A NEW APPROACH TO WAVELET ENTROPY: APPLICATION TO POSTURAL SIGNALS

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ABSTRACT

This study proposes a new approach for quantifying complexity of physiological signals characterized by a spectral distribution in modes. Our approach is inspired by wavelet entropy but based on a modal representation: Synchrosqueezing transform. It is calculated for each time sample within the cone of influence of the decomposition. This index is first validated and discussed on simulated multicomponent signals. Finally, it is applied to assess postural control and ability at using all the sensory resources available. Results show significant differences in our index following an induced change in sensory conditions whereas a conventional approach fails. This index may constitute a promising tool for detection of postural troubles.

***Index Terms*—** Synchrosqueezing transform, Wavelet entropy, Complexity, Postural signals, CoP data.

1. INTRODUCTION

Assessing complexity of a multicomponent physiological signal is a way to give insights into the integrity of the underlying system. Entropy is the most well-spread way to quantify degree of order and therefore complexity. However statistical approaches used for physiological signals, such as sample entropy (SEn), suffer from a lack of reliability [1]. This rises the question of taking into account phenomenons occurring at the different scales at stake. Different methods have been proposed to attempt to integrate these multiple scales within the calculation of entropy. In the case of physiological signals characterized by a spectral distribution in modes such as locomotor or postural ones, a method called Intrinsic Mode Entropy (IME) was proposed thereby combining empirical mode decomposition (EMD) and sample entropy [2]. This approach, however, encounters some drawbacks relative to the use of EMD (*e.g.* mode mixing phenomenon, averaging of modes with different frequency content) and the use of SEn (parameters selection).

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From the point of view of dynamical systems, complex processes are associated with large spectrum, offering a wealth of information contrary to periodic ones which are monotonous, repeatable and deprived of information. The hypothesis of Goldberger about the human body is that aging and disease are characterized by a loss in spectral resources whereas rehabilitation depends on their broadening and the ability to extend itself [3, 4]. Estimating complexity of a dynamical system may then consist in quantifying its spectral wealth.

Our aim is to develop an index of complexity consistent with oscillatory multicomponent signals. The proposed approach is expected to be: (1) adaptive *i.e.* requires minimum a priori assumptions, (2) local to take into account nonstationarities and (3) readable for component identification. Therefore, a novel approach of wavelet entropy (WE) [5] is proposed and based on Synchrosqueezing transform (SQT). The original index is first described. Secondly, changes operated are detailed: (1) the underlying time-frequency representation and (2) the building of a local index. A validation step is performed on a simple composite model and a postural-like multicomponent model. A preliminary study on real CoP signals is finally described.

2. CHANGES FROM WE

2.1. Time-frequency representation from SQT

The WE is originally based on orthogonal discrete wavelet transform (ODWT). As for the orthogonal feature of the decomposition basis, it makes the calculation of energy particularly easy and directly linked to the coefficients of the time-frequency representation (TFR). The main drawback of this method is that it proceeds to a non-adaptable subband filtering whose limits and width are fixed by the sampling frequency. Therefore, the resolution is completely fixed and two modes hold within the same subband cannot be distinguished. In our approach, SQT is used instead of ODWT. It offers a modal representation which is consistent with the spectral content of multicomponent physiological signal by reassigning coefficients of the CWT in frequency.

2.2. Building a local index of complexity

In the case of WE, the monitoring of the temporal evolution is obtained from a segmentation in the temporal domain with a rectangular windowing function. Then the ODWT is calculated on each of these segments following by the WE. The window length must be fixed by the user at the beginning of the analysis involving assumptions about the dynamics of the phenomenon. Concerning CoP data, such a knowledge is not available and *a priori* hypotheses are not desirable. A recent approach inspired by the Welch's method was proposed by Xu et al. to reduce the variability of the estimation [6]. It consists, for each segment, in applying a sliding window on which WE is calculated and averaged.
To avoid a windowing approach, SQT is calculated on the whole signal. A temporal index of entropy is obtained thereby calculating WE_{SQT} at each sample in coherence with the spectral dynamics of the underlying CWT. Indeed, energy is more localized on small scales than on large ones. Suppose that the temporal support of the mother wavelet ψ is compact $[-C, +C]$, the cone of influence of the representation at time b_0 contained all the points whose coordinates (b, a) verifying that b_0 is included in the support of the atom $\psi_{b,a}$ [7]:

$$\begin{aligned} \text{Cone}_{b_0} &= \{(b,a)|b_0 \in [b - Ca; b + Ca]\} \\ &= \{(b,a)||b_0 - b| \leq Ca\} \end{aligned}$$

To take this behavior into account, coefficients located within the cone of influence of the representation are assigned to the considered sample and weighted to favor those close to its center. The contribution of a coefficient at scale a and time b, $W_{b,a}$, is weighted by a factor $\frac{|b-b_0|}{a}.\mathbf{1}_{\text{cone}}$. In this way, the temporal index of complexity corresponds to all the local modes depending on the scattering intrinsic to the CWT. The spectral dynamics of the index depend on the overlap between two consecutive windows and thus on the local behavior of the CWT.

2.3. Calculation of WE_{SQT}

Given s a signal, $C_j(k)$ the coefficient of the SQT and $\text{Supp}_{\text{Cone(k,j)}}$ the support of the cone of influence at time sample k and scale j ($1 \leq j \leq a_{max}$ where $a_{max} = f_0/f_{min}$, f_0 is the central frequency of the mother-wavelet and f_{min} is the inferior boundary of the frequency range), the WE_{SQT} at time sample i is calculated from:

$$E_j(i) = \sum_{k \in \text{Supp}_{\text{Cone(k,j)}}} \frac{|b - b_0|.|C_j(k)|^2}{a}, \text{ energy at scale j} \quad (1)$$

$$E_{\text{tot}}(i) = \sum_{j=1}^{a_{max}} E_j, \text{ total energy} \quad (2)$$

$$p_j(i) = \frac{E_j(i)}{E_{\text{tot}}(i)}, \text{ relative energy at scale j} \quad (3)$$

$$WE_{\text{SQT}}(i) = -\sum_{j=1}^{a_{max}} p_j(i)\ln(p_j(i)), \text{ SQT entropy at time i.} \quad (4)$$

3. WE_{SQT}: VALIDATION ON SIMULATED SIGNALS

3.1. A preliminary composite model

WE_{SQT} is tested on a preliminary composite model which parametrization is consistent with the frequency range of postural data. This step aims at illustrating the functioning of the WE_{SQT} and its ability at detecting changes in complexity.

Method The SQT is calculated on the whole signal: f_0 = 6Hz, f_{min} = 0.11Hz, 64 voices. This signal is the sum of three sines at 1.5 and 3Hz respectively appearing at 0s and 100s. The record lasts 200s and is sampled at 40Hz (Fig.1).

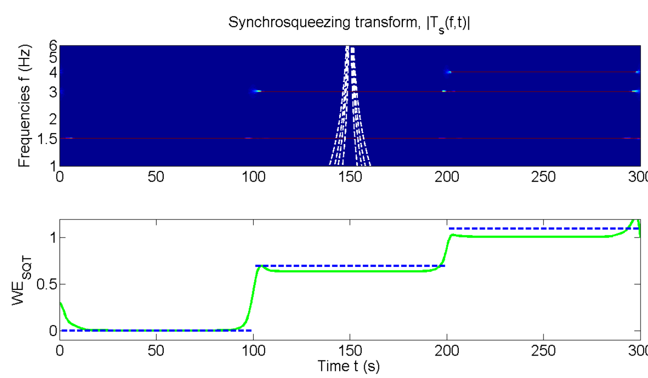

Fig. 1: SQT and WE_{SQT} of a sum of sines appearing gradually at 1.5Hz and 3Hz. Four contour lines of the cone of influence are represented at 150s on the reallocated scalogram. The corresponding WE_{SQT} (plain line) is compared with the expected level of complexity (dotted line).

Observation and discussion WE_{SQT} increases as a new component appears. As expected it is equal to zero in the presence of only one component. WE_{SQT} renders qualitatively variations in complexity but a weak bias may be noted comparatively to the theoretical values from a quantitative point of view. This point will be discussed in further works.

3.2. A postural-like multicomponent model

WE_{SQT} is then tested on a statistical model close to what it may be observed by representing postural signals in a time-frequency plane. This validation step aims at checking that WE_{SQT} is able to reflect changes in complexity independently of an homogenous energy variation throughout the scales thereby facing it to two sets of simulated multicomponent signals.

Method The SQT is calculated on the whole signal: $f_0 =$ 6Hz, $f_{min} = 0.11$Hz, 64 voices. The effect of a change in complexity (resp. energy) is assessed by a Kruskall-Wallis test with repeated measures on WE_{SQT} before (Pre versus after perturbation (Post). Mean values of entropy on the twenty seconds before and after perturbation are compared. Additionally, for each simulation values of WE_{SQT} obtained before the perturbation and the ones obtained after are compared with a paired t-test (Pre versus Post).

Set 1: Variation of complexity with constant energy Ten white gaussian noise were generated (120s at 100Hz) and high-pass filtered (FIR at 3Hz, order 200) on the second part. Energy was normalized on both parts of the recording (Fig. 2).

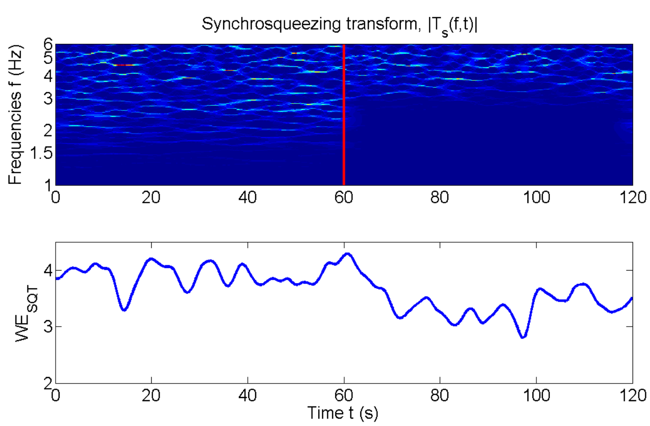

Fig. 2: SQT (up) and WE_{SQT} (down) of a white gaussian noise with variable complexity and constant energy (Set 1).

Set 2: Variation of energy with constant complexity Ten white gaussian noise were generated (120s at 100Hz), band-pass filtered (FIR between [1Hz; 6Hz], order 200) and amplified by a factor two on the second part. Energy was normalized on the two parts of the recording (Fig. 3).

Results and discussion As expected, WE_{SQT} lowers from Pre- to Post-conditions for the first set (KW=13.72, p=0.0002) but no statistically significant difference is observed on the second one (KW=3.29, p=0.0696). Individual comparisons

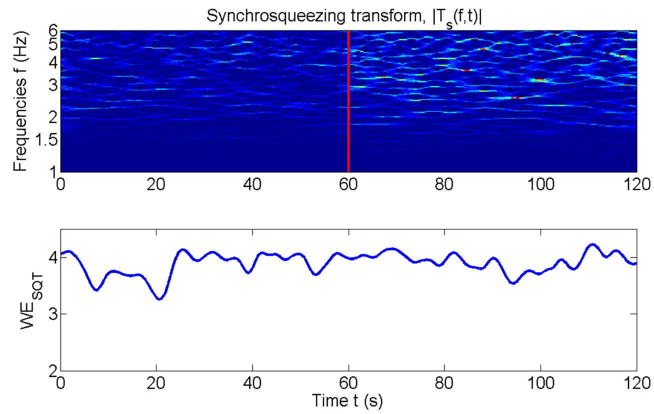

Fig. 3: SQT (up) and WE_{SQT} (down) of a white gaussian noise with variable energy (Set 2).

showed significant difference between values of WE_{SQT} obtained in Pre- and Post-conditions for the whole simulations of the first set confirming a change in complexity after perturbation ($p<0.05$) but none for the second set. Altogether, these results confirm that the developed index allows to show variation of complexity independently of homogenous energy variation throughout the scales. Moreover, our local index remains stable on stationary configuration.

4. APPLICATION TO REAL COP DATA

Maintaining upright stance results from the integration of visual, vestibular and somatosensory information. Assessment of postural control and equilibrium to detect postural troubles is classically based on the measurement of the center of pressure (CoP) displacements with a force platform. Numerous studies about the spectral content of CoP data show that each sensory afferent corresponds to a specific range of frequencies and validate our decision to consider entropy from a spectral point of view [8–14]. Being able to adapt oneself to changes in conditions is testament to proper functioning of the postural system and its capacity at making use of all the resources from the individual, the task and the environment available. Modification of the sensory context (*e.g.* sensory stimulus, fatigue, discomfort) may lead to a re-weighting of their respective loads depending on their availability, confidence or relevance.This phenomenon is called "sensory re-weighting". In terms of signals, one may expect for non-stationarities or transitory phenomenons. This feature added to the distribution through scales of the different sensory contribution reassures the interest of a temporal index of complexity in scale. The present index aims at tracking down changes in postural strategies and detecting sensory modes reweighting. To test it, eight healthy subjects were evaluated in a very simple postural task including an abrupt perturbation.

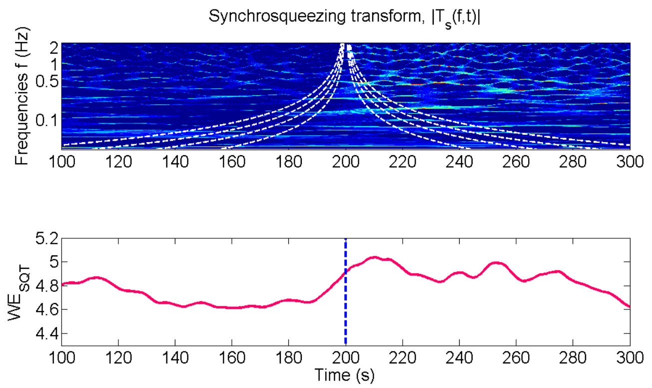

Fig. 4: SQT and WE_{SQT} of the resulting displacements of the CoP placed opposite. The first 200s, the subject has his eyes opened and closed during the following 200s. To avoid boundary effects, only the central 200s are kept. Four contour lines of the cone of influence are represented at the time of perturbation on the reallocated scalogram.

Protocol 8 healthy subjects (age: 34.63 ± 11, BMI: 21.41 ± 1.57) were asked to stand as still as possible fixing a target placed on the wall during 200s (EO) and closing their eyes the 200 following ones (EC). Displacements of the CoP were collected using a force platform (Satel) at a sampling frequency of 40Hz. Because of boundary effects, analysis is performed considering only the central 200s around the pertubation. Considering the sample size, nonparametric statistics are used for mean comparisons (Wilcoxon signed ranks test, $n = 8$, $\alpha = 5\%$). Additionally, for each subject, the values of WE_{SQT} obtained with EO are compared with the ones with EC (Paired T-test, $n = 4000$, $\alpha = 5\%$).

Results and discussion Postural stability is assessed through conventional spatiotemporal parameters: (1) Root mean square (RMS) amplitude (mm), (2) RMS velocity (mm.s^{-1}), (3) area of the 95%-confidence ellipse (mm^2). As described in table 1, all the spatio-temporal parameters increase with EC indicating lower stability with visual deprivation ($p<0.05$, 0.01, 0.01 respectively). Postural control is assessed through WE_{SQT} and SEn (for comparison). No difference is observed for SEn whereas WE_{SQT} increases with EC ($p<0.05$) indicating an increase in complexity and resources used (Tab. 1). Individual comparisons showed significant difference between values of WE_{SQT} obtained with EO and EC for 7 subjects over 8 confirming a change in complexity with visual deprivation ($p<0.05$).

Investigating postural control is based on the assessment of complexity. However most of the approaches consider entropy from a temporal point of view and evaluate the degree of predictability of the time series without taking into account the multiple scales at stake and transitory processes. Multiscale entropy and IME attempts to integrate such an information before applying SEn but the underlying time-scale representations used in addition to the difficulties in parametrizing SEn provide variable and difficult to interpret indexes. The proposed approach takes up complexity from a spectral point of view in consistence with the spectral model of CoP signal depending on its sensory afferents. Besides, the decomposition scheme underlying SQT allows to improve component identification thereby driving the reading of the TFR. Preliminary results on real CoP data were obtained by inducing a non-stationarity during the experiment (deprivation of vision). This simple protocol reflects situations of daily living during which frail persons are exposed to fall risk. The analysis of conventional spatio-temporal parameters confirms that vision deprivation induces instability. An increase in complexity (WE_{SQT}) may be due to the request of other kind of afferent to compensate for visual deprivation. Such an index may be particularly interesting for long-term monitoring or assessment during rehabilitation.

Parameters	EO	EC
RMS amplitude (mm)	3.94 ± 2.09	5.83 ± 3.07*
RMS velocity (mm.s^{-1})	9.29 ± 2.87	17.17 ± 9.67*
95%-confidence ellipse (mm^2)	24.17 ± 8.28	40.19 ± 24.13*
SEn	0.76 ± 0.23	0.83 ± 0.42
WE_{SQT}	4.17 ± 0.48	4.39 ± 0.38*

Table 1: Effect of visual conditions on the displacements of the CoP. Level of significance is set at *: $p< 0.05$.

5. CONCLUSION AND PERSPECTIVES

The index of complexity WE_{SQT} proposed is based on the modal decomposition extracted from SQT. WE_{SQT} is an index of entropy through scales which does not necessitate mode reconstruction and allows considering transitory phenomenons. Besides, WE_{SQT} allows a direct reading in terms of frequency and make the correspondence between modes and the underlying spectral model easier. Finally, for each time sample, an index consistent with the spectral dynamics of the underlying CWT is obtained by reassigning and weighting coefficients located within the cone of influence. The ability of WE_{SQT} at detecting changes in complexity was validated both on simulated signals and real CoP data. It turns out that our index provides qualitatively good results. Future works may be dedicated to the analysis of the bias observed comparatively with theoretical values. They will take into account the presence of spurious coefficients may appear during the reallocation step [15].

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