SUB-NYQUIST 1 BIT SAMPLING SYSTEM FOR SPARSE MULTIBAND SIGNALS

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ABSTRACT

Efficient sampling of wideband analog signals is a hard problem because their Nyquist rates may exceed the specifications of the analog-to-digital converters by magnitude. Modulated Wideband Converter (MWC) is a known method to sample sparse multiband signals below the Nyquist rate and the precision recovery relies on high precision quantization of samples which may take a great bit-budget. This paper proposes an alternative system that optimizes space utilization by applying comparator in sub-Nyquist sampling system. The system first multiplies the signal by a bank of periodic waveforms, and then it performs lowpass filtering, sampling and quantization through the comparator which just keeps the sign information. And we introduce a corresponding algorithm for perfect recovery. The primary design goals are efficient hardware implementation and low bit-budget. We compare our system with MWC to prove its advantages in condition of fixed bit-budget, particularly in low levels of input signal to noise ratio.

Index Terms— Compressive sensing, sub-Nyquist sampling, sparse multiband signal, modulated wideband converter, 1 bit quantization

1. INTRODUCTION

Analog to digital conversion (ADC) lies at the heart of modern signal processing, isolating the delicate interaction with the continuous world. The well-known Shannon sampling theorem states that a real-valued signal can be perfectly reconstructed only if the sampling rate is twice higher than the maximal frequency of the signal [1].

In this paper, we consider the class of multiband signals and Fig.1 depicts a typical communication application, the wideband receiver, in which the received signal follows the multiband model [2]. In today's technology, their Nyquist-rates exceed the capabilities of commercial ADC devices by far. So we focus on sub-Nyquist systems which sample at sub-Nyquist rates. Previous work on multiband signals has shown that it is possible to reduce the sampling rate by acquiring samples from a periodic but nonuniform grid [3]. Multi-coset sampling, a specific strategy of this type, was analyzed in [4], which established that exact recovery is possible when the band locations are known.

In many conditions, the carrier frequencies of signals are unknown or vary with time, a spectrum-blind receiver at the sub-Nyquist rate catches our attention. The conventional spectrum-blind sampling system named as modulated wideband converter (MWC) is proposed in [2] and [5]. The MWC system has prior knowledge of the number of bands and their widths of signals before sampling. The signals can be reconstructed with Simultaneous Orthogonal Matching Pursuit algorithm (SOMP) [6].

In practice, we often must limit the total number of measured bits and the real-valued measurements should be mapped to discrete bits during signal acquisition [7]. The above multiband signal sample systems can be applied to recovery signals with ideal sampling which ignore the problem of limited memory space for big data. The total number of measurement bits is constrained, which suggests a tradeoff between the number of measurements and the number of bits per measurement. Furthermore the input signals may contain noises [8]. Considering that we proposed a sub-Nyquist 1 bit sampling system for sparse multiband signals which is achieved by replacing ADCs with comparators in the MWC systems. We also put forward a robustness algorithm to reconstruct the band locations of the signals from our 1 bit samples. In the condition of the total number of measurement bits limited, numerical simulation experiments justify the robustness of our approach against noise.

The rest of this paper is organized as follows. Section 2 depicts the theoretical background of this work. Our sub-Nyquist 1 bit sampling system and the corresponding reconstruction algorithm are described in section 3. Section 4 presents numerical simulation results in different noise levels and the conclusion is given in section 5. Finally, a review of related work concludes the paper in Section 6.

2. THEORETICAL BACKGROUND

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2.1. The signal model

Let $x(t)$ be a real-valued, finite-energy, continuous-time signal. Throughout this paper, continuous signals are assumed to be bandlimited to $\mathcal{F}=[-1/2T,+1/2T)$. Formally, the Fourier transform of $x(t)$ is defined by

$$X(f)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt \quad (1)$$

where $f\in\mathcal{F}$. The Nyquist frequency is f_{NYQ}. The spectral support sets of Fourier transform $X(f)$ are contained within a union of N disjoint intervals(bands) in $\mathcal{F}$, and each of the bandwidths does not exceed D. A typical sparse multiband signal whose spectral support has Lebesgue measure [1] is illustrated in Fig. 1.

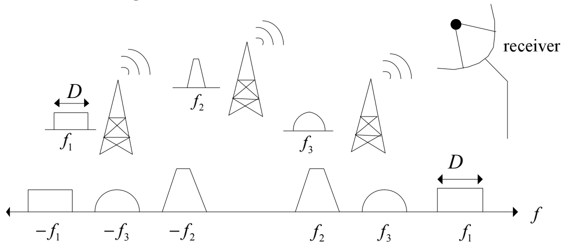

Fig.1. Three RF transmissions with different carries f_i.

2.2. Modulated wideband converter

The MWC is a multi-channel uniform sub-Nyquist sampling strategy for acquiring sparse multiband signals [2]. The system consists of m channels, which is shown in Fig.2. The meanings of the different parameters in the system are shown as following:

Symbol	Explanations
m	The number of sampling channels
T_p	The period of each $p_i(t)$
T_s	The sampling interval
M	number of ± 1 intervals in each period of $p_i(t)$
α_{ik}	the value $p_i(t)$ takes on the kth interval

Table 1. The MWC system parameter declaration

The sparse multiband signal $x(t)$ enters m channels simultaneously. Each of the m channels in the MWC system first multiplies $x(t)$ by a random periodic signal $p_i(t)$, i = 1, . . . , m. After mixing, the spectrum of $\tilde{x}_i(t)$ is truncated by a low pass filter with cutoff $1/(2T_s)$. The filtered signal is sampled and quantized synchronously in all channels at rate $f_s=1/T_s$ (see Fig. 2) after passing the ADC [9]. The sampling rate $1/T_s$ matches the cutoff of $H(f)$ Then, we obtain time and amplitude discrete samples $y_i[n]$. We define $p_i(t)$ as follow,

$$p_i(t)=\alpha_{ik},k\frac{T_p}{M}\le t\le(k+1)\frac{T_p}{M},0\le k\le M-1 \quad (2)$$

with $a_{ik}\in\{+1,-1\}$, and $p_i(t+nT_p)=p_i(t)$ for $n\in\mathbb{Z}$.

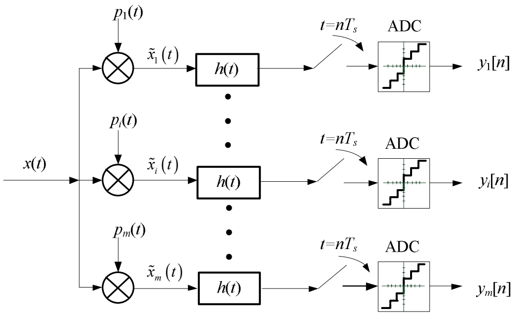

Fig.2. The framework of modulated wideband converter

2.3. The frequency analysis

Let $x(t)$ be a sparse multiband signal. We define

$$\begin{aligned} f_p &= 1/T_p, \mathcal{F}_p=[-f_p/2,+f_p/2] \\ f_s &= 1/T_s, \mathcal{F}_S=[-f_s/2,+f_s/2] \end{aligned} \quad (3)$$

Consider the ith channel. Since $p_i(t)$ is periodic, it has a Fourier expansion as (4) and c_{il} defined in (5) are the Fourier series coefficients of $p_i(t)$.

$$p_i(t)=\sum_{l=-\infty}^{\infty}c_{il}e^{j\frac{2\pi}{T_p}lt} \quad (4)$$

$$c_{il}=\frac{1}{T}\int_0^{T_p}p_i(t)e^{-j\frac{2\pi}{T_p}lt}dt \quad (5)$$

The Fourier transform of the analog multiplication $\tilde{x}_i(t)=x(t)p_i(t)$ is evaluated as,

$$\begin{aligned} \tilde{\mathbf{X}}_i(f) &= \int_{-\infty}^{+\infty}\tilde{x}_i(t)e^{-j2\pi ft}dt \\ &= \sum_{l=-\infty}^{\infty}c_{il}X(f-lf_p) \end{aligned} \quad (6)$$

Therefore, the input to $H(f)$ is a linear combination of f_p-shifted copies of $X(f)$. Filtering $\tilde{\mathbf{X}}_i(f)$, results that only frequencies in interval $\mathcal{F}_s$ are contained in the uniform sequence $y_i[n]$. Thus, the discrete-time Fourier transform (DTFT) of the ith sequence $y_i[n]$ is expressed as

$$\begin{aligned} Y_i(e^{j2\pi fT_s}) &= \sum_{n=-\infty}^{\infty}y_i[n]e^{-j2\pi fnT_s} \\ &= \sum_{l=-L_0}^{+L_0}c_{il}X(f-lf_p), f\in\mathcal{F}_S \end{aligned} \quad (7)$$

We define $\mathbf{z}(f)=\mathbf{X}(f-lf_p)$, where $-L_0\le l\le L_0$, $L_0=[\frac{f_{NYQ}+f_s}{2f_p}]-1$. So we write (7) in matrix form as

$$\mathbf{y}(f)=\mathbf{A}\mathbf{z}(f) \quad (8)$$

where $f \in \mathcal{F}_S$, $\mathbf{y}(f)$ is a vector of length m and $\mathbf{A}$ is an $m \times L$ matrix. Equation (8) refers to an infinite measurement vectors (IMV) problem,

$$\mathbf{z}_i(f) = \mathbf{X}(f + (i - L_0 - 1) f_p), 1 \le i \le L, f \in \mathcal{F}_s \quad (9)$$

where $L = 2L_0 + 1$.

2.4 Signal reconstruction

Theorem 1[10]: Let $x(t)$ be a sparse multiband signal and assume $T_p = M / f_{NYQ}$ for an integer M and $T_s = T_p$. If

1. $M \le f_{NYQ} / D$
2. $m \ge 2N$ for non-blind reconstruction or $m \ge 4N$ for blind
3. $\mathbf{S} = \{a_{ik}\}$ is such that every $4N$ columns are linearly independent

then, for every $f \in \mathcal{F}_S$, the vector $\mathbf{z}(f)$ is the unique $2N$-sparse solution of (8).

With finite number of snapshots, denoted by d, MWC falls into the framework of solving a multiple measurement vectors (MMV) problem, which is a known NP-hard problem [6]. Simultaneous Orthogonal matching pursuit algorithm (SOMP) can be used to recover the support S=supp(z[n]) by solving (10).

$$\mathbf{y}[n] = \mathbf{A}\mathbf{z}[n] \quad (10)$$

where $\mathbf{z}[n] = [\mathbf{z}_{-L}[n], ..., \mathbf{z}_L[n]]^T, n = 1, 2, ..., d$, and $\mathbf{z}_l[n]$ is the inverse DTFT of $\mathbf{z}_l[f]$. The sequence $\mathbf{z}_l[n]$ denotes f_p-rate samples sequence of the lth spectrum slice of $x(t)$, $\mathbf{y}[n] = [\mathbf{y}_1[n], ... \mathbf{y}_m[n]]^T$. Once the support S has been identified, we can recovery the multiband signal by solving (11) with the support S,

$$\mathbf{z}_S[n] = \mathbf{A}_S^\dagger \mathbf{y}[n] \,;\, z_i[n] = 0, \;\; i \notin S \quad (11)$$

where $\mathbf{A}_S^\dagger = (\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H$.

3. THE PROPOSED APPROACH

3.1 The proposed system

In many potential applications, we must constrain the total number of measurement bits, which suggests a tradeoff between the number of measurements and the number of bits per measurement as mentioned above [11]. In order to alleviate the pressure of hardware, we attempt to sacrifice quantization precision to improve the number of measurements. So we proposed a novel sampling system named as sub-Nyquist 1 bit sampling system for sparse multiband signals as shown in Fig.3.

The comparator can obtain the sign of each sample and represent it with ± 1. Signal $x(t)$ pass through the above system, then the samples become $\bar{y}_i[n] = sign(y_i[n])$.

$$\bar{y}_i[n] = \begin{cases} +1 & y_i[n] > 0 \\ -1 & y_i[n] \le 0 \end{cases} \quad (12)$$

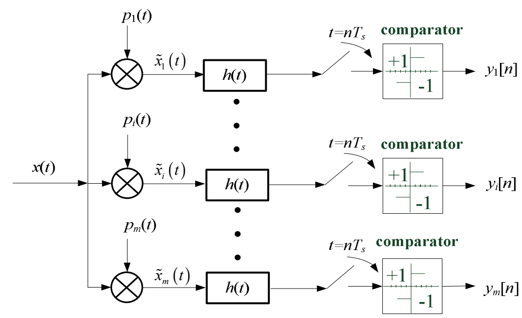

Fig.3. Sub-Nyquist 1 bit sampling system for sparse multiband signals

3.2. Algorithm description

The problem of reconstructing a sparse signal from its 1 bit measurements was introduced in [11,12]. In this paper, we utilize the sign information of samples to recover original signals which is beyond the capacity of SOMP algorithm.

So we propose a specialized reconstruction algorithm which is named simultaneous Binary Iterative Hard Thresholding l_2 Norm algorithm (SBIHTl_2) which is some kind of extension on the 1 bit compressive sensing recovery method-Binary Iterative Hard Thresholding l_2 Norm (BIHTl_2) [12]. [12] also shows the BIHTl_2 algorithm has favorable performance compared to other 1-bit CS algorithms in noisy regime.

The sparse multiband signal $x(t)$ passes through the sub-Nyquist 1-bit sampling system and we obtain the 1 bit samples $\bar{\mathbf{y}}$ at the part of system output. We recover the frequency spectrum support set of the multiband signals S by solving (13) with the SBIHTl_2 algorithm.

$$\bar{\mathbf{y}}[n] = \mathbf{A}\mathbf{z}[n] \quad (13)$$

The main procedure of the SBIHTl_2 algorithm can be described as follows.

Input: $\mathbf{A}$, $\bar{\mathbf{y}}[n]$, m, N; **Output:** support S;

Initialization:

$$\mathbf{z}_0 = \frac{\mathbf{A}^T \bar{\mathbf{y}}}{\left\| \mathbf{A}^T \bar{\mathbf{y}} \right\|_0} \quad \{\text{Initial approximation } \mathbf{z}_0\}$$

$$\mathbf{y}_0 = \bar{\mathbf{y}}.\times \max\{\bar{\mathbf{y}}.\times(\mathbf{A}\mathbf{z}_0),\ 0\} \quad \{\text{Initial approximation } \mathbf{y}_0\}$$

Iteration: at the i-th iteration, go through the next steps.

1) compute: $\mathbf{a}^{i+1} = \frac{1}{m} \mathbf{A}^T (\bar{\mathbf{y}} - \mathbf{A}\mathbf{z}_i)$;

2) update: $\mathbf{z}_{i+1} = \mathbf{z}_i + \mathbf{a}^{i+1}$;

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3) update:

$$\mathbf{z}_{i+1} = \eta(\text{sum}(\text{abs}(\mathbf{z}_{i+1}),2),2N)./sqrt(diag(\mathbf{A}^T\mathbf{A}))$$

4) update: $\mathbf{y}_{i+1} = \mathbf{A}\mathbf{z}_{i+1}$;

compute the different number between $\mathbf{y}_i$ and $\bar{\mathbf{y}}$ by computing $numb = nnz(abs(\bar{\mathbf{y}} - sign(\mathbf{y}_{i+1})))$;

5) if $numb > 0$ or $i < tol$, i=i+1 and return step 1;

6) output $\mathbf{z}_{i+1}$ and obtain the support set S=supp($\mathbf{z}_{i+1}$).

where $\eta(\mathbf{v}, 2N)$ computes the best $2N$-term approximation of $\mathbf{v}$ by thresholding [13].

4. NUMERICAL RESULTS

To evaluate the empirical performance of our proposed system (see Fig. 3), we can simulate the action of the system on test signals contaminated with Gaussian white noise. The meaning of $\mathcal{B}$ is the bit-budget occupied by samples,

$$\mathcal{B} = md \times B \tag{14}$$

where B is the bit-depth of each samples and the total number of samples is $m \times d$ and m is the number of system channels.

More precisely, we evaluate the performance on 500 noisy test signals of the form $x(t) + e(t)$. The multiband signals consist of $N = 3$ pairs of bands, each of width $D = 50$ MHz, constructed using the formula

$$x(t) = \sum_{i=1}^{3} \sqrt{E_i D} \sin c(D(t-\tau_i)) \cos(2\pi f_i(t-\tau_i)) \tag{15}$$

where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. The energy coefficients E_i and carrier frequency f_i are random and the time offsets are τ_i={0.7 0.4 0.3} μsecs.

We additionally add and scale Gaussian noise so that the test signal has the desired input signal to noise ratio (ISNR). The experiments are performed as described previously, for $\mathcal{B} = [150d, 550d]$ and bit-depth B=1, 4, 8 with the SBIHTl_2 and SOMP recovery algorithms. Since the sample ratio f_s is unchanged, the number of samples in each channel d is fixed. Then, increasing the number of channels is equal to increasing the number of measurements. The success is defined when exact support recovered. Fig.4,5 depict the experiment for the input ISNR=20, 5dB respectively.

These simulations demonstrate that the 1 bit quantization framework performs significantly better than the multi-bit quantization framework for low ISNRs. When the total number of measurement bits is fixed and noise is present on the input signals, we analyze the quantization influence at low ISNR inspired by [14]. For lower ISNR, it is beneficial to choose smaller bit-depths B for more measurements. Additionally, the sub-Nyquist 1 bit sampling system for sparse multiband signals performs competitively with or better than MWC system which applies SOMP algorithm for all ISNRs tested.

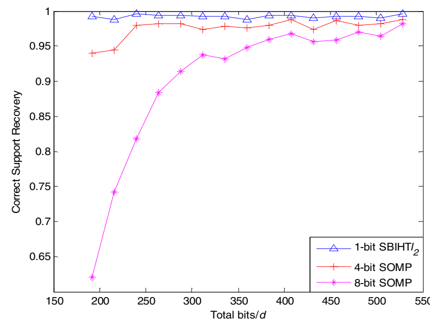

Fig.4. Reconstruction probability as a function of total bits/d for ISNR=20dB

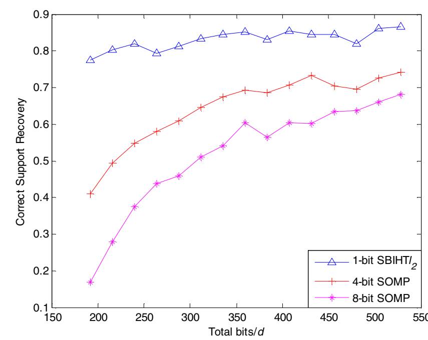

Fig.5. Reconstruction probability as a function of total bits/d for ISNR=5dB

5. CONCLUSIONS

The main contribution of this paper is proposing a novel sub-Nyquist 1 bit sampling system for sparse multiband signals to reduce hardware complexity. We also develop an efficient recovery algorithm-SBIHTl_2 which can reconstruct the support of multiband signals with the signs of samples. We can summarize that considering the quantization effect in condition of fixed bit-budget $\mathcal{B}$, at low ISNR, we should take more number of channels with small bit-depth of each measurement. And the work of replacing ADCs with comparators to improve system performance in noise environments has not been considered in the earlier studies. In practice, our proposed system is expected to be used in cases where the bit-budgets are significantly lower than in a conventional multi-channel sampling system and large input noise levels.

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