UNCOVERING HARMONIC CONTENT VIA SKEWNESS MAXIMIZATION - A FOURIER ANALYSIS

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ABSTRACT

Blind adaptation with appropriate objective function results in enhancement of signal of interest. Skewness is chosen as a measure of impulsiveness for blind adaptation to enhance impacting sources arising from defective rolling bearings. Such impacting sources can be modelled with harmonically related sinusoids which leads to discovery of harmonic content with unknown fundamental frequency by skewness maximization. Interfering components that do not possess harmonic relation are simultaneously suppressed with proposed method. An experimental example on rolling bearing fault detection is given to illustrate the ability of skewness maximization in uncovering harmonic content.

Index Terms— Adaptive filters, harmonic analysis, higher order statistics, rolling element bearings

1. INTRODUCTION

Adaptive filters can be trained to meet a variety of objectives. A classical objective, known as least mean squares (LMS), minimizes the mean squared error between the adaptive filter output and a target signal [1]. However, LMS requires knowledge of the source signal for adaptation. In some applications, exact knowledge of this target signal is not available and must be replaced with more vague information. In such a case, a proper figure of statistical characteristics of the desired signal may suffice. Such an adaptation process can be defined as *blind adaptation*. Typical examples for such blind processing can be provided from digital communications, seismic deconvolution and image restoration [2].

Characterizing the statistical properties of the desired signal is done by choosing an appropriate objective function. Depending on the physical phenomenon, a proper measure of merit must be selected. One of the higher order statistics, normalized third order moment or *skewness*, can be utilized to measure asymmetry in order to reveal impulsiveness in blind adaptation. To quantify asymmetry in the probability density function (PDF), skewness has previously been used in different signal processing applications such as speech polarity detection [3] and vocal source characterization [4].

Depending on the concept of blind adaptation, impacting signals buried in noise were enhanced through adaptive filtering by using the skewness as objective in [5]. Objective surface characteristics and convergent behaviour of skewness with a deterministic and periodic impulsive signal model were analysed by a recent work [6]. This paper takes further steps in this direction with computationally and memory efficient algorithms drawn from the ideas of the blind adaptive filtering community. The presentation here is directed towards rolling bearing diagnostics, but the broader application is meant to be seen as *enhancement of harmonically related content* of unknown fundamental frequency under distortion, noise and severe interference.

The analysis in limited dimensions in [6] with the aforementioned signal model is extended and generalized in order to understand the mathematical reasoning behind how maximizing skewness results in promoting harmonically related components while attenuating the ones without harmonic relation. Through such an analysis, a signal pre-processing method for defect detection in rolling bearings is presented as an application example and supported with an experiment on mechanical vibrations from industry. Such pre-processing enables enhancement of impulsiveness by reducing the effects of linear amplitude and phase distortion.

2. BACKGROUND

2.1. Prior Work

The relation between the presented study and the blind deconvolution problem is not specifically discussed, however, a number of the ideas and concepts presented here are drawn from that research community.

Minimum Entropy Deconvolution (MED) was suggested

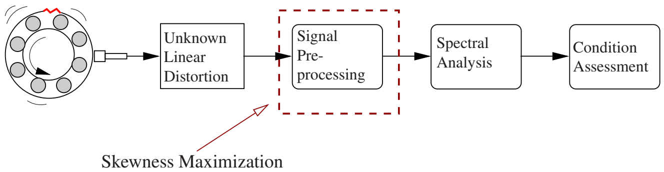


Fig. 1. Skewness maximization as a signal pre-processing approach for rolling bearing diagnostics.

by Wiggins [7] to recover a seismic trace by minimizing the entropy of an observed signal through maximization of the *varimax norm*. Donoho [8] generalized the method and investigated the concept of choosing a proper objective function to decrease Gaussianity. This groundwork idea was later employed for defect detection problem for condition monitoring purposes [9] and for deconvolution of impacting signals [10].

Rolling bearings are widely used in rotating machinery to ease rotational movement. Impacting signals can emerge from defective bearings due to the contact of rotating elements to a fault on a bearing race. Various mechanisms, such as undesired vibrations, may appear to mask the desired impulsive signature arising from a possible defect, making fault detection a challenging problem in practice [11, 12]. Such impulsive signatures can be modelled as harmonic series [6] where undesired vibrations are regarded as sinusoidal interference. In such a problem setting, defect signatures can be recovered by suppressing the undesired sinusoids while enhancing the harmonic content. As shown in Fig. 1, the outcome from such an algorithm can then be introduced to spectral analysis for condition assessment.

Examples of applications where harmonic content detection is essential can be given from power systems where neural networks [13] and wavelet transform [14] may be used. Harmonic analysis can be useful in pitch detection problems as well [15]. Period histogram and product spectrum were examined in [16] for fundamental frequency measurement.

The presented study here shows how maximizing skewness enhances harmonically related components. Forthcoming sections will explore this ability by analysing the relation between skewness and harmonic content.

2.2. Skewness of Harmonic Content

A continuous-time deterministic signal model of an harmonic series is represented as

$$s(\theta, t) = \sum_{r=1}^{N} \alpha_r \cos(r\omega_0 t) \tag{1}$$

with $\alpha_r \in \mathbb{R}$ and

$$\theta = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_N \end{bmatrix}^T \tag{2}$$

is the parameter vector of harmonic amplitudes. Arbitrary phase will also be introduced to the signal model after some derivations on this simplified case.

Employing this signal model can be justified by considering the Fourier series. It is known that periodic series of impulses in time domain have a spectral representation of equal amplitude harmonically related sinusoids. Thus, it is natural to think of impulsive signatures arising from rolling bearings with the given signal model.

Stating the problem in the opposite way results in an application in which the discovery of an impulsive signal of unknown fundamental period is desired. This signal of interest may be obscured, or even completely buried by linear amplitude and phase distortion, possible disturbances, noise and interference. Then the challenge becomes to uncover buried harmonic content in an efficient way by exploiting the knowledge of the PDF, time and frequency domain representation of an impulse train.

The skewness ϕ of the impulse train $s(\theta, t)$ is

$$\phi_{(s(\theta,t))} = \frac{E\{s^3(\theta,t)\}}{(E\{s^2(\theta,t)\})^{3/2}}, \tag{3}$$

in which the expectation operation on deterministic and periodic $s(\theta, t)$ can be replaced by integration over time. To obtain an expression of the skewness of $s(\theta, t)$, we begin with expanding the numerator

$$\frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} s^3(\theta, t)\, dt. \tag{4}$$

As the integrand is a polynomial strictly of order 3, all monomial terms will also be of order 3. Each of these monomials fall into one of the three classes listed below. Let $b, c \in \mathbb{Q}$, $i, j, k \in [1, N] \subset \mathbb{Z}$ and $i < j < k$:

i) The elements of the first class are strictly cubed sinusoids of a single frequency, therefore

$$\frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \alpha_i^3 \cos^3(i\omega_0 t)\, dt = 0, \tag{5}$$

since any odd-order moment of a symmetric function is zero.

ii) The second class consists of cross-terms of two harmonics, where

$$\frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \alpha_i^2 \alpha_j \cos^2(i\omega_0 t) \cos(j\omega_0 t)\, dt = \\ = \begin{cases} b\left(\alpha_i^2 \alpha_j\right), & \text{if } j = 2i \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

iii) Finally, the third class consists of cross-terms of three harmonics, which yields

$$\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\alpha_i\alpha_j\alpha_k\cos(i\omega_0 t)\cos(j\omega_0 t)\cos(k\omega_0 t)\,dt =$$
$$= \begin{cases} c\left(\alpha_i\alpha_j\alpha_k\right), & \text{if } k = i \pm j \\ 0, & \text{otherwise.}\end{cases} \tag{7}$$

These three classes determine the structure of skewness for the given signal class of harmonic series. Therefore, a necessary condition for the harmonic series to give non-zero skewness is to include *harmonically related content* satisfying (6) or (7). A fundamental notion obtained at this point is that harmonically related content (as defined above) is necessary for periodic signals with non-zero skewness.

As we wish to broaden the class of considered signals $s(\theta, t)$, as well as to account for eventual effects of a transfer function, we now extend (1) to include a phase term in each harmonic, such as

$$s_e(\theta_e, t) = \sum_{r=1}^{N}\alpha_r\cos(r\omega_0 t + \gamma_r). \tag{8}$$

that is collected in the parameter vector of amplitudes α_r and phases γ_r

$$\theta_e = \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \alpha_N & \gamma_1 & \gamma_2 & \cdots & \gamma_N\end{bmatrix}^T.$$

Repeating the expansion of the numerator of (3), we obtain a structure for skewness of the signal $s_e(\theta_e, t)$ with arbitrary phase:

ia) The cubed sinusoids of a single frequency with phase component will be zero,

$$\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\alpha_i^3\cos^3(i\omega_0 t + \gamma_i)\,dt = 0.$$

iia) The cross-terms of two harmonics yield

$$\begin{cases}\alpha_i^2\alpha_j(-\frac{3}{4}\cos(\gamma_j) + \frac{3}{2}\cos^2(\gamma_i)\cos(\gamma_j) \\ +\frac{3}{2}\cos(\gamma_i)\sin(\gamma_i)\sin(\gamma_j)), & \text{if } j = 2i \\ 0, & \text{otherwise.}\end{cases}$$

iiia) The cross-terms of three harmonics give

$$\begin{cases}\frac{3}{2}\alpha_i\alpha_j\alpha_k(-\sin(\gamma_i)\sin(\gamma_j)\cos(\gamma_k) \\ +\sin(\gamma_i)\cos(\gamma_j)\sin(\gamma_k) \\ +\cos(\gamma_i)\sin(\gamma_j)\sin(\gamma_k) \\ +\cos(\gamma_i)\cos(\gamma_j)\cos(\gamma_k)), & \text{if } k = i \pm j \\ 0, & \text{otherwise.}\end{cases}$$

After these derivations, it can be concluded that maximum skewness can be achieved by aligning the phase as $\gamma_r = 0, \pm\,\pi, \pm 2\pi$. Therefore, an algorithm that aims to maximize the skewness of a given harmonic content with arbitrary phase should align each harmonic as proposed.

Further analysis will be conducted through theoretical investigation of the harmonic relation and a practical example. As phase alignment is necessary for skewness maximization, we make the presentation more transparent by dropping the explicit phase terms, considering only phase aligned signals.

Before presenting the study on the non-harmonic content, it is worth mentioning about the impact of additive and symmetric noise (e.g. Gaussian noise) that is uncorrelated with the source signal in skewness maximization. As it is a known fact that any odd-order moment of a symmetric signal is zero, and due to the assumption of noise being uncorrelated with the source signal, the outcome of the integral in (4) will not include a noise term. With consideration of the denominator in (3), it can be shown that the convergence properties of skewness are not altered by additive symmetric noise.

2.3. Effect of Non-Harmonic Content on Skewness

The concept of increasing skewness by promoting harmonic content can be illustrated by a signal model

$$s_d(\theta_d, t) = \sum_{r=1}^{N}\alpha_r\cos(r\omega_0 t) + \beta\cos(\omega_p t), \tag{9}$$

where ω_p, without being an integer multiple of the fundamental, is the unknown frequency of a non-harmonically related component, $\beta \in \mathbb{R}$ is the scaling coefficient and $\theta_d = \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \alpha_N & \beta\end{bmatrix}^T$ is the coefficient vector. Utilizing the structure for skewness of harmonic series derived in Sec. 2.2 and with the same assumptions on i, j, k, the skewness of the disturbed signal $s_d(\theta_d, t)$ becomes,

$$\phi_{(s_d(\theta_d,t))} = \frac{\frac{3}{4}\alpha_i^2\alpha_j + \frac{3}{2}\alpha_i\alpha_j\alpha_k}{\left(\frac{1}{2}\sum_{r=1}^{N}\alpha_r^2 + \frac{1}{2}\beta^2\right)^{\frac{3}{2}}}. \tag{10}$$

It is apparent from (10) that in order to increase skewness, β must be attenuated. After this basic outcome, we can now introduce an arbitrary number of mutually non-harmonic disturbances that will give a signal model

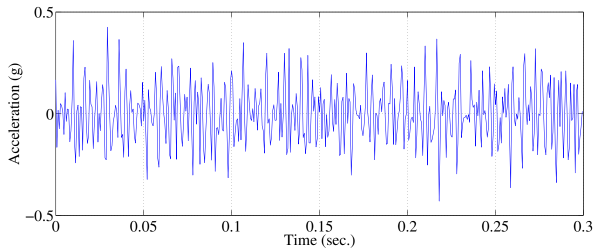
$$s_d(\theta_d, t) = \sum_{r=1}^{N}\alpha_r\cos(r\omega_0 t) + \sum_{p=1}^{M}\beta_p\cos(\omega_p t), \tag{11}$$

with $\beta_p \in \mathbb{R}$ and a coefficient vector

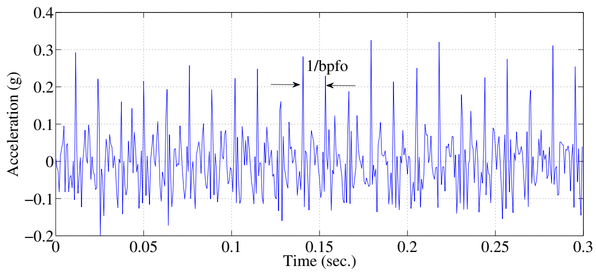
$$\theta_d = \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \alpha_N & \beta_1 & \beta_2 & \cdots & \beta_M\end{bmatrix}^T,$$

where skewness becomes

$$\phi_{(s_d(\theta_d,t))} = \frac{\frac{3}{4}\alpha_i^2\alpha_j + \frac{3}{2}\alpha_i\alpha_j\alpha_k}{\left(\frac{1}{2}\sum_{r=1}^{N}\alpha_r^2 + \frac{1}{2}\sum_{p=1}^{M}\beta_p^2\right)^{\frac{3}{2}}}. \tag{12}$$

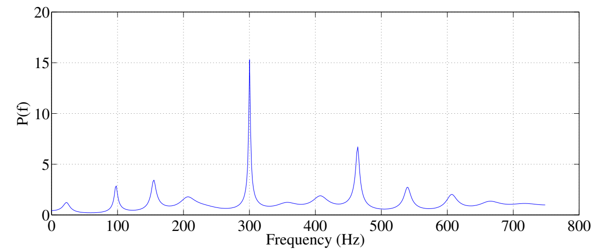


(a) Observed signal.

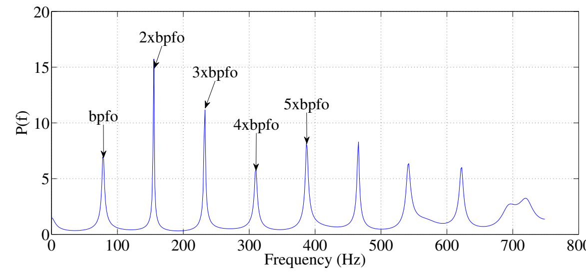


(b) Output signal after skewness maximization.

Fig. 2. Input and output signals for the experimental example. Outer race ball-pass period is marked.



(a) Frequency spectrum of the observed signal.



(b) Frequency spectrum of the output signal.

Fig. 3. Input and output signal AR frequency spectra. Outer race ball-pass frequency (bpfo = 77.5 Hz) and its harmonics are marked.

It is readily seen in (12) that increasing skewness is possible by amplifying the harmonically related components satisfying (6) and (7), while attenuating the ones without such relation. As mentioned in Sec. 2.2, we can state the problem in the opposite way to propose an approach where the goal is to uncover harmonic content with unknown fundamental period by maximizing the skewness of the given signal. Such an approach will be exploited in the next section to illustrate an experimental example on an industrial setting to support the analytical derivations.

3. EXPERIMENTAL EXAMPLE WITH MECHANICAL VIBRATIONS

Undesired vibrations in a rotating machinery may arise from misalignment or wear of rotating parts. An interference produced by these parts typically manifests itself at a multiple of rotational frequency.

Such disturbances can be modelled as sinusoids interfering to the signal of interest as in (11). To experiment such a case, an example from industry is presented.

The measurement signal shown in the upper plot of Fig. 2 was collected using an accelerometer magnet mounted to a fan. This measurement represents a typical observation of a vibration signal from an industrial environment. There is no sign of an impulsive signature that may correspond to a bearing defect. The estimated auto-regressive (AR) frequency spectrum P(f) [17] for this observed signal depicted in the upper plot of Fig. 3 shows an apparent sinusoid at 300 Hz, which is a multiple of the shaft rotation frequency (25 Hz). The outer race ball-pass frequency (bpfo) and inner race ball-pass frequency (bpfi) are calculated [18] to be 77.5 Hz and 122.5 Hz respectively, where there are no visible components at these frequencies and their harmonics stronger than the disturbance in the frequency spectrum.

The output signal shown in the bottom plot of Fig 2 was achieved by processing the observed signal by a 225-tap adaptive FIR filter (f) to maximize the skewness of its output through gradient ascent. The adaptation was accomplished in an iterative manner as $f^{k+1} = f^k + \mu\nabla_{\phi(f^k)}$ (with iteration index k) where $\mu > 0$ is a small, constant step size and $\nabla_{\phi(f^k)}$ is the gradient of skewness estimated with respect to the filter f. The gradient equations can be found in [6]. AR spectrum P(f) for the processed signal is presented in the bottom plot of Fig 3, where the 300 Hz disturbance was suppressed while the outer race defect frequency and its harmonics were amplified. The spectrum achieved after enhancing the skewness, thus promoting harmonic relation, provides information about the bearing status by identifying the existence of a defect. Detection of faults on rolling element bearings to prevent machine failure is an important aspect in predictive maintenance in industry.

The example illustrates that maximizing skewness through gradient ascent can be utilized to promote harmonic content with unknown fundamental frequency and to suppress non-harmonic components (sinusoidal interference) in an industrial setting.

4. CONCLUSION

Emerging from the concept of blind adaptation, skewness enhancement can be used to recover an unobservable impulsive signal of unknown fundamental period. This signal of interest may be obscured, or even completely buried by linear amplitude and phase distortion, possible disturbances, noise and interference. Modelling such an impacting source signal as an harmonic series enables the investigation of ability of skewness to discover buried harmonic content in an efficient way.

Aforementioned signal model with the structure of skewness for harmonic series was used to explore the theoretical basis for skewness maximization to promote harmonic content. It was shown that maximizing skewness is possible by promoting harmonically related components while suppressing the non-harmonics within a deterministic and periodic signal class.

An experimental example that supported the analytical results was performed. The effectiveness of the proposed method in application to adaptive filtering through gradient ascent was simply demonstrated. Skewness maximization resulted in discovery of harmonic content and suppression of a component without harmonic relation.

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