFIGURATION USING MULTIPLE REGULAR TETRAHEDRA FOR HIGH-RESOLUTION 2-D DOA ESTIMATION

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ABSTRACT

This paper presents a novel 3-D array configuration using multiple regular tetrahedra which enables high resolution 2-D DOA estimation. The proposed array configuration has better DOA estimation performance as that of the conventional 3-D array configuration for uncorrelated waves, and can be rearranged into the cuboid array configuration which can estimate DOAs of correlated waves. Performance of the proposed 3-D array configuration is evaluated through a computer simulation.

Index Terms— direction of arrival estimation, array antenna, array signal processing

1. INTRODUCTION

Mobile communication environments are often recognized as the combination of many small cells like micro- or femtocells [1]-[4]. In such an environment, one base station covers smaller areas and supports a smaller number of mobile terminals in comparison with the old system in order to achieve higher wireless communication speed. Then the elevation angle from mobile terminals to a base station becomes relatively larger than the conventional situation because of the small cell radius. Therefore, the conventional cell design in the 2-D plane has now been replaced by a new cell design in 3-D space, and 2-D Direction-Of-Arrival (DOA) estimation (for both azimuth and elevation angles) has consequently become more important.

Planar (2-D) antenna arrays like the Uniform Rectangular Array (URA) or Uniform Circular Array (UCA) are often employed for 2-D DOA estimation. Planar arrays can often estimate azimuth angles well but cannot estimate elevation angles accurately because of the small antenna aperture in the elevation direction. In order to improve elevation angle estimation accuracy, one may put more number of array elements, or develop a three-dimensional (3-D) array structure by putting multiple planar arrays in vertical direction to make long antenna aperture in elevation direction. [5]–[7]. However they often require very large hardware and computational cost.

The authors have recently proposed a simple way of developing a 3-D array configuration not by increasing the number of array elements but by simply modifying the height of some array elements in the planar array [8]. Also the authors have proposed a cuboid array configuration [9] which can estimate DOAs of uncorrelated waves more accurately, and estimate DOAs of correlated or coherent waves using Spatial Smoothing Preprocessing (SSP). The detailed array configurations in [8],[9] are determined so that their Cramer-Rao Lower Bound (CRLB) [10] becomes as small as possible.

Here we recall that triangular array configurations typically have good antenna characteristics [11],[12]. An example 3-D extension of triangular array configurations will be the regular tetrahedron configuration. Therefore the 3-D array configuration using regular tetrahedron may lead better DOA estimation accuracy.

This paper presents a novel 3-D array configuration using multiple regular tetrahedra which has almost the same or better DOA estimation accuracy with that of the conventional 3-D array configuration [8],[9]. The proposed configuration includes the cuboid array configuration by arranging two regular tetrahedra with same barycenter, therefore that we can estimate DOAs of correlated waves. Performance of the proposed array configurations is evaluated through a computer simulation of DOA estimation.

2. PRELIMINARIES

This section briefly summarizes the formulation and the minimization methods for CRLB. Assume that K waves from far-field are received by P-elements array antenna under an Additive White Gaussian Noise (AWGN) environment, and the corresponding DOAs are set up as in Fig. 1.

Let $\boldsymbol{s}_k(t)$ and (θ_k, ϕ_k) for $k = 1, 2, 3, ..., K$ denote the k-th incident signal and the corresponding DOA, respectively. The array input vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k, \phi_k)\boldsymbol{s}_k(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{a}(\theta_k, \phi_k) = [a_1(\theta_k, \phi_k), ..., a_P(\theta_k, \phi_k)]^T$ is the steering vector of k-th incident wave, and $\boldsymbol{n}(t) = [n_1(t), ..., n_P(t)]^T$ is the Gaussian noise of zero-mean and the variance σ^2. Then the CRLB matrix $\boldsymbol{V}$ can be formulated as

$$\begin{aligned} \boldsymbol{V} &= \begin{bmatrix} \mathrm{var}(\omega_1) & & 0 \\ & \ddots & \\ 0 & & \mathrm{var}(\omega_K) \end{bmatrix} \\ &= \frac{\sigma^2}{2N_s}\mathrm{Re}\left[(\boldsymbol{B}^H\boldsymbol{P}_N\boldsymbol{B}) \odot \boldsymbol{S}^T\right]^{-1} \quad (2) \\ \boldsymbol{S} &= E\left[\boldsymbol{s}(t)\boldsymbol{s}(t)^H\right] \quad (3) \\ \boldsymbol{P}_N &= \boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H \quad (4) \\ \boldsymbol{A} &= [\boldsymbol{a}(\theta_1, \phi_1), \boldsymbol{a}(\theta_2, \phi_2), ..., \boldsymbol{a}(\theta_K, \phi_K)] \quad (5) \\ \boldsymbol{s}(t) &= [s_1(t), s_2(t), ..., s_K(t)]^T \quad (6) \\ \boldsymbol{B} &= [\boldsymbol{b}(\theta_1), \boldsymbol{b}(\theta_2), \ldots, \boldsymbol{b}(\theta_K)] \quad (7) \\ \boldsymbol{b}(\omega) &= \frac{\partial \boldsymbol{a}}{\partial \omega} \quad (8) \end{aligned}$$

where $\odot$, $\mathrm{Re}[\cdot]$, N_s, $E[\cdot]$ and $\boldsymbol{I}$ respectively denote Hadamard product, real part, the number of snapshots, the ensemble average and the identity matrix. The parameter ω in (8) can be either θ or ϕ.

In the case of one wave source, the expression of CRLB in (2) can be reduced to:

$$\mathrm{var}(\omega) = \frac{\sigma^2}{2N_sP_s} \cdot \frac{1}{AR} \cdot \left(\frac{\lambda}{2\pi}\right)^2 \quad (9)$$

where P_s is the signal power, and the term AR is a real scalar which depends only on array configuration. This means that minimizing CRLB is equivalent to maximizing the value of AR. Indeed, AR for elevation θ and that for azimuth ϕ of 9 ($= 3 \times 3$) elements URA are respectively written as follows.

$$AR_\theta = 6(d_x\cos\theta\cos\phi)^2 + 6(d_y\cos\theta\sin\phi)^2 \quad (10)$$

$$AR_\phi = 6(d_x\sin\theta\cos\phi)^2 + 6(d_y\sin\theta\sin\phi)^2 \quad (11)$$

3. CONVENTIONAL URA-BASED 3-D ARRAY CONFIGURATION

The authors have already proposed a URA-based 3-D array configuration [8] as in Fig. 2(a). This array configuration is obtained by simply modifying the height of some array elements in URA. We have also confirmed that the estimation accuracy of the array configuration in Fig. 2 becomes higher than that of URA [8].

Besides, the authors have proposed a cuboid array configuration [9] as in Fig. 2(b), which can estimate DOAs of uncorrelated waves more accurately, and estimate DOAs of correlated or coherent waves using SSP. The parameter AR of the configuration in Fig. 2(b) in terms of elevation θ and

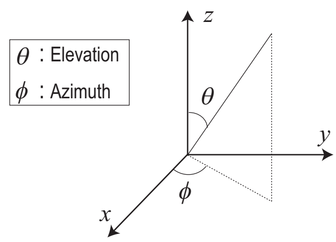


Fig. 1. Azimuth and elevation angles setup.

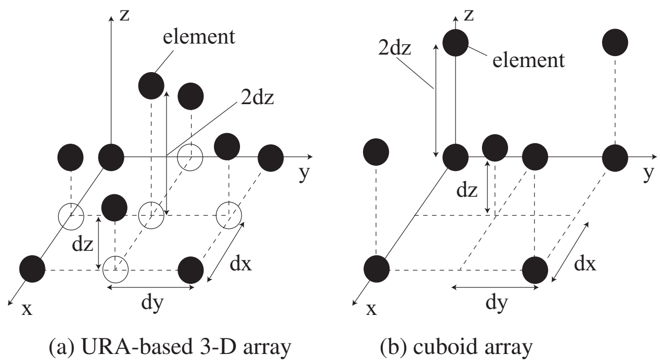


Fig. 2. Example configurations of (a) URA-based 3-D array [8], and (b) cuboid array [9] of 9 elements.

azimuth ϕ can be written as follows.

$$\begin{aligned} AR_{\theta\_cuboid} &= 8(d_x\cos\theta\cos\phi)^2 + 8(d_y\cos\theta\sin\phi)^2 \\ &\quad + 8(d_z\sin\theta)^2 \quad (12) \\ AR_{\phi\_cuboid} &= 8(d_x\sin\theta\cos\phi)^2 + 8(d_y\sin\theta\sin\phi)^2 \quad (13) \end{aligned}$$

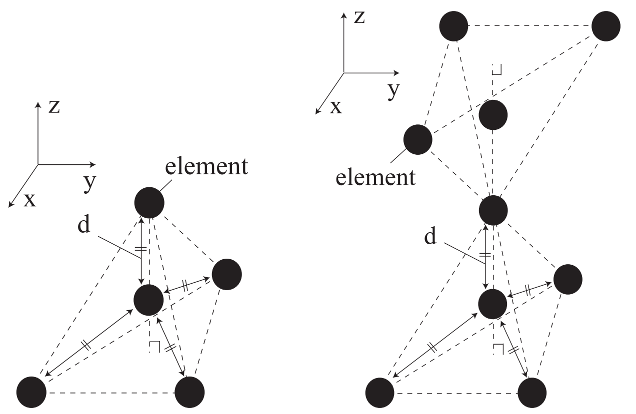
Comparing (12) and (13) with (10) and (11), we see that the parameter AR of the cuboid configuration in Fig. 2(b) becomes always larger than that of URA for both elevation θ and azimuth ϕ, which consequently leads to smaller CRLB.

4. 3-D ARRAY CONFIGURATION USING MULTIPLE REGULAR TETRAHEDRA

In this section, we propose a novel 3-D array configuration using multiple regular tetrahedra.

The array configuration with big array aperture is often desired in order to realize a good DOA estimation accuracy. Therefore we propose a 3-D array configuration formed by combining multiple regular tetrahedra along z-axis, to improve the estimation accuracy of elevation angles.

For simplicity, we here consider the case of combining two regular tetrahedra. A tetrahedron has four apexes; we can form the 3-D array configuration which has five elements by locating the antenna elements at each apex and their barycenter as in Fig. 3(a). Then combine two (or more) of the configuration in Fig. 3(a) so that two tetrahedra are overlapped



(a) a regular tetrahedron (b) two regular tetrahedra

Fig. 3. Configuration of regular tetrahedra array antenna

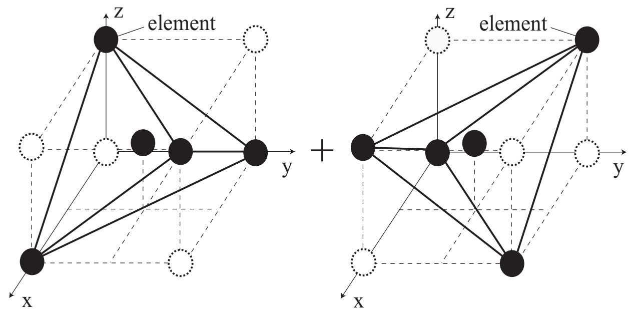


Fig. 4. The cuboid array configuration formed by two regular tetrahedra

at their end-points, we can form a novel 3-D array configuration which has 9 (or more) elements in total. For example, by putting two regular tetrahedra along the z-axis, one is placed with its upside down, then we can arrange the 3-D configuration as in Fig. 3(b). This configuration has bigger array aperture along z-axis than the conventional 2-D and 3-D array configurations with the same number of array elements, therefore it is expected to improve the elevation estimation accuracy. Also this configuration has the biggest array aperture along z-axis of all the array configurations which are formed by combining two regular tetrahedra. The parameter AR of the configuration in Fig. 3(b) in terms of elevation θ and azimuth ϕ can be written as follows.

$$AR_{\theta\_proposed} = 8(d_x \cos\theta \cos\phi)^2 + 8(d_y \cos\theta \sin\phi)^2 + 38(d_z \sin\theta)^2 \quad (14)$$

$$AR_{\phi\_proposed} = 8(d_x \sin\theta \cos\phi)^2 + 8(d_y \sin\theta \sin\phi)^2 \quad (15)$$

where $d_x = d_y = d_z = d/\sqrt{3}$ so as to be compared with the cuboid array in Fig. 2(b). We see that the parameters AR of the proposed array in (14) and (15) look almost the same with that of the cuboid-array in (12) and (13) at a glance, but the coefficients of the term $(d_z \sin\theta)^2$ in AR_θ are very different. This will lead larger AR_θ and smaller CRLB.

In addition to this configuration, we can consider another array configuration which is formed by combining at another position of elements. By combining two regular tetrahedra with the same barycenter as in Fig. 4, we can arrange the cuboid array configuration. Therefore the cuboid array can be considered as a particular case of multiple regular tetrahedra array where two regular tetrahedra are overlapped with the same barycenter. Even the cuboid configuration has smaller array aperture along the z-axis than the configuration of Fig. 3(b), it can estimate DOAs of correlated sources by using SSP as discussed in [9].

Table 1. Specifications of simulation

	Fig. 6	Fig. 7
array configuration	as in Fig. 3(b)	
interelement spacing	0.5λ	
# of incident signals	1	2
correlation	–	uncorrelated
# of snapshots	200	
SNR	0dB	0 to 20dB
DOA, (θ, ϕ)	θ : 10° to 80° ϕ : fixed to 0°	$(20^\circ, -100^\circ)$ $(60^\circ, 120^\circ)$
DOA estimation algorithm	MUSIC method [13]	
# of trials	1,000	

5. SIMULATION

We evaluate the DOA estimation accuracy of the proposed 3-D array configuration in this section. First we discuss the maximum interelement spacing that can preserve high DOA estimation accuracy. Then we evaluate the DOA estimation accuracy in terms of the angular dependency in the case of one wave source, and the accuracy in terms of the SNR dependency in the case of multiple uncorrelated waves. The specifications of the simulation are listed in Table 1.

We consider the multiple tetrahedra array configuration of Fig. 3(b), which has the biggest array aperture along the z-axis of all the array configurations which are formed by combining two regular tetrahedra.

5.1. Interelement spacing

DOA estimation accuracy generally becomes higher as the antenna aperture becomes larger. However, an excessively long interelement spacing will cause grating-lobes (pseudo peaks in the MUSIC spectrum) which degrade DOA estimation accuracy. Therefore we first evaluate the behavior of Root Mean Square Error (RMSE) of the DOA estimation for various interelement spacing values to find out the maximum possible spacing length which still preserves high DOA estimation ac-

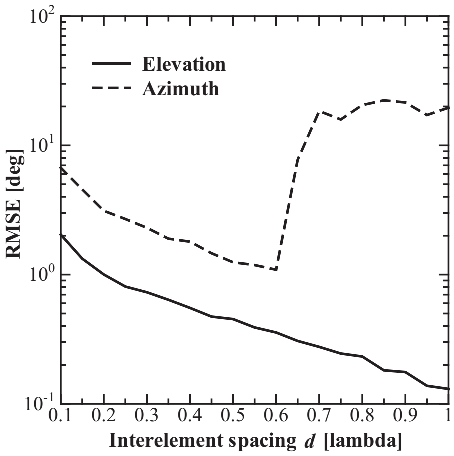


Fig. 5. Behavior of RMSE for various values of the interelement spacing d in regular tetrahedron array of Fig. 3(b)

curacy. Figure 5 shows the behavior of RMSE where the interelement spacing d in regular tetrahedron array of Fig. 3(b) is changed from 0.1λ to λ. All the other parameters are set to be the same as in Table 1. We see from Fig. 5 that the DOA estimation accuracy becomes worse when d is greater than 0.6λ. This is because the grating lobes arise when the distance between two adjacent elements exceeds a half wavelength, therefore it has the same characteristics with that of the conventional planar array.

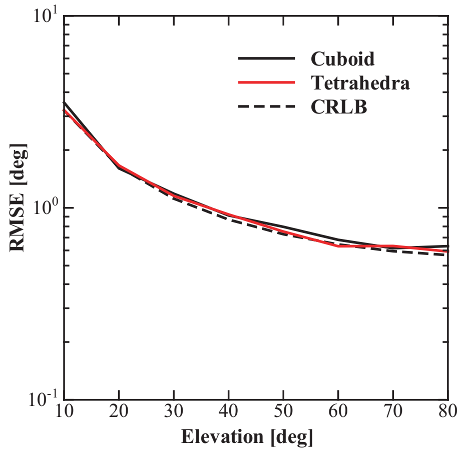
5.2. Angular Dependency

Hereafter the distance between two adjacent elements d is $d = 0.5\lambda$, and we compare it with the cuboid array configuration [9] with $d_x = d_y = 2d_z = \frac{0.5\lambda}{\sqrt{3}}$ to satisfy the equivalent condition.

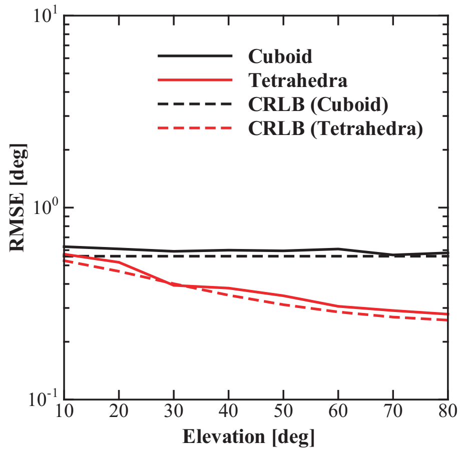
Figure 6 shows the behavior of RMSE for the case of changing the elevation angle θ from $10°$ to $80°$, while the azimuth is fixed to $\phi = 0°$. We see from Fig. 6(a) that the DOA estimation accuracy of azimuth angles is almost equivalent (their CRLBs are completely the same). Also we can observe from Fig. 6(b) that the DOA estimation accuracy of elevation angles is improved by the proposed array configuration, especially when the DOA is close to the horizontal plane (θ approaches to $90°$).

5.3. SNR Dependency

Then we evaluate the basic performance of the proposed tetrahedra array in the case of uncorrelated multiple incident signals. Figure 7 shows the behavior of RMSE in terms of the SNR characteristics in the case of two uncorrelated incident signals from $(20°, -100°)$ and $(60°, 120°)$. We see from Fig. 7 that the DOA estimation accuracy of azimuth angles of both



(a) Behavior of RMSE of azimuth ϕ



(b) Behavior of RMSE of elevation θ

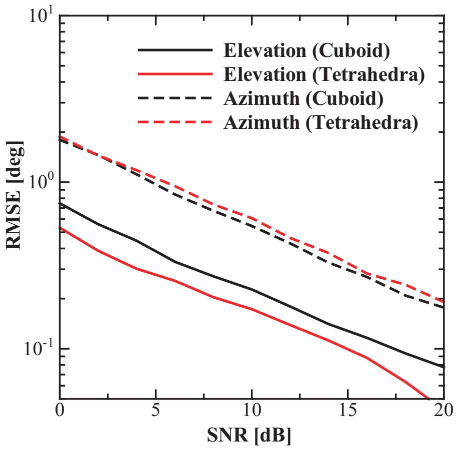
Fig. 6. Behavior of RMSE for various values of elevation changed from $10°$ to $80°$, while the azimuth is fixed to $\phi = 0°$

the first and second waves is almost equivalent, but that of elevation angles of both waves is improved in comparison with the cuboid array. Moreover, we see from Fig. 7 that the elevation estimation accuracy of the second wave (close to horizontal level) is very much improved. Therefore we could show that the configuration of Fig. 3 had better DOA estimation characteristics for single and multiple uncorrelated sources than the cuboid configuration [9], and in particular it has much better characteristics when the DOAs are close to the horizontal plane (θ approaches to $90°$).

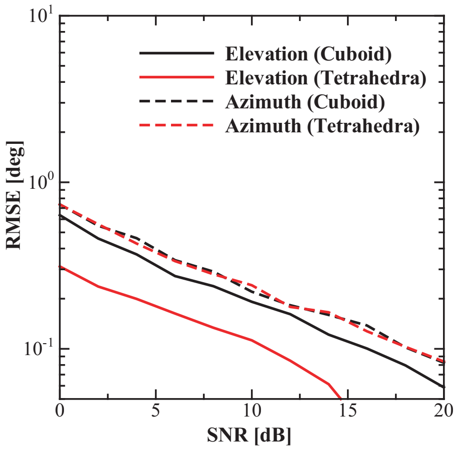
6. CONCLUDING REMARKS

This paper proposed a novel 3-D array configuration using multiple regular tetrahedra for accurate 2-D (azimuth and elevation) DOA estimation.

The proposed 3-D array configuration estimates DOAs of



(a) RMSE of 1st wave $(\theta_1, \phi_1) = (20°, -100°)$



(b) RMSE of 2nd wave $(\theta_2, \phi_2) = (60°, -120°)$

Fig. 7. Behavior of RMSE in the sense of SNR characteristics for the case of two uncorrelated incident signals from $(20°, -100°)$ and $(60°, 120°)$

uncorrelated waves as accurately as the conventional 3-D array configuration, and more accurately than the planar array configuration. Also the proposed configuration includes the cuboid array configuration by arranging two regular tetrahedra with same barycenter, means that the proposed configuration can be regarded as an extension of cuboid array that can estimate DOAs more accurately.

One may say that such 3-D cuboid array configurations cannot be feasible; however, the array elements of the proposed configuration can be installed using a spherical dipole antenna [14] which has an omni radiation pattern.

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