ENHANCED RADAR IMAGING VIA SPARSITY REGULARIZED 2D LINEAR PREDICTION

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ABSTRACT

ISAR imaging based on the 2D linear prediction uses the l2 norm minimization of the prediction error to obtain 2D autoregressive (AR) model coefficients. However, this approach causes many spurious peaks in the resulting image. In this study, a new ISAR imaging method based on the 2D sparse AR modeling of backscattered data is proposed. The 2D model coefficients are obtained by the l2- norm minimization of the prediction error penalized by the l1 norm of the prediction coefficient vector. The resulting 2D prediction coefficient vector is sparse, and its use yields radar images with reduced side lobes compared to the classical l2- norm minimization.

***Index Terms*—** radar imaging, autoregressive modeling, linear prediction, sparsity, regularization

1. INTRODUCTION

An ISAR image represents a 2D spatial distribution of the scattering centers of an object to be imaged. Robustness to noise and clutter and insensitivity to relative aspect angle between the radar and the target makes ISAR images appropriate for identification applications. Conventional ISAR imaging uses polar format algorithm which is based on the 2D inverse Fourier transform of the Cartesian frequency domain backscattered data. Unfortunately, the range resolution depends on the bandwidth of the radar while the cross-range resolution depends on the aspect angle region. Thus for the limited data case where only narrow bandwidth and small aspect angle data is available, polar format algorithm yields poorly resolved images [1-3].

Subspace decomposition based spectral estimation techniques such as MUSIC and AR have been widely used in the past to obtain high resolution images [2-4]. However these methods require the estimation of the covariance matrix which tends to be ill conditioned for the limited data case and its eigenvalue decomposition. In MUSIC method, the ISAR image is obtained using the eigenvectors of the noise subspace. The performance of the method depends heavily on the distinction between the eigenvalues related to signal and noise subspaces. This distinction may not be obvious for complex targets which are characterized by a large number of scattering centers and additional processing may be required for their estimation. In AR based method the available data are modeled as a 2D autoregressive model and ISAR image is obtained by the 2D AR based power spectrum. A common way to estimate the model coefficients is to minimize l2-norm of the residual, the difference between the observed signal and the predicted signal, yields the well known linear least-squares problem which requires the inversion of the covariance matrix. However for high model orders, this matrix tends to be ill-conditioned resulting in many spurious peaks and high side lobes in the ISAR images. Although it is possible to refine these images by SVD truncation, this approach also needs a precise distinction between signal and noise subspaces. Moreover, SVD truncation while decreasing the background side lobes, leads to a decrease in the dynamic range of the ISAR image reducing the distinction of the target from the background areas [1].

Recently, regularization methods with sparsity prior using l0-norm penalty functions have found many applications [5,6] . Since l0-norm minimization is a NP hard problem, it can be approximated by the use of 1-norm as penalty functions which produces sparse results with few number of non-zero coefficients. Regularization problems which involve l1-norm penalty functions are not differentiable, thus unlike l2-norm case they do not have a closed form solution, but they can be transformed into convex quadratic problems and solved by convex optimization methods. In [6] the sparsity based regularization of AR models is used in the framework of linear prediction of speech signals. The authors defined sparse AR models by including l1-norm penalty or sparse residuals by l1-norm minimization of the residual. In this work, we extend their results to the 2D AR modeling and apply to ISAR image formation problem.

The paper is organized as follows. Section 2 reviews ISAR imaging using 2D linear prediction, section 3 introduces sparse linear predictors obtained by sparsity based regularization. In section 4 several results for both simulated and experimental targets are presented. The proposed method is compared with conventional l2-norm minimization as well as SVD truncation. General conclusions are given in section 5.

2. PROBLEM FORMULATION

The received echo signal from a target consisted of d scattering centers at frequency f_m and aspect angle $\theta_n (n = 0,1,\ldots\ldots, N-1,\ m = 0,1,\ldots\ldots, M-1)$ can be given as [2],

$$E(f_m, \theta_n) = u(f_m, \theta_n) + \sum_{k=1}^{d} a_k \exp\left(-j\frac{4\pi f_m}{c}(x_k \cos\theta + y_k \sin\theta)\right) \quad (1)$$

Where a_k is the amplitude of the kth scattering center, x_k and y_k are the coordinates of the kth scattering center, $u(f_m, \theta_n)$ is the additive Gaussian noise with zero mean and variance σ^2 and c is the speed of the light. In order to obtain a focused ISAR image, one can transform the frequency-aspect data $y(f_m, \theta_n)$ to spatial frequency domain (f_x, f_y) using the relations

$$f_x = \frac{2f}{c}\cos\theta \quad (2.a)$$

and

$$f_y = \frac{2f}{c}\sin\theta \quad (2.b)$$

After polar formatting (1) can be expressed as,

$$E(m,n) = \sum_{k=1}^{d} a_k \exp\left(-j2\pi\left(x_k f_x(m) + y_k f_y(n)\right)\right) + u(m,n) \quad (3)$$

With

$$f_x(m) = f_x(0) + m\Delta f_x \qquad m = 1,2,\ldots\ldots, M$$
$$f_y(n) = f_y(0) + m\Delta f_y \qquad n = 1,2,\ldots\ldots, N$$

Where $f_x(0)$ and $f_y(0)$ are the starting values of f_x and f_y, respectively. M and N are the number of interpolated data samples. Using 2D linear prediction, measured field at a Cartesian frequency (f_x, f_y) can be predicted using quarter-plane models [2] as,

1st quadrant (forward prediction in both f_x and f_y)

$$\hat{E}_1(m,n) = -\sum_{\substack{i=0 \\ i=j\neq 0}}^{L} \sum_{j=0}^{L} a_{ij} E(l-i, n-j) \quad (4)$$

3rd quadrant (backward prediction in both f_x and f_y)

$$\hat{E}_3(l,n) = -\sum_{i=0}^{L} \sum_{j=0}^{L} \tilde{a}_{ij}\, E(l+i, n+j) \quad (5)$$
$$i = j \neq 0$$

Since $a_{ij} = a_{ij}^*$, (5) can be written as,

$$\hat{E}_3^*(m,n) = -\sum_{i=0}^{L} \sum_{j=0}^{L} a_{ij} E^*(l+i, n+j) \quad (6)$$
$$i = j \neq 0$$

By combining (4) and (6) one can obtain,

$$\boldsymbol{Ea} = -\boldsymbol{e} \quad (7)$$

Where $\boldsymbol{E}$ is a $2(N-L)^2$ by $(L+1)^2 - 1$ matrix, **a** is a vector of length $(L+1)^2 - 1$ and $\boldsymbol{e}$ is a vector of length $2(N-L)^2$. The prediction coefficient vector by the least-square solution of (7) as [2],

$$\boldsymbol{a} = -(\boldsymbol{E}^H \boldsymbol{E})^{-1} \boldsymbol{E}^H \boldsymbol{e} \quad (8)$$

Similarly, using 2nd and 4th quadrant models which consist of using backward and forward prediction on either of f_x and f_y, a second set of equations can be obtained as,

$$\widetilde{\boldsymbol{E}}\boldsymbol{b} = -\tilde{\boldsymbol{e}} \quad (9)$$

Where $\widetilde{\boldsymbol{E}}$ is a $2(N-L)^2$ by $(L+1)^2 - 1$ matrix, **b** is a vector of length $(L+1)^2 - 1$ and $\tilde{\boldsymbol{e}}$ is a vector of length $2(N-L)^2$. Finally the radar image, ie. locations of the scattering centers, is given by the peaks of

$$P(x,y) = \frac{1}{\left|1 + \sum_{i=0}^{L}\sum_{j=0}^{L} a_{ij}\, z_1^{-i} z_2^{-j}\right|^2 + \left|1 + \sum_{i=0}^{L}\sum_{j=0}^{L} b_{ij}\, z_1^{-i} z_2^{-j}\right|^2} \quad (10)$$
$$i = j \neq 0$$

With $z_1 = \exp\left(j\frac{4\pi}{c}\Delta f_x x\right)$ and $z_2 = \exp\left(j\frac{4\pi}{c}\Delta f_y y\right)$ for $-\frac{c}{\Delta f} < x, y < \frac{c}{4\Delta f}$.

Since the radar image has more peaks than the scattering centers, SVD can be used to eliminate the spurious peaks. However, this method needs the consuming eigenvalue decomposition and estimation of the scattering center numbers for the cases where the distinction between the eigenvalues is not obvious.

3. REGULARIZED LINEAR PREDICTION

Using eqns. (7) and (9), two prediction errors can be defined as,

$$\boldsymbol{e}_{2-4} = \widetilde{\boldsymbol{E}}\boldsymbol{b} + \tilde{\boldsymbol{e}} \quad (11.b)$$

$$\boldsymbol{e}_{1-3} = \boldsymbol{E}\boldsymbol{a} + \boldsymbol{e} \qquad (11.a)$$

The calculation of prediction coefficients may be thought as an optimization problem which consists of finding the prediction coefficients using a set of observed complex signals so that the prediction error is minimized. The resulting minimization problem may be stated as,

$$\boldsymbol{a} = \min_{\boldsymbol{a}} \|\boldsymbol{e}_{1-3}\|_p^p + \lambda\|\boldsymbol{a}\|_k^k = \min_{a} \|\boldsymbol{E}\boldsymbol{a} + \boldsymbol{e}\|_2^2 + \lambda\|\boldsymbol{a}\|_k^k \qquad (12.a)$$

$$\boldsymbol{b} = \min_{\boldsymbol{b}} \|\boldsymbol{e}_{2-4}\|_2^2 + \lambda\|\boldsymbol{b}\|_k^k = \min_{\boldsymbol{b}} \left\|\widetilde{\boldsymbol{E}}\boldsymbol{b} + \tilde{\boldsymbol{e}}\right\|_2^2 + \lambda\|\boldsymbol{b}\|_k^k \qquad (12.b)$$

Where $\|\boldsymbol{x}\|_p$ is the lp-norm defined as,

$$\|\boldsymbol{x}\|_p = (\sum_{n=1}^{N} |x(n)|^p)^{1/p} \quad for\ p>1 \qquad (13)$$

And λ is the regularization parameter considering $p = 2$ and $\lambda = 0$ will lead us to the classical least-square solution [5,6]. As it is mentioned in the preceding section l2-norm minimization approach has several issues, especially for high orders many spirous peaks appear requiring the use of SVD.

It is possible to induce the sparsity constraints on the prediction coefficients to obtain l1-norm regularized least-square solution as,

$$\boldsymbol{a} = \min_{\boldsymbol{a}} \|\boldsymbol{E}\boldsymbol{a} + \boldsymbol{e}\|_2^2 + \lambda\|\boldsymbol{a}\|_1^1 \qquad (14.a)$$

$$\boldsymbol{b} = \min_{\boldsymbol{b}} \left\|\widetilde{\boldsymbol{E}}\boldsymbol{b} + \tilde{\boldsymbol{e}}\right\|_2^2 + \lambda\|\boldsymbol{b}\|_1^1 \qquad (14.b)$$

We should mention that sparseness is often measured as the cardinality, ie. l0-norm $\|a\|_0$. Therefore, by defining the prediction coefficients as in eqn. 14.a and 14.b, we would like to minimize the number of non-zero prediction coefficients. Since this is an NP hard problem, general approach is to use more tractable l1-norm $\|b\|_1$. The minimization problems b can be efficiently solved using optimization packages such as cvx or l1-magic [7].

3.1. Selection of the regularization parameter

The regularization parameter λ controls the tradeoff between the sparsity of the predictor and the sparsity of the residual. We have used the Lcurve method for the determination of the λ parameter [8]. Lcurve is a log-log plot of the regularized solution against the squared norm of the regularized residual for a range of values of the regularization parameter. The optimal value of λ is calculated as the maximum curvature of the curve $(\|\boldsymbol{E}\boldsymbol{a} + \boldsymbol{e}\|_p^p, \|\boldsymbol{a}\|_k^k)$.

4. EXPERIMENTAL RESULTS

The proposed method is applied to simulated and experimental data downloaded from [9,10]. The radar images obtained using l2-norm minimization, l2-norm minimization with SVD truncation as well as noisy data results have been included for comparison purposes.

Simulated data "mig25" have the following specifications: central frequency f_c = 9GHz, bandwidth B =531 MHz, view angle Ω =3.67°, data size is 64x64 pixels. The prediction order is chosen as $L = 25$ for all methods.

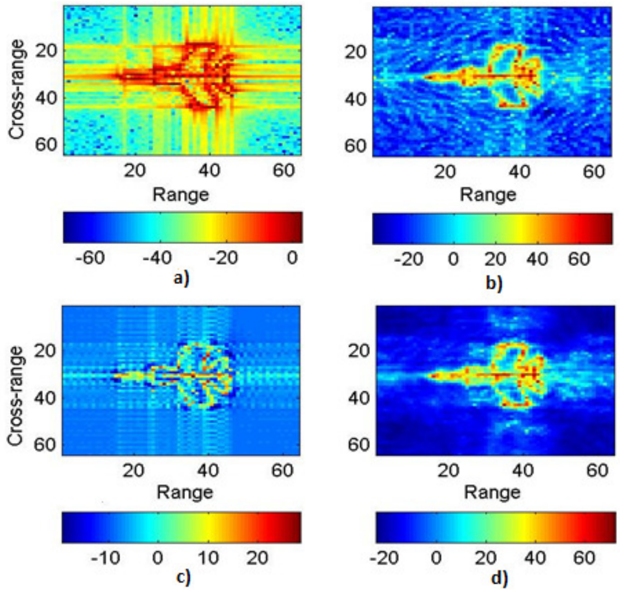

Figure 1. ISAR images using a) polar-format algorithm, b) l2 minimization c) l2 minimization with SVD truncation, d) proposed l2 minimization with l1 penalty, λ=1.0

Fig1. shows the resulting ISAR images for classical Fourier transform based polar format algorithm, l2-norm minimization, l2-norm minimization with SVD truncation and l2-norm minimization with l1-norm penalty function, respectively. The regularization parameter is chosen as λ=1.0

As it is seen in Fig1.a polar format algorithm result has poor resolution and high sidelobes. The classical l2-norm minimization showed in Fig1.b yields images with very high lobes in the background area. It is possible to reduce the amount high lobes using SVD truncation as shown in Fig1.c The regularization approach based on and l1- norm penalty provides clearer images as shown in Fig 1.d. The use of l1-norm penalty decreases spurious peaks and gives a smoother background region.

Since SVD truncation depends on the distinction of the small and large eigenvalues or on the knowledge of scattering centers number, its performance may be affected for realistic targets with large number of scattering centers. Fig 2 shows clearly the effect of the choice of large eigenvalues on the results. As expected a large choice of scattering centers results in images with high sidelobes wherein a small choice may lead to the underestimation of the scattering centers.

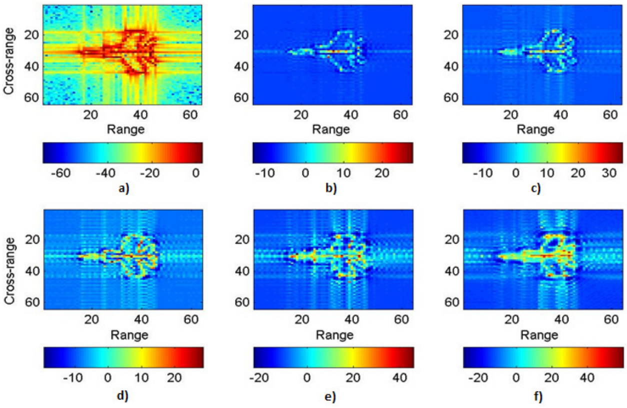

Figure 2. Radar images obtained using a) polar-format algorithm and radar images using l2-norm minimization with SVD truncation with modelling level 25 and number of scattering centers b) 30, c) 50, d) 70, e) 90, f) 110

The impact of regularization parameter can be observed from Fig 3. A small choice approximates the l2- norm minimization while a large choice decreases the resolution.

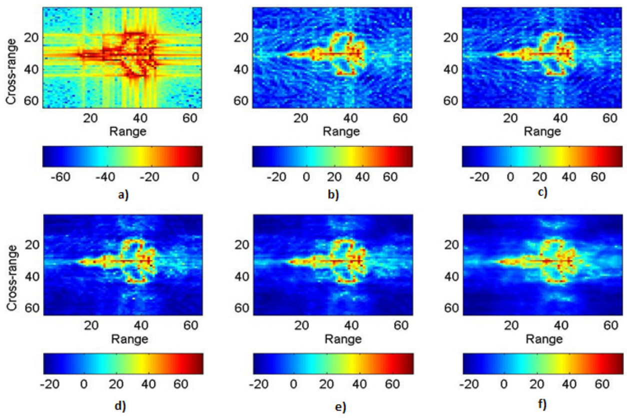

Figure3. ISAR images for mig25 data using a) polar-format algorithm,the proposed method with regularization parameters b) λ =0.001, c) λ =0.1, d) λ =0.5, e) λ =1, f) λ =4

All the methods have also been applied to the experimental "esairbus" data of [10] with the following specifications: central frequency f_c = 4 GHz, band width B =122MHz, view angle Ω = 2.08 °; number of scattrer is 20, data size 32x32 pixels. The prediction order is chosen as 12.

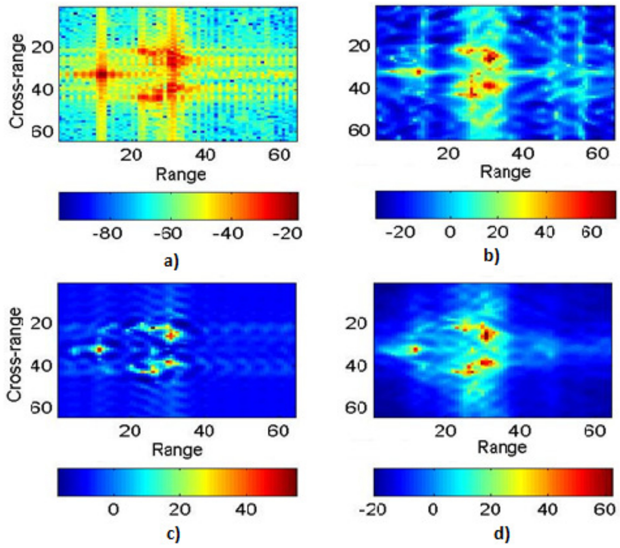

Figure 4. ISAR images using a) polar-format algorithm, b) l2-norm minimization c) l2-norm minimization with SVD truncation, d) proposed l2-norm minimization with l1-norm penalty, λ=0.5

Fig 4 shows the resulting radar images for polar format algorithm, conventional l2 norm minimization, l2-norm minimization combined with SVD truncation, and the proposed l2-norm minimization with l1 norm regularization methods with λ=0.5. As in the previous example, l2-norm minimization with SVD truncation and the proposed method have been able to decrease the high sidelobes observed in classical l2-norm minimization. However, unlike our method SVD trunctation result has a decreased dynamic range and scattering centers are more obvious in the proposed method's result.

An additive white Gaussian noise (AWGN) at a signal-to-noise ratio (SNR) of 10 dB is added to the phase history data to compare the performance of the methods mentioned above. The ISAR images obtained for the low quality data can be seen in Fig 5.And Fig.6. As expected, all the results have been deterioriated for the noisy case, especially l2-norm minimization based ones, however the proposed method presents better results compared to them. Again, similar to the simulated data case, the dynamic range of the proposed method is higher compared to the l2-norm minimization with SVD truncation and the distinction of the scattering centers from the background is better.

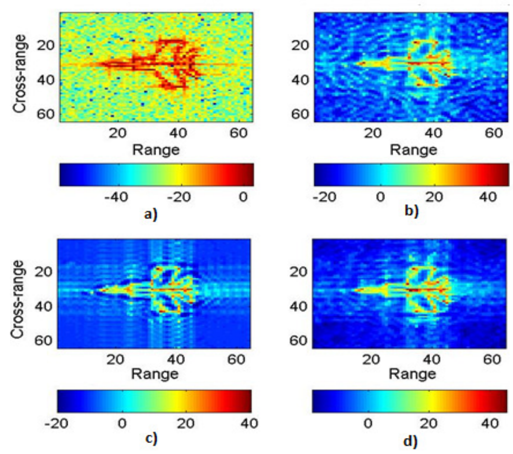

Figure5. ISAR images for mig25 data using a) polar format algorithm b) l2-norm minimization (modeling level=20), c) l2-norm minimization with SVD truncation (modeling level=20 and number of scatterers=70), d) l2- norm minimization with l1-norm penalty function (modeling level=20) with λ=4.0 (S/N=10dB)

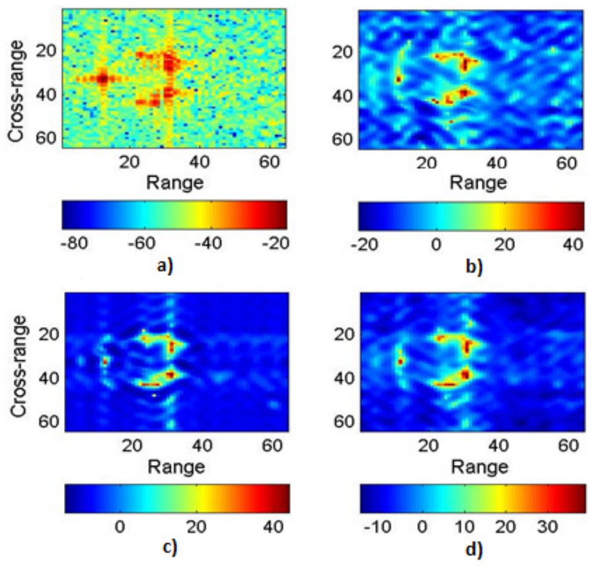

Figure6. ISAR images for esairbus data using a) polar format algorithm b) l2-norm minimization, c) l2-norm minimization with SVD truncation (number of scatterers=20), d) l2- norm minimization with l1-norm penalty function with λ=0.5 (S/N=10dB, modeling level=10)

5. CONCLUSIONS

A new high resolution ISAR imaging method based on the 2D regularized linear prediction is presented. Unlike the classical l2-norm minimization, the 2D AR coefficient vector is found by the minimization of the new cost function including a l1-norm penalization term. Since the optimization problem do not have a closed form solution, we have used an optimization package in the minimization step. The resulting coeffient vector is sparse and yields radar images with reduced spurious peaks, thus reduced side lobes. The choice of scattering centers is not crucial as in l2-norm minimization with SVD truncation where a large number choice leads to the loss of some scattering centers or to the ineffective decrease of the spurious peak levels. Results of the simulated and experimental targets show that the proposed method gives enhanced radar images with higher dynamic ranges which are expected to increase the classification rate in the target recognition application. Current research is focused on this subject.

REFERENCES

[1] J. I. Park, K.T. Kim, "A Comparative study on ISAR imaging algorithms for radar target identification", *Progress in Electromagnetic Research*, Vol. 108, 155-175, 2010.

[2] Gupta I.J., "High resolution radar imaging using 2-D linear prediction", *IEEE Transactions on Antennas and Propagation*, Vol. 22, 31-37, Jan. 1994.

[3] K.T. Kim, S.W. Kim, H.T. Kim, "Two dimensional ISAR imaging using full polarization and superresolution processing techniques", *IEE Proceedings Radar, Sonar and Navigation*, Vol. 145, 240-246, Aug. 1998.

[4] I. Erer and A.H. Kayran, "Superresolution ISAR Imaging Using 2-D Autoregressive Lattice Filters", *Microwave and Optical Technology Letters*, 32, 81-85, (2002).

[5] D. Gracobello, M. G. Christensen, M.N. Murthi, S.H. Jensen, M. Moonene, "Sparse Linear prediction and its applications to speech processing", *IEEE Trans on Audio Speech and LanguageProcessing*, Vol. 20, No.5, 2012.

[6] D. Giacobello, M.G. Christensen, J. Daohl, S.H. Jensen, M. Moonen, "Sparse Linear Predictors for speech processing", *Proceedings Interspeech 2008*, 1351-1356

[7] P. C. Hansen, D.P. O'leary, "The use of the L-curve in the regularization of discrete ill-posed problems", *SIAM on Sci. Comp.*, Vol. 14, No. 6, 1478-1503, 1993.

[8] M. Grant , S. Boyd, CVX: Matlab software for Disciplined Convex Programming (web page and software)2008 [online]. Available : http//Stanford.edu/boyd/cvx.

[9] V. C. Chen, H. Ling., Time-frequency transforms for radar imaging and signal analysis, Boston: Artech House, 2002.

[10] Ozdemir, C. (). Inverse Synthetic Aperture Radar Imaging with MATLAB Algorithms. New Jersey: Wiley, 2012.