ALTERNATING MAXIMIZATION ALGORITHM FOR THE BROADCAST BEAMFORMING

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ABSTRACT

Semidefinite relaxation (SDR) is a powerful approach to solve nonconvex optimization problems involving rank condition. However its performance becomes unacceptable for certain cases. In this paper, a nonconvex equivalent formulation without the rank condition is presented for the broadcast beamforming problem. This new formulation is exploited to obtain an alternating optimization method which is shown to converge to the local optimum rank one solution. Proposed method opens up new possibilities in different applications. Simulations show that the new method is very effective and can attain global optimum especially when the number of users is low.

Index Terms— Transmit beamforming, multicast beamforming, semidefinite relaxation, convex optimization

1. INTRODUCTION

In this paper, the problem of broadcasting common information to geographically spread users with an antenna array is considered. This problem is typically observed in communications [1], [2], [3], and radar [4]. The traditional approach is to transmit the signal isotropically. As a more efficient approach, single group multicast beamforming exploits the channel state information (CSI) to generate simultaneous beams for users. Multicast beamforming is now a part of enhanced multimedia broadcast multicast service (EMBMS) of the long term evolution (LTE) standard [5].

The optimization problem for multicast beamforming is nonconvex and NP hard in general. The single group multicast beamforming under minimum received signal-to-noise ratio (SNR) constraint was initially studied in [2] and [6]. An important variation to this problem is max-min fair beamforming which is also considered in the same works. In max-min fair beamforming design, the goal is to maximize the minimum received SNR over all users. Although this problem is NP hard, a near-optimal solution is usually found by semidefinite relaxation [1], [2], [6], [7]. In these approaches, nonconvex rank constraint is dropped and the problem is solved using convex optimization. Unfortunately SDR does not guarantee rank one solution in general [1]. When the rank constraint is not satisfied, randomization is used to obtain a better solution [2]. However there are some cases where randomization does not work well, especially when the number of users is large.

In this paper, rank conditioned nonconvex optimization problem is converted to an equivalent form. This equivalent form does not include the rank condition and it is nonconvex due to the bilinear cost maximization. This new problem is solved using alternating maximization [8], [9]. Iterations are used to obtain a solution in the neighborhood of the SDR with rank > 1 solution. It is shown that the proposed method, alternating maximization algorithm, (AMA), is guaranteed to converge to a local optimum if it is not the optimum. Simulations show that the proposed algorithm converges to the upper bound when the number of users is small in which case global optimum is obtained. In general, the proposed method is very effective in comparison to the alternatives including randomization [2].

2. SYSTEM MODEL

Consider a base station equipped with M transmit antennas to transmit a common signal to N receivers, each having a single antenna. Assume that the antennas are identical. The transmitted signal can be written as,

$$\mathbf{x}(t) = s(t)\mathbf{w} \tag{1}$$

where $s(t)$ is the source signal and $\mathbf{w}$ is the $M \times 1$ complex beamformer weight vector. The received signal at the k^{th} receiver is given as,

$$y_k(t) = \mathbf{h}_k^H \mathbf{x}(t) + n_k(t) \quad k = 1, \dots N \tag{2}$$

where $\mathbf{h}_k$ is the $M \times 1$ complex channel vector for the k^{th} receiver and n_k is additive noise uncorrelated with the source signal. Noise variance is σ_k^2. Signal-to-noise ratio (SNR) for the k^{th} receiver is,

$$SNR_k = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{h}_k|^2}{\sigma_k^2} \tag{3}$$

where σ_s^2 is the source signal variance. $\sigma_s^2 = 1$ is selected without loss of generality throughout the paper.

In "$max\text{-}min$" transmit beamforming, beamforming weight vector, $\mathbf{w}$, is chosen to maximize the minimum transmitted power to any user. Considering P_{tot} as the total power, *max-min* problem can be written as follows,

$$\begin{aligned} &\max_{\mathbf{w}\in\mathbb{C}^M} \; t \\ s.t. \quad & \mathbf{w}^H\mathbf{R}_k\mathbf{w} \geq t\gamma_k\sigma_k^2, \quad k=1,...,N \\ & \mathbf{w^H w} = P_{tot} \end{aligned} \tag{4}$$

where γ_k is the power proportion for the k^{th} target and $\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k\mathbf{h}_k^H\}$ is the correlation matrix of the channel vector. Let us define $\mathbf{W} = \mathbf{w}\mathbf{w}^H$. The above problem can be written as,

$$\begin{aligned} &\max_{\mathbf{W}\in\mathbb{C}^{M\times M}} \; t \\ s.t. \quad & Tr\{\mathbf{R}_k\mathbf{W}\} \geq t\gamma_k\sigma_k^2, \quad k=1,...,N \\ & Tr\{\mathbf{W}\} = P_{tot} \\ & \mathbf{W} \succeq 0 \\ & rank(\mathbf{W}) = 1 \end{aligned} \tag{5}$$

This problem can be solved by convex optimization with semidefinite relaxation [1], [2] where the rank condition is dropped. Let us denote the solution found by semidefinite relaxation as $\hat{\mathbf{W}}$ and the principal eigenvector of $\hat{\mathbf{W}}$ as $\hat{\mathbf{w}} = \mathcal{P}(\hat{\mathbf{W}})$. If the solution matrix $\hat{\mathbf{W}}$ has rank one then $\hat{\mathbf{w}}\hat{\mathbf{w}}^H = \hat{\mathbf{W}}$ and $\hat{\mathbf{w}}$ is the optimum beamforming weight vector. If the rank of $\hat{\mathbf{W}}$ is greater than one, then randomization should be used. In SDR, rank one solution is not guaranteed, and it may return unacceptable solutions in certain problems such as (5). In this paper, an alternative and effective approach is presented for the solution of (5).

3. EQUIVALENT PROBLEM

Semidefinite relaxation is an effective approach which has found widespread applications in a variety of fields. The major limitation of semidefinite relaxation is the absence of the rank one condition leading to suboptimal solutions in general. In this paper, the original problem in (5) with rank condition is converted into an equivalent form which admits more flexible and manageable solutions. In order to obtain this equivalent form, the following theorem is used.

Theorem 1: For $M \times M$ Hermitian symmetric, positive semidefinite matrices $\mathbf{W_1}$ and $\mathbf{W_2}$, $Tr\{\mathbf{W_1W_2}\}$ is upper bounded by $Tr\{\mathbf{W_1}\}Tr\{\mathbf{W_2}\}$, i.e. $Tr\{\mathbf{W_1W_2}\} \leq Tr\{\mathbf{W_1}\}Tr\{\mathbf{W_2}\}$. This upper bound is reached if and only if $\mathbf{W_1}$ and $\mathbf{W_2}$ are rank one matrices and $\mathbf{W_2} = \alpha\mathbf{W_1}$ where α is a positive scalar.

Proof: For the proof of the Theorem 1, the following lemma is used.

Lemma 1 ([10], page 176): Let $\mathbf{A} \in \mathbb{C}^{n\times p}$, $\mathbf{B} \in \mathbb{C}^{p\times m}$ be given and $q = \min\{n,p,m\}$. The ordered singular values of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{AB}$ can be written as, $\sigma_1(\mathbf{A}) \geq ... \geq \sigma_{\min\{n,p\}}(\mathbf{A}) \geq 0$, $\sigma_1(\mathbf{B}) \geq ... \geq \sigma_{\min\{p,m\}}(\mathbf{B}) \geq 0$, and $\sigma_1(\mathbf{AB}) \geq ... \geq \sigma_{\min\{n,m\}}(\mathbf{AB}) \geq 0$, respectively. Then,

$$\sum_{i=1}^{k}\sigma_i(\mathbf{AB}) \leq \sum_{i=1}^{k}\sigma_i(\mathbf{A})\sigma_i(\mathbf{B}), \quad k=1,2,...,q. \tag{6}$$

Since $\mathbf{W_1}$ and $\mathbf{W_2}$ are $M\times M$ Hermitian symmetric positive semidefinite matrices, their singular values are equal to their eigenvalues. Let $\lambda_1(\mathbf{W_1}) \geq ... \geq \lambda_M(\mathbf{W_1}) \geq 0$, $\lambda_1(\mathbf{W_2}) \geq ... \geq \lambda_M(\mathbf{W_2}) \geq 0$, and $\lambda_1(\mathbf{W_1W_2}) \geq ... \geq \lambda_M(\mathbf{W_1W_2}) \geq 0$ be the ordered eigenvalues of $\mathbf{W_1}$, $\mathbf{W_2}$ and $\mathbf{W_1W_2}$ respectively. Then,

$$Tr\{\mathbf{W_1W_2}\} = \sum_{i=1}^{M}\lambda_i(\mathbf{W_1W_2}) \leq \sum_{i=1}^{M}\lambda_i(\mathbf{W_1})\lambda_i(\mathbf{W_2}) \tag{7}$$

$$\leq \sum_{i=1}^{M}\lambda_i(\mathbf{W_1})\sum_{i=1}^{M}\lambda_i(\mathbf{W_2}) = Tr\{\mathbf{W_1}\}Tr\{\mathbf{W_2}\} \tag{8}$$

Note that $\sum_{i=1}^{M}\lambda_i(\mathbf{W_1})\lambda_i(\mathbf{W_2}) = Tr\{\mathbf{W_1}\}Tr\{\mathbf{W_2}\}$ if $\sum_{i=1}^{M}\sum_{p=1,p\neq i}^{M}\lambda_i(\mathbf{W_1})\lambda_p(\mathbf{W_2}) = 0$. This means that $\mathbf{W_1}$ and $\mathbf{W_2}$ have only one nonzero eigenvalue. Hence, they are rank one matrices. Therefore $\mathbf{W_1} = \lambda_1(\mathbf{W_1})\mathbf{u_1}\mathbf{u_1}^H$ and $\mathbf{W_2} = \lambda_1(\mathbf{W_2})\mathbf{u_2}\mathbf{u_2}^H$ can be written where $\mathbf{u_1}$ and $\mathbf{u_2}$ are unit norm vectors. In this case,

$$Tr\{\mathbf{W_1W_2}\} = \lambda_1(\mathbf{W_1})\lambda_1(\mathbf{W_2})|\mathbf{u_1}^H\mathbf{u_2}|^2 \tag{9}$$

$Tr\{\mathbf{W_1W_2}\} = \lambda_1(\mathbf{W_1})\lambda_1(\mathbf{W_2})$ only when $\mathbf{u_1} = \mathbf{u_2}e^{j\gamma}$. In this case $\mathbf{W_2} = \frac{\lambda_1(\mathbf{W_2})}{\lambda_1(\mathbf{W_1})}\mathbf{W_1}$ and hence the theorem is proved.

In the following part, rank condition is to be embedded into the optimization problem.

Corollary 1: For two Hermitian symmetric, positive semidefinite matrices $\mathbf{W_1}$ and $\mathbf{W_2}$, the condition in (10) implies rank one matrices, i.e., $\mathbf{W_2} = \frac{\lambda_1(\mathbf{W_2})}{\lambda_1(\mathbf{W_1})}\mathbf{W_1}$.

$$Tr\{\mathbf{W_1}\}Tr\{\mathbf{W_2}\} - Tr\{\mathbf{W_1W_2}\} \leq 0 \tag{10}$$

Theorem 2: The optimum solution of the following optimization problem in (11) and (5) are the same, namely $\mathbf{W_1^\star} = \mathbf{W_2^\star} = \mathbf{W}^\star$ where $\mathbf{W}^\star$ is the optimum solution of (5):

$$\begin{aligned} &\max_{\mathbf{W_1},\mathbf{W_2}\in\mathbb{C}^{M\times M}} \; t_1 + t_2 \\ s.t. \quad & Tr\{\mathbf{R}_k\mathbf{W_1}\} \geq t_1\gamma_k\sigma_k^2, \quad k=1,...,N \\ & Tr\{\mathbf{R}_k\mathbf{W_2}\} \geq t_2\gamma_k\sigma_k^2, \quad k=1,...,N \\ & Tr\{\mathbf{W_1}\} = Tr\{\mathbf{W_2}\} = P_{tot} \\ & \mathbf{W_1} \succeq 0, \mathbf{W_2} \succeq 0 \\ & P_{tot}^2 - Tr\{\mathbf{W_1W_2}\} \leq 0 \end{aligned} \tag{11}$$

Proof: $\mathbf{W_1^\star}$ and $\mathbf{W_2^\star}$ are rank one matrices due to $P_{tot}^2 - Tr\{\mathbf{W_1W_2}\} \leq 0$ which is the condition in (10). Hence

$\mathbf{W_1^\star} = \mathbf{W_2^\star}$ since $\lambda_1(\mathbf{W_1^\star}) = \lambda_1(\mathbf{W_2^\star}) = P_{tot}$ by Corollary 1. Since $\mathbf{W_1^\star}$ and $\mathbf{W_2^\star}$ independently solve the same problem, $\mathbf{W_1^\star} = \mathbf{W_2^\star} = \mathbf{W}^\star$.

The only nonconvex constraint in (11) is $P_{tot}^2 - Tr\{\mathbf{W_1 W_2}\} \leq 0$. Fortunately this constraint can be moved into the objective function using exact penalty approach [11], [12], [13]. This modification does not change the optimum solution of the problem. In the following theorem, the equivalency of the new form and (11) are established.

Theorem 3: The problem in (11) is equivalent to the problem in (12) for $\mu > \mu_0$ with μ_0 being a finite positive value in the sense that both problems have the same optimum solution. Furthermore any local maximum of the problem in (12) is also a local maximum of the problem in (11).

$$\begin{aligned} \max_{\mathbf{W_1},\mathbf{W_2}\in\mathbb{C}^{M\times M}} \quad & t_1 + t_2 - \mu \max(0, P_{tot}^2 - Tr\{\mathbf{W_1 W_2}\}) \\ s.t. \quad & Tr\{\mathbf{R}_k \mathbf{W_1}\} \geq t_1 \gamma_k \sigma_k^2, \quad k = 1, ..., N \\ & Tr\{\mathbf{R}_k \mathbf{W_2}\} \geq t_2 \gamma_k \sigma_k^2, \quad k = 1, ..., N \\ & Tr\{\mathbf{W_1}\} = Tr\{\mathbf{W_2}\} = P_{tot} \\ & \mathbf{W_1} \succeq 0, \mathbf{W_2} \succeq 0 \end{aligned} \tag{12}$$

Proof: Constraints in (11) are all continuous functions. The feasible sets of (11) and (12) are both closed and bounded and hence they are compact due to the finite dimensional space [14]. Therefore $\max(0, P_{tot}^2 - Tr\{\mathbf{W_1 W_2}\})$ corresponds to an exact penalty function [12], [13]. As a consequence of the definition of exact penalty function [13], Theorem 3 becomes a valid statement.

Note that $\max(0, P_{tot}^2 - Tr\{\mathbf{W_1 W_2}\}) = P_{tot}^2 - Tr\{\mathbf{W_1 W_2}\}$, and (12) can be expressed as,

$$\begin{aligned} \max_{\mathbf{W_1},\mathbf{W_2}\in\mathbb{C}^{M\times M}} \quad & t_1 + t_2 + \mu Tr\{\mathbf{W_1 W_2}\} \\ s.t. \quad & Tr\{\mathbf{R}_k \mathbf{W_1}\} \geq t_1 \gamma_k \sigma_k^2, \quad k = 1, ..., N \\ & Tr\{\mathbf{R}_k \mathbf{W_2}\} \geq t_2 \gamma_k \sigma_k^2, \quad k = 1, ..., N \\ & Tr\{\mathbf{W_1}\} = Tr\{\mathbf{W_2}\} = P_{tot} \\ & \mathbf{W_1} \succeq 0, \mathbf{W_2} \succeq 0 \end{aligned} \tag{13}$$

Alternating maximization can be used to solve the problem in (13) [8], [9]. At the iteration k, with the fixed $\mathbf{W_1^{k-1}}$, we can obtain a new $\mathbf{W_2^k}$ to maximize the objective function while satisfying the SNR conditions. Then we alternate the fixed variable and update $\mathbf{W_1^k}$ while fixing $\mathbf{W_2^k}$. This alternating optimization is continued until convergence. The following lemma describes the convergence property of this algorithm.

Lemma 2: The problem in (13) converges when the alternating maximization is used.

Proof: Consider the problem in (13) with $\mathbf{W_1^{k-1}}$ fixed and $\mathbf{W_2}$ as variable. This problem is convex and the optimum solution $\mathbf{W_2^\star} = \mathbf{W_2^k}$ is found. Let $f(\mathbf{W_1}, \mathbf{W_2})$ be the cost function in (13). Since at each iteration the optimum solution can be found, the sequence $\{f(\mathbf{W_1^k}, \mathbf{W_2^k})\}$ increases, i.e.,

$$f(\mathbf{W_1^k}, \mathbf{W_2^k}) \geq f(\mathbf{W_1^{k-1}}, \mathbf{W_2^k}) \geq f(\mathbf{W_1^{k-1}}, \mathbf{W_2^{k-1}}) \tag{14}$$

In addition, $f(\mathbf{W_1^k}, \mathbf{W_2^k})$ is upper bounded by some finite value, i.e.,

$$\begin{aligned} f(\mathbf{W_1^k}, \mathbf{W_2^k}) &= t_1^k + t_2^k + \mu Tr\{\mathbf{W_1^k W_2^k}\} \\ &\leq \max_k Tr\{\mathbf{R}_k \mathbf{W_1^k}\} + \max_k Tr\{\mathbf{R}_k \mathbf{W_2^k}\} + \\ &\quad \mu Tr\{\mathbf{W_1^k}\} Tr\{\mathbf{W_2^k}\} \\ &\leq \max_k Tr\{\mathbf{R}_k\} Tr\{\mathbf{W_1^k}\} + \max_k Tr\{\mathbf{R}_k\} Tr\{\mathbf{W_2^k}\} + \\ &\quad \mu Tr\{\mathbf{W_1^k}\} Tr\{\mathbf{W_2^k}\} \\ &= 2P_{tot} \max_k Tr\{\mathbf{R}_k\} + \mu P_{tot}^2 \end{aligned} \tag{15}$$

Since the sequence $\{f(\mathbf{W_1^k}, \mathbf{W_2^k})\}$ is increasing and upper bounded by $2P_{tot} \max_k Tr\{\mathbf{R}_k\} + \mu P_{tot}^2$, the sequence $\{f(\mathbf{W_1^k}, \mathbf{W_2^k})\}$ converges to some $B \leq 2P_{tot} \max_k Tr\{\mathbf{R}_k\} + \mu P_{tot}^2$ [9].

4. ALTERNATING MAXIMIZATION ALGORITHM

In the previous parts, the problems in (5) and (13) are shown to be equivalent in the sense that they have the same optimum solutions. Furthermore, it is shown that (13) can be solved with alternating maximization. The convergence of this approach is guaranteed. However, there is no guarantee that optimum solution is achieved after the convergence. Nevertheless the solution is much better than the alternatives including the semidefinite relaxation [1] and randomization [2]. The steps for AMA can be presented as follows,

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Alternating Maximization Algorithm (AMA)

Let $\lambda_{max}(\mathbf{W})$ be the maximum eigenvalue of the matrix $\mathbf{W}$.
Initialization: $k = 0$,
Set a proper μ and $\mathbf{W_1^0} = \hat{\mathbf{W}}$ where $\hat{\mathbf{W}}$ is the solution of (5) with semidefinite relaxation. Solve the problem in (13) for $\mathbf{W_2^0}$.
Iterations: $k = k + 1$
1) Solve (13) for $\mathbf{W_1^k}$ while fixing $\mathbf{W_2}$ as $\mathbf{W_2^{k-1}}$. If $rank(\mathbf{W_1^k}) = 1$ go to step 6.
2) If $\lambda_{max}(\mathbf{W_1^k}) \geq \lambda_{max}(\mathbf{W_2^{k-1}}) + \beta$ (improved solution), where β is a proper positive threshold value (Ex: $P_{tot}/100$), keep the value of μ same. Otherwise, increase μ (Ex: $\mu \to 2\mu$)
3) Solve (13) for $\mathbf{W_2^k}$ while fixing $\mathbf{W_1}$ as $\mathbf{W_1^k}$. If $rank(\mathbf{W_2^k}) = 1$, go to step 6.
4) If $\lambda_{max}(\mathbf{W_2^k}) \geq \lambda_{max}(\mathbf{W_1^k}) + \beta$, keep the value of the μ same. Otherwise, increase μ.
5) Check the convergence ($\|\mathbf{W_2^k} - \mathbf{W_2^{k-1}}\|_F \leq \epsilon$ where ϵ is a proper threshold value). If AMA is converged go to step 6 else go to step 1.
End:
6) Take the beamformer weight vector as the principal eigenvector of the matrix $\mathbf{W_i^k}$ (where $i = 1$ or 2 depending on termination).

AMA searches the rank one solution in the neighborhood of the solution found by the semidefinite relaxation at the initial step of the algorithm. This process does not guarantee to find the global optimum solution. However, the performance of AMA is significantly better than the alternative solutions in general. In fact, for small number of users, there is almost zero gap between the solution found by semidefinite relaxation which returns rank > 1 and the AMA. This shows that for such problems, AMA finds the optimum solution.

Convergence to a local optimum solution is guaranteed throughout the iterations. Since the iteration steps are with small increments, the global optimum is found as long as it is in the neighborhood of the result returned by the SDR in the initial step.

5. SIMULATION RESULTS

In this part, proposed method, AMA, is evaluated by comparing its performance with SDR-U which is the SDR solution with rank>1, SDR with principal eigenvector, SDR-P, and SDR with randomization, SDR-Rand. For SDR-U, "$t = \min\{\frac{SNR_k}{\gamma_k}\}$" value after semidefinite relaxation in (5) is given. In this case, solution, $\mathbf{W}$, has usually rank > 1. Note that SDR-U result is presented only to give a loose upper bound. This bound may not be achieved for rank=1 solutions in general. SDR-P is the case where the principal eigenvector for the SDR-U solution is used and the value of "t" is plotted. SDR-Rand uses the best solution of the three randomization methods with $30NM$ trials each, as described in [2] after SDR-P.

Proposed method, AMA, is implemented using convex programming solver CVX [15]. Both far field line-of-sight and Rayleigh fading transmit channels are considered. There are $M = 16$ antennas and total power is set as $P_{tot} = 10W$. The threshold parameter is selected as $\beta = 0.1$. The power proportion γ_k takes values from $\{1, 2, 3, 4\}$ set equally likely for the users. Simulation results are presented for 100 independent trials for different channel realizations. It should be noted that rank=1 solution is obtained in all of the experiments described in this section. The SNR values are given in linear scale.

In Fig. 1, a uniform circular transmit array (UCA) is used for $N = 12$ users. SDR-U stands for the upper bound that can be achieved for "t". AMA is very close to this bound and has the same value with the bound in many cases. SDR-Rand cannot reach to the bound in general. The performance of SDR-P is unacceptable in general. This figure shows that AMA finds the global optimum in general.

In Fig. 2, a similar setup with Fig. 1 is used and only the number of users is changed, i.e., $N = 32$. In this figure, AMA has the best performance with a large gap between SDR-P and SDR-Rand. As N is increased, SDR-U may result high rank (Ex: rank=4) and AMA cannot get very close to the SDR-U result but its performance is very good in comparison.

In Fig. 3, channel realizations are obtained from identically distributed Rayleigh fading model. The "t" values in Fig. 3 are slightly lower than Fig. 2 due to model difference. The loss in SDR-Rand is large in comparison to AMA.

In Fig.4, average iteration number is presented for different number of users and antenna elements by averaging 100 different channel realizations. Results of both Rayleigh fading channel and line of sight (LOS) channel with uniform circular array are shown in the figure. Average iteration number in Rayleigh channel case is larger than LOS channel. As the number of users increases, the problem becomes more complex and the number of iterations for convergence increases.

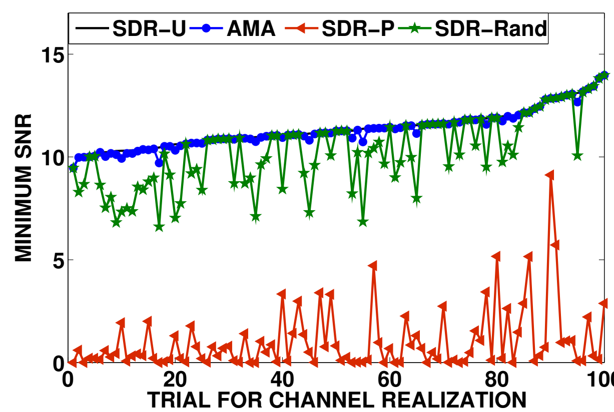

Fig. 1. Optimization parameter "t" (minimum SNR) for AMA, SDR-U, SDR-P and SDR-Rand for 100 different far field channel realizations with M=16 element UCA and N=12.

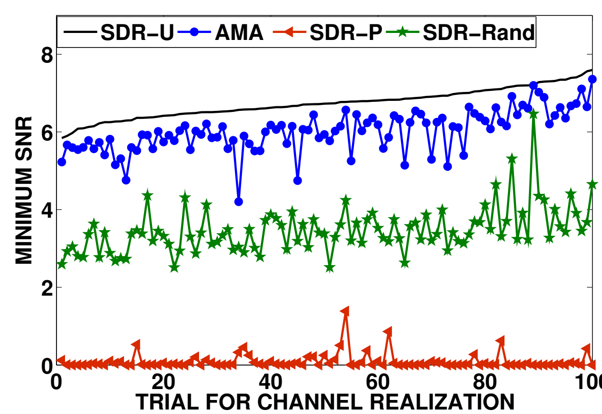

Fig. 2. Optimization parameter "t" (minimum SNR) for AMA, SDR-U, SDR-P and SDR-Rand for 100 different far field channel realizations with M=16 element UCA and N=32.

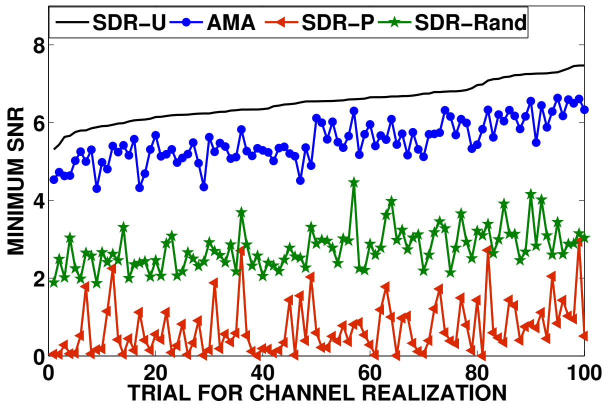

Fig. 3. Optimization parameter "t" (minimum SNR) for AMA, SDR-U, SDR-P and SDR-Rand for 100 different Rayleigh channel realizations with M=16 and N=32.

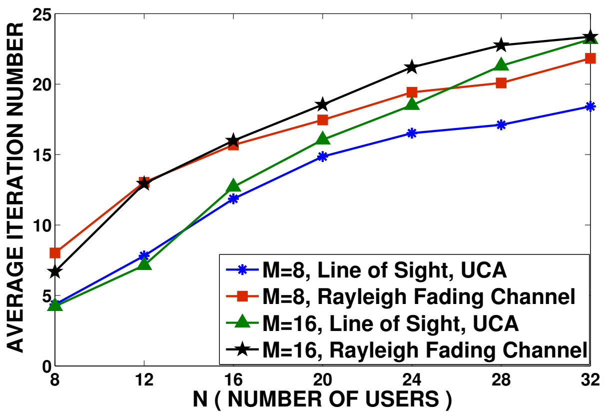

Fig. 4. Average iteration number for different number of users and antennas.

6. CONCLUSION

In this paper, an alternative to semidefinite relaxation for rank conditioned problems is considered. An equivalent nonconvex formulation is obtained which does not include the rank condition. This new form of the optimization problem is solved for single group multicast beamforming application using alternating maximization. It is shown that the proposed solution is guaranteed to converge and returns very good solutions compared to the alternative methods.

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