A FAST AND ACCURATE ADAPTIVE NOTCH FILTER USING A MONOTONICALLY INCREASING GRADIENT

Yosuke SUGIURA

Tokyo University of Science
Faculty of Science and Technology

ABSTRACT

In this paper, we propose a new adaptive notch filter algorithm to achieve the fast and accurate narrow-band noise reduction. In the proposed algorithm, we introduce the monotonically increasing function into the gradient, which provides the fast convergence far away from the optical frequency. We additionally introduce the enhancement function into the gradient to design the steepness of the gradient curve. The proposed gradient can adjust the trade off between the convergence speed and the estimation accuracy more flexibly. Several computational simulations show that the proposed algorithm can simultaneously provide fast convergence and high accurate estimation compared with the conventional NLMS algorithm.

Index Terms— Adaptive Notch Filter, Gradient Based Algorithm, Monotonically Increasing Gradient, Fast and Accurate Estimation

In speech and audio signal processing, biomedical signal processing, and many other signal processing fields, one of the most important issues is to remove an undesired narrow-band noise embedded in a wide-band signal. Examples of such noise are a background engine noise in voice communications, an acoustic feedback in hearing aids, a hum noise in electrocardiograms, and so on. In many practical situations, the noise frequency is unknown and often changes with time. In these situations, it is required to detect narrow-band the noise quickly and estimate the noise frequency fast and accurately.

An adaptive notch filter is one of the effective methods to remove the narrow-band noise [1–3]. The adaptive notch filter is composed by a second-order IIR filter, and its amplitude response has a steep rejection characteristic at the frequency of the zero (notch frequency). Thus, if we get the exact noise frequency, we can completely remove the narrow-band noise by adjusting the notch frequency to be equal to the noise frequency. For the estimation of the noise frequency, we especially use a Least Mean-Square (LMS) algorithm which is a simple gradient based algorithm.

In the gradient method, both the convergence speed and the estimation accuracy are the important properties which determine the performance of the algorithm. Note that we define the estimation accuracy as the ability to decrease the variance of the estimation error after convergence. Both the properties depend on the characteristics of the gradient curve. The high estimation accuracy is performed when the value of the gradient is small in the vicinity of the noise frequency, and the fast convergence speed is provided by the large absolute value in other frequency band. In the LMS algorithm, the absolute value of the gradient becomes large in the vicinity of the noise frequency and small in other frequency band. Thus, the LMS algorithm significantly degrades the convergence speed and the estimation accuracy, simultaneously.

To solve the problem, several methods have been presented. A typical one is a Normalized LMS (NLMS) algorithm [4]. Although the NLMS algorithm improves the characteristic of the gradient, the improvement is limited in the narrow frequency band nearby the noise frequency. There are other methods combining multiple notch filters which have different bandwidth parameters or different step-size parameters [5–8]. Although these methods provide the fast and accurate estimation, all the adaptations are derived by using the NLMS algorithm. To achieve the more fast and accurate estimation, it is required to fundamentally improve the characteristics of the gradient curve.

In this paper, we propose a new gradient based algorithm for the fast and accurate adaptive notch filter. In the proposed algorithm, the fast estimation is achieved by introducing a monotonically increasing gradient. Additionally, the high estimation accuracy is achieved by adjusting the gradient curve to be gentle in the vicinity of the noise frequency. Several computational simulations for removing sinusoidal signals shows that the proposed method can provide fast convergence and high accurate estimation, simultaneously.

1. REVIEW OF ADAPTIVE NOTCH FILTER AND CONVENTIONAL NLMS ALGORITHM

For the narrow-band noise reduction, we often use an adaptive notch filter, which has a steep rejection characteristic at the notch frequency. Figure 1 shows the structure of a second-order adaptive IIR notch filter [1, 3]. The notch filter can be realized using only three multipliers by sharing the multipliers. Here, the signal $x(n)$ is the input signal, $e(n)$ is the

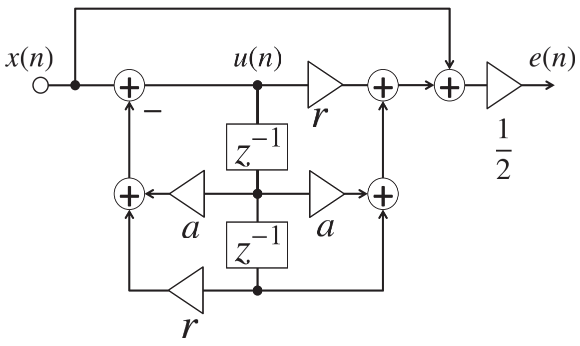

Fig. 1. Structure of an adaptive IIR notch filter.

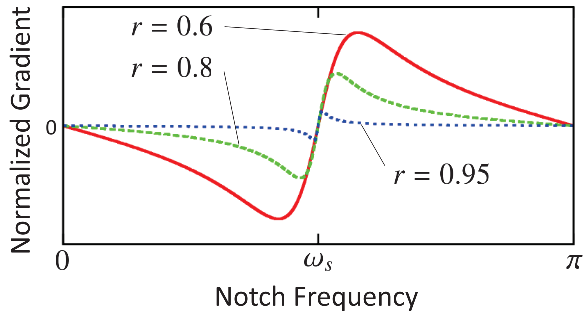

Fig. 2. Gradient curves for the NLMS algorithm with $\omega_s = \pi/2$ and $r = 0.6, 0.8, 0.95$.

output signal, and $u(n)$ is the signal described by $u(n) = x(n) - au(n-1) - ru(n-2)$. The input signal $x(n)$ is described by using a narrow-band noise $s(n)$ and an additive wide-band desired signal $w(n)$, i.e.,

$$x(n) = w(n) + s(n). \tag{1}$$

We assume that $w(n)$ is given by a white Gaussian signal with the mean 0 and the variance σ^2, and $s(n)$ is given by a sinusoidal signal with the frequency ω_s, the amplitude p, and the phase ϕ.

The transfer function of the adaptive notch filter is given by

$$H(z) = \frac{1}{2}\left(1 + \frac{r + az^{-1} + z^{-2}}{1 + az^{-1} + rz^{-2}}\right), \tag{2}$$

where r $(-1 < r < 1)$ is a parameter which controls the rejection bandwidth. Specifically, the bandwidth becomes narrow along with the increasing value of r toward 1. The parameter a determines the notch frequency ω_N. The relation between a and ω_N is given by

$$a = -(1 + r)\cos(\omega_N). \tag{3}$$

The adaptive notch filter can remove the noise by adjusting the notch frequency to be equal to the noise frequency.

An NLMS algorithm is one of the simple estimation algorithms in the notch filter [4]. The updating equation is given by

$$a(n+1) = a(n) - \mu\frac{E[e(n)u(n-1)]}{E[u^2(n-1)]}, \tag{4}$$

where μ is a step size parameter which satisfies $0 < \mu < 2$.

Both the estimation accuracy and the convergence speed depend on the gradient $E[e(n)u(n-1)]/E[u^2(n-1)]$. Figure 2 shows the gradient curves of the NLMS algorithm with $\omega_s = \pi/2$ and $r = 0.6, 0.8, 0.95$. As seen from this figure, the absolute value of the gradient becomes large in the vicinity of ω_s for each r. Although the large absolute value of the gradient provides the fast convergence speed, it also causes the low estimation accuracy in the vicinity of ω_s. To obtain the high estimation accuracy, we should set μ to the sufficiently small value. In this case, however, the convergence speed is significantly degraded, since the absolute value of the gradient is small far away from ω_s. In the NLMS algorithm, thus, it is difficult to simultaneously achieve the high estimation accuracy and the fast convergence speed.

2. PROPOSED ALGORITHM USING MONOTONICALLY INCREASING GRADIENT

To simultaneously achieve the high estimation accuracy and the fast convergence speed, it is required to achieve the ideal gradient curve, that is, we should design the gradient so that its absolute value becomes small in the vicinity of ω_s and large far away from ω_s. In this paper, we introduce a monotonically increasing gradient as one of the gradients which satisfies such ideal shapes. The monotonically increasing gradient can be achieved by

$$\Phi(a) = \mathrm{sgn}\big(E[e(n)u(n-1)]\big)\frac{E[e^2(n)]}{E[u^2(n-1)]}, \tag{5}$$

where $\mathrm{sgn}(\cdot)$ is the sign function. In (5), $E[e^2(n)]/E[u^2(n)]$ is a quasi-convex function which has only one minimum at ω_s, and it is multiplied by the sign given by

$$\mathrm{sgn}\big(E[e(n)u(n-1)]\big) = \begin{cases} -1, & 0 \le \omega < \omega_s \\ 0, & \omega = \omega_s \\ 1, & \omega_s < \omega < \pi \end{cases}. \tag{6}$$

Figure 3 shows the gradient curve $\Phi(a)$ with $\omega_s = \pi/2$ and $r = 0.6, 0.8, 0.95$. We see from this figure that $\Phi(a)$ is a monotonically increasing function and it satisfies the ideal shape.

In Fig. 3, every gradient curves for various r are gentle in the relatively wide frequency range centered on ω_s. It causes the high deterioration of the convergence speed. To improve the convergence speed, we enhance the value of $\Phi(a)$ only

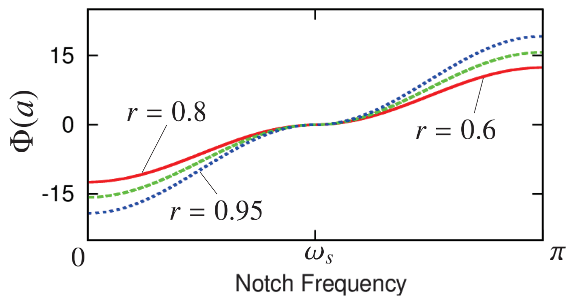

Fig. 3. Gradient curves $\Phi(a)$ with $\omega_s = \pi/2$ and $r = 0.6, 0.8, 0.95$.

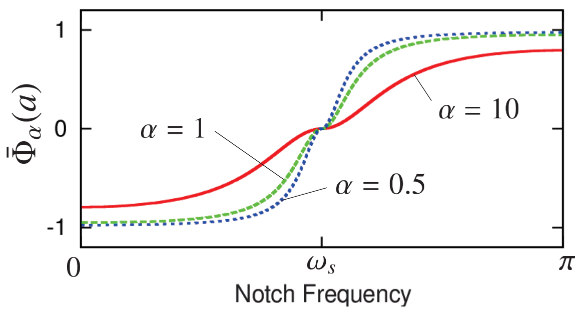

Fig. 4. Gradient Curves $\bar{\Phi}_\alpha(a)$ with $\omega_s = \pi/2$, $r = 0.95$, and $\alpha = 0.5, 1, 10$.

nearby ω_s. In this process, we employ the following enhancement function,

$$\varsigma_\alpha(y) = \frac{y}{\alpha + |y|}. \tag{7}$$

This function can enlarge the value of y around $y = 0$. The parameter α $(0 < \alpha)$ determines the degree of the enhancement. Specifically, the degree of the enhancement increases when α approaches 0. By substituting $y = \Phi(a)$ for $\varsigma_\alpha(y)$, we obtain the flexible gradient expressed by,

$$\bar{\Phi}_\alpha(a) = \mathrm{sgn}\big(E[e(n)u(n-1)]\big)\frac{E[e^2(n)]}{\alpha E[u^2(n-1)] + E[e^2(n)]}. \tag{8}$$

Figure 4 shows the flexible gradient curve $\bar{\Phi}_\alpha(a)$ with $\omega_s = \pi/2$, $r = 0.95$, and $\alpha = 0.5, 1, 10$. When the value of α is large, the curve shape becomes close to the original curve shape $\Phi(a)$. When the value of α is small, the steepness around ω_s in the gradient increases, so that the convergence speed is improved. Note that the extremely small value of α causes the excessive steepness in the gradient, i.e., the gradient curve becomes close to the sign function. In this case, the serious degradation is occurs in the estimation accuracy. Thus, we should appropriately set α in consideration of speed and accuracy.

Comb.	**1**	**2**	**3**	**4**	**5**	**6**	**7**
α	0.05	0.1	0.5	1	10	50	100
$\bar{\mu}$	0.05	0.1	0.5	1	10	50	100

Table 1. Combinations of α and $\bar{\mu}$ under the same estimation accuracy.

Based on the gradient method, the proposed updating equation is obtained by

$$a(n+1) = a(n) - \bar{\mu}\bar{\Phi}_\alpha\big(a(n)\big). \tag{9}$$

When the value of the step size parameter $\bar{\mu}$ is sufficiently small, $a(n)$ converges to $a_s = -(1+r)\cos(\omega_s)$ with $n \to \inf$, since the gradient $\bar{\Phi}_\alpha\big(a\big)$ becomes 0 at $a = a_s$.

3. SIMULATION

In this section, we confirm the convergence performance of the proposed algorithm in (9) through several computational simulations for the sinusoidal noise reduction. In the following simulations, we use the input signal composed of the desired white Gaussian signal and the sinusoidal noise with $\mathrm{SNR} = 0$. We set the noise frequency to $\pi/10$, and changed it to $\pi/2$ at the half of the signal length. The default notch frequency is $\pi/2$.

Firstly, we showed the convergence behaviors of notch frequency ω_N with varied combinations of α and $\bar{\mu}$. For the evaluation of the convergence behaviors, we used the MSE of ω_N calculated by

$$\mathrm{MSE} \quad = \quad 10\log_{10}(\omega_s - \omega_N)^2 \ \ [\mathrm{dB}]. \tag{10}$$

The small value of the MSE means that the algorithm provides the high estimation accuracy. The parameter settings are shown in the following. We set $r = 0.95$, and we prepared seven combinations of α and $\bar{\mu}$ such that the MSE values after convergence are equal for each combination, i.e., the each combination has the same estimation accuracy. The combinations of α and $\bar{\mu}$ are shown in Table 1. As seen from this table, the values of α and $\bar{\mu}$ are equal for each combination. For calculating the expectation value in the algorithm, $\hat{q}(n) = E[q(n)]$, we used

$$\hat{q}(n) = \beta\hat{q}(n-1) + (\beta - 1)q(n), \tag{11}$$

where β $(0 < \beta < 1)$ is the forgetting factor, and all calculations of the expectation value were done with $\beta = 0.8$. The convergence behaviors with 500 trials are shown in Figure 5. From this figure, we can see that the every convergence speed are almost same, except for the very large or very small values of α, such as $\alpha = 100, 0.1, 0.05$. To elicit the performance of the proposed algorithm, we should find the appropriate range of the values of α from the preliminary simulation.

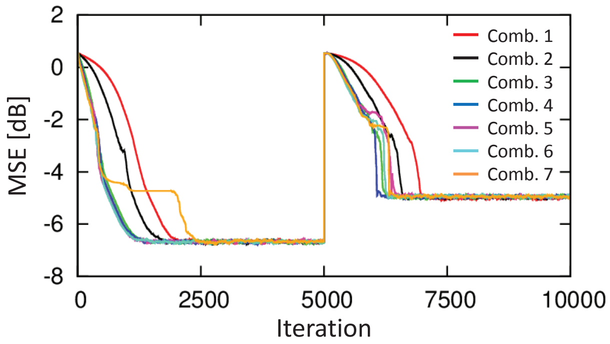

Fig. 5. MSE curves for the proposed algorithm with combinations of Table 1.

Secondly, we compared the convergence speed of the proposed algorithm with one of the NLMS algorithm in (4), under the same estimation accuracy. For the evaluation, we use the MSE and the improvement of the SNR (ISNR). The ISNR is calculated by

$$\begin{aligned}\text{ISNR} \quad &= \quad 10\log_{10}\frac{\sum_{n=jL}^{(j+1)L-1} w(n)^2}{\sum_{n=jL}^{(j+1)L-1}\{e(n)-w(n)\}^2}\\ &\quad -10\log_{10}\frac{\sum_{n=jL}^{(j+1)L-1} w(n)^2}{\sum_{n=jL}^{(j+1)L-1} s(n)^2} \quad [\text{dB}],\end{aligned}$$

where L denotes the length of the calculation block. The large value of the ISNR means that the algorithm can remove the noise efficiently. In the proposed algorithm, we set $r = 0.95$, $\alpha = 1$, and $\bar{\mu} = 0.1$. Also, in the conventional algorithm, we set $r = 0.95$ and $\mu = 0.3$ to achieve the same convergence speed with the proposed algorithm. Figure 6 shows the MSE of the notch frequency with 500 trials, where we see that the value of the MSE in the proposed algorithm is up to 3.7dB lower than the conventional one. Figure 7 shows the ISNR with $L = 10$. As can be seen, the value of the ISNR in the proposed method is up to 8dB higher than the conventional one. These results shows that the proposed algorithm can, compared with the NLMS algorithm, estimate the noise frequency with high accuracy and remove the noise more efficiently.

In the third simulation, we compared the convergence speed of the proposed algorithm with one of the conventional algorithm under the same estimation accuracy. The parameter setting is same in the second simulation except of the conventional step size parameter, $\mu = 0.015$. Figure 8 shows the MSE with 500 trials. From this figure, we can verify that the convergence speed of the proposed method is up to 5th as fast as the conventional one. Figure 9 shows the ISNR with $L = 100$, where we see that the proposed algorithm can remove the noise fast compared with the conventional algorithm.

These simulation results present that the proposed algorithm can simultaneously achieve the high estimation accuracy and the fast convergence.

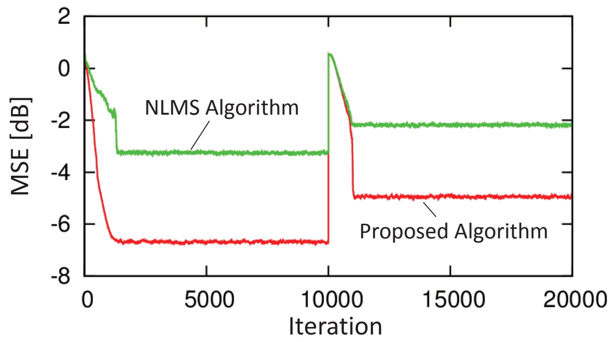

Fig. 6. MSE curves for the proposed algorithm and the NLMS algorithm under the same convergence speed.

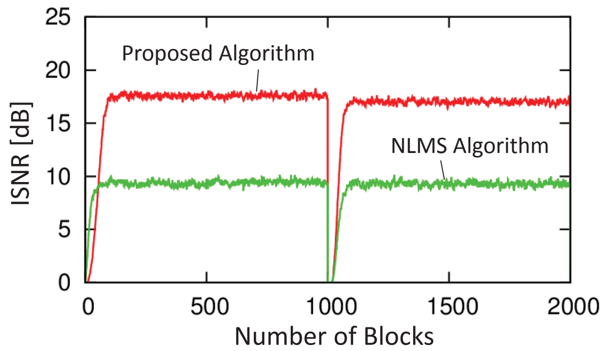

Fig. 7. ISNR curves for the proposed algorithm and one of the NLMS algorithm under the same convergence speed.

4. CONCLUSION

In this paper, we had proposed a new gradient based adaptive algorithm for the fast and accurate adaptive notch filter. The proposed method achieved the fast and accurate estimation by introducing an monotonically increasing gradient. To flexibly design the steepness of the gradient, we additionally introduced the enhancement function into the gradient. Several computational simulations for removing sinusoidal signals had shown that the proposed method can provide fast convergence and high accurate estimation, simultaneously.

In future works, we derive the convergence condition of the step size parameter μ and the enhancement parameter α. We also investigate the more fast algorithm by using the variable μ and α or introducing the variable r.

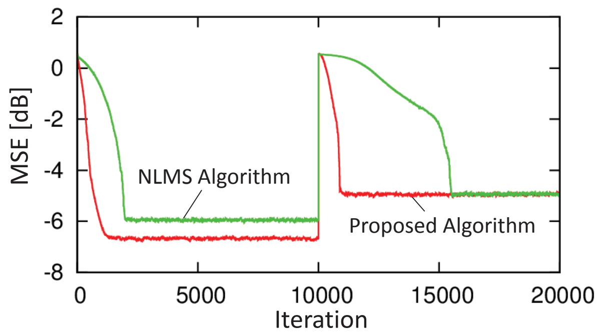

Fig. 8. MSE curves for the proposed algorithm and the NLMS algorithm under the same estimation accuracy.

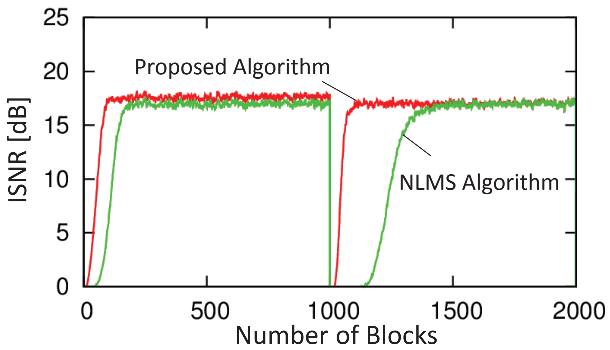

Fig. 9. ISNR curve for the proposed algorithm and the NLMS algorithm under the same estimation accuracy.

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