T WAVE ALTERNANS DETECTION IN ECG USING EXTENDED KALMAN FILTER AND DUALRATE EKF

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ABSTRACT

T Wave Alternans (TWA) is considered as an indicator of Sudden Cardiac Death (SCD). In this paper for TWA detection, a method based on a nonlinear dynamic model is presented. For estimating the model parameters, we use an Extended Kalman Filter (EKF). We propose EKF6 and dualrate EKF6 approaches. Dualrate EKF is suitable for modeling the states which are not updated in all time instances. Quantitative and qualitative evaluations of the proposed method have been done on TWA challenge database. We compare our method with that proposed by Sieed et al. in TWA challenge 2008. We also compare our method with our previous proposed approach (EKF25-4obs). Results show that the proposed method can detect peak position and amplitude of T waves in ECG precisely. Mean and standard deviation of estimation error of our method for finding position of T waves do not exceed four samples (8 msec).

***Index Terms*—** Electrocardiogram (ECG), T Wave Alternans (TWA), Extended Kalman Filter (EKF), Dualrate EKF.

1. INTRODUCTION

CardioVascular Diseases (CVD) are one of the major causes of mortality in humans [1]. A great part of these deaths occurs suddenly and is known as Sudden Cardiac Death (SCD) which has a high incidence. Implantable Cardioverter Defibrillators (ICD) are the most effective way of preventing SCD. However, implanting an ICD is an invasive procedure. Various non-invasive indices have been proposed to predict SCD such as QRS duration, QT dispersion, heart rate variablity and etc. T wave alternans (TWA) is one of the most promising non-invasive indices for SCD prediction [1]. It is a pattern in ECG characterized by two (rarely more) distinct forms of T waves appearing in alternation (like Fig.1). Several algorithms have been proposed to automatically detect TWA, employing linear and nonlinear signal processing techniques. Details of them can be found in [2].

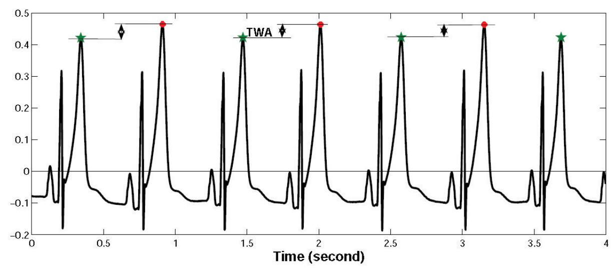

Fig. 1. Visible TWA in ECG signal [3].

A nonlinear dynamical model for generation of synthetic ECG signals has been developed by McSharry et al. [4]. Up to now, many researches extended and modified this model. In this paper and based on this model, we propose a new framework for detecting the T wave alternans in ECG signal. By introducing a simple AR model for each of the parameters of T Gaussian function and considering separate states for PQRS and T waves, new EKF structure (EKF6) is constructed. Firstly we use EKF6 approach for estimating the states, then we use the estimated states for finding the peak position and amplitude of T waves in ECG. We observe that in ECG signals with TWA, the amplitude of T waves in odd and even beats are different. We also propose "dualrate EKF6" approach, which assumes that some states (such as peak position of T wave, ...) are not necessary to be updated in all time instances and can only be updated in certain instances. It is for the first time that EKF-based frameworks have been used for T wave alternans detection. And also dualrate EKF-based approach is used for ECG analysis for the first time in this paper. For validation of our method, we will use ECG signals from TWA challenge database [5]. The proposed method is compared with that proposed by Sieed et al. [3, 6] in TWA challenge 2008. We also compare our method with our previous proposed approach (EKF25-4obs) [7]. Results show that the proposed methods can detect peak position and amplitude of T waves in ECG precisely.

This work partially supported by scholarship of French Embassy and the European project ERC-2012-AdG-320684-CHESS. GIPSA-lab is partner of the LabEx PERSYVAL-Lab (ANR–11-LABX-0025)

Due to space limitations, basics of EKF are not discussed in this paper. Details of them can be found in [8–10]. Previous EKF-based approaches are discussed in section 2. In section 3, we explain our proposed method ("EKF6" and "dualrate EKF6" approaches) for T wave alternans detection. In section 4, we present the results. Finally, discussion and conclusions are provided in section 5.

2. PREVIOUS EKF-BASED APPROACHES

A synthetic ECG generator has been proposed by McSharry et al. [4], which is based on a nonlinear dynamic model. Sameni et al. [8] transformed this model and proposed an Extended Kalman Filter (EKF) algorithm which was firstly used for ECG denoising. Discrete state-equations of this model (EKF2) are as follows:

$$\begin{cases} \varphi_{k+1} = (\varphi_k + \omega_k \delta) \ \ mod(2\pi) \\ z_{k+1} = -\sum_i \delta \frac{\alpha_{i,k}\omega_k}{b_{i,k}^2} \Delta\theta_{i,k} \exp(-\frac{\Delta\theta_{i,k}^2}{2b_{i,k}^2}) + z_k + \eta_k \end{cases} \tag{1}$$

where φ_k is the phase of ECG, ω_k is the beat-to-beat angular frequency of the RR interval. In this model it is assumed that z_k is a state variable which is a sum of 5 Gaussian functions ($i \in \{P, Q, R, S, T\}$). Each Gaussian function is defined with three main parameters: $\alpha_{i,k}$, $b_{i,k}$ and $\theta_{i,k}$ terms correspond to the amplitude, width and location of the Gaussian functions and $\Delta\theta_{i,k} = (\varphi_k - \theta_{i,k})\, mod(2\pi)$. δ is the sampling time, η_k is a random additive noise that models the inaccuracies of the dynamic model. Details of this model can be found in [8].

Sayadi et al. modified the EKF2 framework and proposed "EKF17" approach used for ECG denoising, compression [11] and beat segmentation of normal ECG signals [12]. They also proposed "EKF4" approach used for PVC detection [13] and ECG denoising [14].

Akhbari et al. [9] introduced a simple AR model for angular velocity of ECG (ω_k), considered it as a state of model and proposed "EKF3" and "EKF3-2" approaches used for ECG denoising. They also proposed EKF25 frameworks for ECG fiducial points extraction [7, 15]. In fact 25 parameters of ECG signal were considered as states of an EKF and peak, onset and offset of all characteristic waves (QRS complex, P and T waves) of ECG signal were found by this approach. For observations, they considered two cases: in [15] two observations were used, while four observations were used in [7]. The later, also was used for R-peak detection in non-invasive fetal ECG (fECG) signals which are acquired from multiple electrodes on mother's abdomen [16].

3. OUR PROPOSED METHOD

3.1. EKF6 Approach

In this paper by taking the idea of previous EKF-based approaches, we propose a new framework. We consider separate states for PQRS and T waves and also define an AR model for parameters of T Gaussian function and consider them as states. Discrete state and observation equations of our proposed model are defined in (2) and (3), respectively.

$$\begin{cases} \varphi_{k+1} = (\varphi_k + \omega_k \delta) \ \ mod(2\pi) \\ PC_{k+1} = -\sum_i \delta \frac{\alpha_{i,k}\omega_k}{b_{i,k}^2} \Delta\theta_{i,k} \exp(-\frac{\Delta\theta_{i,k}^2}{2b_{i,k}^2}) + PC_k + \eta_{PC_k}, \\ i \in \{P, Q, R, S\} \\ T_{k+1} = -\delta \frac{\alpha_{T,k}\omega_k}{b_{T,k}^2} \Delta\theta_{T,k} \exp(-\frac{\Delta\theta_{T,k}^2}{2b_{T,k}^2}) + T_k + \eta_{T_k} \\ \alpha_{T,k+1} = \alpha_{T,k} + u_{1,k} \\ b_{T,k+1} = b_{T,k} + u_{2,k} \\ \theta_{T,k+1} = \theta_{T,k} + u_{3,k} \end{cases} \tag{2}$$

$$\begin{cases} \Phi_k = \varphi_k + v_{1k} \\ PPCC_k = PC_k + v_{2k} \\ TT_k = T_k + v_{3k} \end{cases} \tag{3}$$

In (2), the first state is the phase of the ECG. PC_k and T_k are the PQRS and T waves of ECG which are separately considered as states. The parameters of the PQRS wave Gaussian functions are considered as process noise but the parameters of the T wave Gaussian function are considered as states 4 to 6 with first order AR dynamics but without corresponding observations. In fact in this approach we consider 6 parameters of ECG as states, so we call it "EKF6". The system state and process noise vectors are defined as:

$$\begin{aligned} &\underline{x}_k = [\varphi_k, PC_k, T_k, \alpha_{T,\,k}, b_{T,\,k}, \theta_{T,\,k}]^T \\ &\underline{w}_k = [\omega_k, \eta_{PC_k}, \eta_{T_k}, \alpha_{i,\,k}, b_{i,\,k}, \theta_{i,\,k}, u_{1,\,k}, u_{2,\,k}, u_{3,\,k}]^T, \\ &i \in \{P, Q, R, S\} \end{aligned} \tag{4}$$

In (3), the first equation corresponds to phase observation and others correspond to ECG observation in PQRS and T intervals, respectively: $\underline{y}_k = [\Phi_k, PPCC_k, TT_k]^T$ and $\underline{v}_k = [v_{1k}, v_{2k}, v_{3k}]^T$. Details of observation definitions can be found in [7].

Fig. 2 shows the blockdiagram of our proposed approach for finding the peak position of T waves. In this blockdiagram, $\hat{T}(\theta)$ is the estimated T wave by EKF6 model (third estimated state) and $T(\theta)$ is constructed from estimated T Gaussian parameters (states 4 to 6) as following:

$$T(\theta) = \hat{\alpha}_T \exp(-\frac{(\theta - \hat{\theta}_T)^2}{2\hat{b}_T^2}) \tag{5}$$

The proposed method for finding the peak position and amplitude of T waves, consists of the following steps:

- Considering the estimated T wave ($\hat{T}(\theta)$) and finding its maximum value location by (6). These points are called T_P and are the first candidate group for final peak position of T waves.

$$T_P = \underset{\theta}{\operatorname{argmax}}\, \hat{T}(\theta) \tag{6}$$

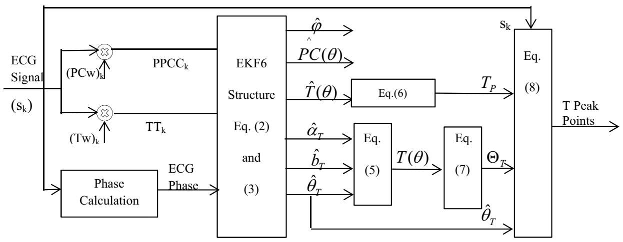

Fig. 2. Blockdiagram of proposed EKF6 approach for finding peak position of T waves.

- Construct $T(\theta)$ Gaussian function (by (5)) and find the location of maximum of absolute of this function by (7). These points are called Θ_T and are the second candidate group for final peak position of T waves.

$$\Theta_T = \underset{\theta}{\operatorname{argmax}}\ |T(\theta)| \tag{7}$$

- $\hat{\theta}_T$ is 6^{th} estimated state by EKF6 and can be considered as third candidate group for final peak position of T waves.
- Using a decision rule like (8) to find the final peak position of T wave (T_{peak}), where s_k is the observed (original) ECG signal. T_{amp} is the amplitude of T wave peaks.

$$\begin{aligned} T_{\text{peak}} &= \underset{T_P,\Theta_T,\hat{\theta}_T}{\operatorname{argmax}}\ (s_k(T_P), s_k(\Theta_T), s_k(\hat{\theta}_T)) \\ T_{\text{amp}} &= s_k(T_{\text{peak}}) \end{aligned} \tag{8}$$

3.2. Dualrate EKF6 Approach

Dualrate Kalman filter is a kind of Kalman filter used in cases which we have observations which are not in the same rate. For example, Kuure-Kinsey et al. used a dualrate Kalman filter for continuous glucose monitoring [17]. Their model has two measured outputs: sensor output and reference blood glucose (fingerstick) measurements, each on a different time scale. The sensor measurements are the fast time scale, with order of magnitude in minutes. The fingerstick measurements are more infrequent and on the slow time scale with order of magnitude in hours. For estimating the model parameters, they considered one dynamic equation and two observation equation sets as below [17]:

$$\begin{aligned} X_{k+1} &= \Phi X_k + \Gamma w_k \\ y_{f,k} &= C_{fast} X_k + v_{f,k} \\ y_{s,k} &= C_{slow} X_k + v_{s,k} \end{aligned} \tag{9}$$

Update at fast and slow sample time are given by predictor/corrector equations in (10) and (11), respectively [17].

$$\begin{aligned} \hat{X}_{k|k-1} &= \Phi \hat{X}_{k-1|k-1} \\ \hat{y}_{f,k|k-1} &= C_{fast} \hat{X}_{k|k-1} \\ \hat{X}_{k|k} &= \hat{X}_{k|k-1} + K_k^{fast}(y_{f,k} - \hat{y}_{f,k|k-1}) \end{aligned} \tag{10}$$

$$\begin{aligned} \hat{X}_{k|k-1} &= \Phi \hat{X}_{k-1|k-1} \\ \hat{y}_{s,k|k-1} &= C_{slow} \hat{X}_{k|k-1} \\ \hat{X}_{k|k} &= \hat{X}_{k|k-1} + K_k^{slow}(y_{f,k} - \hat{y}_{s,k|k-1}) \end{aligned} \tag{11}$$

For solving the model which has different measurement rates, we can consider two different observations and predictor/corrector equations (as above) or we can actually let Kalman filter run as normal, that is with the same rate for both corrector/predictor parts, but in some time instances we force the effective Kalman gain to become zero [18]. This can be done by simply multiplying K_k by a factor, say a, so that the corrected estimate is:

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + aK_k(y_k - \hat{y}_{k|k-1}) \tag{12}$$

$a = 1$ is default value, to be used when $\hat{X}_{k|k}$ be updated by the measurement (via the innovation process), and $a = 0$ is used when $\hat{X}_{k|k}$ shall not be updated, implying that $\hat{X}_{k|k}$ is equal to $\hat{X}_{k|k-1}$ [18]:

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + 0K_k(y_k - \hat{y}_{k|k-1}) = \hat{X}_{k|k-1} \tag{13}$$

In this paper beside EKF6 approach, we also implement "dualrate EKF6" approach. In fact, in dualrate approach we assume that $\alpha_{T,k}$, $b_{T,k}$ and $\theta_{T,k}$ states in (2) are not updated in all time instances and are only updated in certain time points beside the peak position of T (i_T). i_T is a rough approximation of T-peak positions which are found by adding a constant value to the location of R-peaks. So in dualrate EKF6 approach, we assume that in the Kalman gain matrix (K_k), the corresponding elements of $\alpha_{T,k}$, $b_{T,k}$ and $\theta_{T,k}$ states have only values in certain time instances (i_T) and we force these elements to zero in other instances.

4. RESULTS

For validation of our method, we use TWA challenge database. It contains 100 multichannel ECG records sampled at 500 Hz. By following the procedure of Fig.2 and equations (5)-(8), peak position and amplitude of T waves have been detected.

Sieed et al. achieved the best score in TWA challenge 2008 [19]. They first detected the local maximum value in each ECG beat and considered it as R-peaks (normally R is the highest peak in an ECG beat of lead-I or II) and after that defined a interval between two consecutive R-peaks and found the local maximum value in this interval and considered it as T-peaks [6]. We compare our proposed method with Sieed et al. method and also with our previous proposed approach (EKF25-4obs) [7]. For a few records of TWA challenge database, the position of T wave is defined by "ecgpuwave" software [20]. We calculate estimation error which is defined as time differences between results of "ecgpuwave" software (considered as ground truth) and our proposed method. Table 1 shows the mean (m) and standard deviation (SD) of estimation error of EKF6, dualrate EKF6, EKF25 and Sieed et al. approaches for finding the T-peak position. We can see that EKF6 and dualrate EKF6 can detect T-peak position better than other methods; their 'm" and "SD" values do not exceed four samples (8 msec).

Table 1. Mean and SD of errors (msec) between estimated T-peak position and annotations defined by "ecgpuwave" software

	$m \pm SD\ (\mu V)$			
Record	EKF6	Dualrate EKF6	EKF25	Sieed et al.
twa01	6.9 ± 1.2	7 ± 1.3	8.3 ± 1.9	9.6 ± 0.8
twa10	2.5 ± 4.3	2.3 ± 4.6	10.2 ± 3.7	7.2 ± 3.4
twa91	7.2 ± 1.2	7.2 ± 1.6	8.6 ± 2.4	9.8 ± 1.2
twa93	2.8 ± 7.3	2.7 ± 7.6	11.2 ± 7.6	9 ± 4.2

After estimating the T waves successfully, then we separate odd and even T-peaks; mean and standard deviation of their amplitudes are calculated. Results are given in table 2. The difference between T- amplitude of odd and even beats is known as T wave alternans. Mean and standard deviation of T wave alternans are also given in table 2.

TWA challenge database includes different records from various databases such as long-term ST database, sudden cardiac death, normal sinus rhythm and synthetic ECG. Here we use three different kinds of ECG of this database:

- Records twa01, twa29, twa70, twa91 and twa97 are synthetic ECG with TWA and from table 2, we can see that these records have significant TWA and all the approaches have reasonable results; results of EKF6 and dualrate EKF6 are as good as other methods.
- Records twa10 and twa93 are normal sinus rhythm. In table 2, we can see that all methods have a large variance and more than their mean; which are not reasonable. Since these two records are normal, maybe they can not be considered as a signal with TWA and maybe their odd and even T waves may not appearing in alternations.
- Records twa52 and twa81 are synthetic ECG without TWA and from table 2, we can see that they have no significant TWA as we expected.

5. DISCUSSION AND CONCLUSIONS

In this paper, we proposed a method for finding the peak position and amplitude of T waves and consequently the T wave alternans in ECG signals. By introducing a simple AR model for each of the parameters of T Gaussian function and considering separate states for PQRS and T waves, new EKF structure (EKF6) is constructed. We also propose dualrate EKF6 approach and assume that parameters of T Gaussian function ($\alpha_{T,k}$, $b_{T,k}$ and $\theta_{T,k}$) are not updated in all time instances and are only updated in certain instances beside the T peak points. For validation of our proposed approach, we use TWA challenge database and compare our results with the results of the team (Sieed et al.) which had the best scores in challenge 2008 and also with our previous proposed approach (EKF25-4obs). We see that our proposed methods can detect the T-peak position better than other methods and 'm" and "SD" values of their estimation error do not exceed four samples (8 msec). For TWA amplitude calculation, all methods have good and reasonable results for synthetic ECG signals with TWA (twa01, twa29, twa70, twa91 and twa97). For signals which are normal (twa10 and twa 93), results are not reasonable as we expected and for synthetic ECG signals without TWA (twa52 and twa81), results show no significant TWA as we expected. So our proposed methods can distinguish between different kinds of signals (signals with TWA, normal signals and signals without TWA) and their results are as good as other mentioned methods. Future work will include intensive experiments on more TWA signals using this approach.

Table 2. Comparison of mean and SD of T wave amplitudes for odd and even beats and TWA amplitude obtained by different approaches.

	$m \pm SD\ (\mu V)$			
Record	Method	odd beats	even beats	TWA Amp.
twa01	EKF6	338.6 ± 1.8	328.2 ± 0.4	10.7 ± 1.7
	D.R.EKF6	339.2 ± 1.9	328.2 ± 0.4	11 ± 1.9
	EKF25	338.9 ± 1.4	325.6 ± 3.6	13.2 ± 3.4
	Sieed et al.	337.3 ± 0.7	325.5 ± 0.9	11.7 ± 0.9
twa29	EKF6	489 ± 13.3	455 ± 2.3	34 ± 11.6
	D.R.EKF6	492.2 ± 8	454.6 ± 2.3	37.6 ± 6.6
	EKF25	491 ± 5.4	452.2 ± 4.3	38.9 ± 4.4
	Sieed et al.	494 ± 2.6	452.3 ± 4.2	41.7 ± 4.8
twa70	EKF6	565.1 ± 3.4	557 ± 2.3	8.1 ± 3.2
	D.R.EKF6	562.2 ± 4	554.4 ± 3	7.85 ± 5.6
	EKF25	566 ± 3.8	558.7 ± 1.4	7.2 ± 3.7
	Sieed et al.	567.6 ± 1.4	559.2 ± 1.6	8.5 ± 1.2
twa91	EKF6	336.5 ± 2.5	321 ± 1	15.5 ± 2
	D.R.EKF6	336.8 ± 2.5	320.4 ± 1.8	16.4 ± 2.5
	EKF25	333 ± 5.6	318.3 ± 5.4	14.8 ± 5
	Sieed et al.	334.2 ± 3.2	318.2 ± 2.7	16 ± 2.8
twa97	EKF6	493 ± 3.4	479.8 ± 1.6	13.2 ± 4.2
	D.R.EKF6	492.7 ± 3.4	479.8 ± 1.6	13 ± 4.2
	EKF25	492 ± 5.3	475.7 ± 6	16.4 ± 9.8
	Sieed et al.	489.9 ± 4	475.4 ± 4	14.5 ± 6
twa10	EKF6	492.8 ± 15	484.7 ± 16.7	8 ± 18.8
	D.R.EKF6	492.8 ± 15	484.3 ± 17	8.5 ± 19.2
	EKF25	493.5 ± 17.9	487.9 ± 14.8	5.6 ± 13.2
	Sieed et al.	495.5 ± 16.1	492.4 ± 13.2	3.2 ± 14.5
twa93	EKF6	776.7 ± 67	825 ± 73.3	48.3 ± 76.7
	D.R.EKF6	776.3 ± 66.7	824.5 ± 72.7	48.2 ± 75.3
	EKF25	764.6 ± 133.2	827.5 ± 72	62.8 ± 82.3
	Sieed et al.	808.8 ± 62.2	832.7 ± 77	23.8 ± 73.2
twa52	EKF6	677.6 ± 3	676.8 ± 2.8	0.73 ± 4.8
	D.R.EKF6	677.1 ± 4.8	677.5 ± 2.5	-0.36 ± 5.5
	EKF25	675.6 ± 5.6	674.3 ± 5	1.26 ± 8.6
	Sieed et al.	675 ± 2.4	673.8 ± 3.3	1.26 ± 4.1
twa81	EKF6	677.5 ± 2.3	677.3 ± 2	0.17 ± 2.4
	D.R.EKF6	678 ± 1.6	677.7 ± 1.9	0.27 ± 1.5
	EKF25	677.5 ± 2.5	677.5 ± 2.5	-0.05 ± 2.4
	Sieed et al.	673.4 ± 4.5	673.8 ± 3.5	-0.38 ± 4.7

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