LOW COMPLEXITY MULTIUSER MIMO SCHEDULING FOR WEIGHTED SUM RATE MAXIMIZATION

Ganesh Venkatraman, Antti Tölli, Janne Janhunen, and Markku Juntti

Center for Wireless Communications (CWC), University of Oulu, Oulu, FI-90014,
Email: {gvenkatr, antti.tolli, janne.janhunen, markku.juntti}@ee.oulu.fi

ABSTRACT

The paper addresses user scheduling schemes for the multi-user multiple-input multiple-output (MU-MIMO) transmission with the objective of sum rate maximization (SRM) and the weighted counterpart in a single cell scenario. We propose a low complex product of independent projection displacements (PIPD) scheduling scheme, which performs the user selection for the MU-MIMO system with significantly lower complexity in comparison with the existing successive projections (SP) based designs. The PIPD scheme uses series of independent vector projections to evaluate the decision metrics. In addition, we also propose a heuristic algorithm of weighted scheduling, addressing the weighted sum rate maximization (WSRM) objective, which can be used with any scheduling algorithm. The performance of the weighted scheduling schemes are studied with the objective of minimizing the queues.

I. INTRODUCTION

The radio access technologies aim at achieving higher throughput owing to the increasing demands from the higher layers. The use of multi-antenna transmission favors the spatial multiplexing of data by applying transmit precoders for the multi-user multiple-input multiple-output (MU-MIMO) transmission. In order to avail the benefit of the MU-MIMO scheme, the multiplexed users should have channel vectors as linearly independent as possible. The linear independency condition arises due to the use of linear precoder design at the transmitter in order to de-couple the user streams at the receiver end. The selection of users with this constraint is performed by the schedulers in order to utilize the resources efficiently. After selecting the user set, the precoder design can either be based for example zero-forcing beamforming (ZFBF) design [1] or some general iterative transmitter-receiver optimization design based on e.g. weighted sum rate maximization (WSRM) schemes as in [2].

The scheduling algorithms based on the sum rate maximization objective for MU-MIMO are discussed thoroughly in the literature. The search based on successive projections (SP) scheme for single-antenna receiver is discussed in [3] and the extension for multi-antenna receivers in [4], which selects the users by choosing the user with the maximum gain on to the orthogonal subspace to the space spanned by the already chosen user channel vectors. Similar algorithms with addressing the complexity are proposed in [5], where computationally efficient algorithms are proposed. Selection based on the volume maximization metric are discussed in [6]. The performance of the volume based selection is identical to the SP or block diagonal (BD) scheme, since both schemes uses Gram-Schmidt (GS) procedure. The user selection for MU-MIMO based transmission scheme are classified and analyzed briefly in [7], aiming at minimizing the average beamforming power.

Since the transmissions are driven by the backlogged packets, queues play a major role in the algorithm designs. Earlier works on the schedulers to empty the queued packets are focused on the single-transmit single-receive cases. The simple and easy to implement round-robin (RR) scheduling is also analyzed for the single-transmit single-receive systems owing to its starvation-free resource allocation for all the users. Scheduling schemes to minimize the queued packets at the transmitter are considered comprehensively in [8]. It also discusses the constraints like finite buffer size, packet delays in an optimization framework. Similar work is also shown in [9] by sharing the available power to each user using geometric programming (GP) formulation.

In this paper, we propose and analyze scheduling schemes to maximize both sum rate and the weighted sum rate performance in a MU-MIMO transmission scenario. We propose a product of independent projection displacements (PIPD) scheduling algorithm, which provides a low complexity scheduling algorithm for a downlink MU-MIMO user selection problem. The loss in the sum rate performance is negligible while the complexity reduction is high as comparing to earlier projections based algorithms. In addition, we also discuss a heuristic procedure, which provides a modification for the channel vectors to address the weighted sum rate objective. The proposed heuristic algorithm is used with the existing and the proposed schemes to analyze the queue minimizing scheduling problem.

II. SYSTEM MODEL

We consider a single-cell downlink MU-MIMO transmission with N_T transmit antennas and K single antenna users.

The transmission is carried out by multiplexing user signals in the spatial dimensions. Since the users are transmitted on the same time-frequency resource, the received signal y_k of the user k with interference is given by

$$y_k = \mathbf{h}_k \mathbf{s}_k + \sum_{i \in \mathcal{U}_t \setminus \{k\}} \mathbf{h}_k \mathbf{s}_i + n_k, \tag{1}$$

where $\mathcal{U}_t$ is the set having the indices of the multiplexed users at a given scheduling instant. The transmitted signal for user k is denoted by $\mathbf{s}_k = \mathbf{m}_k d_k$, where $\mathbf{m}_k \in \mathbb{C}^{N_T \times 1}$ is the transmit precoder and d_k denotes the data meant for user k with unit energy as represented by $\mathbf{E}[|d_k|^2] = 1$. The channel between the user k to the cell is given by $\mathbf{h}_k \in \mathbb{C}^{1 \times N_T}$ and n_k is the complex Gaussian noise added at the receiver, drawn from $\sim \mathcal{CN}(0, N_0)$, zero mean and variance N_0.

Let $\mathcal{U}$ be the set consisting of all users in the system. The objective of the scheduler is to select users for the set $\{\mathcal{U}_t \subset \mathcal{U}\}$ at each scheduling instant to maximize the WSRM objective. The precoders $\mathbf{m}_i$ are designed to maximize the sum rate while satisfying the total transmit power $P_{\max}$ as

$$\sum_{i \in \mathcal{U}_t} \mathrm{Tr}\left(\mathbf{m}_i \, \mathbf{m}_i^H\right) \leq P_{\max}. \tag{2}$$

Let Q_k be the number of backlogged packets to be transmitted for the user k at any given scheduling instant. The queues are updated with the fresh arrivals a_k, which are drawn from the Poisson distribution with the average arrival rate A_k. Let t_k the number of packets transmitted from the cell for user k. The queue update for user k at i^{th} instant is

$$Q_k(i) = \max\left\{ Q_k(i-1) - t_k(i-1), \, 0 \right\} + a_k(i), \tag{3}$$

where $Q_k(i-1)$ and $t_k(i-1)$ denote the queued and the transmitted packets at $(i-1)^{\text{th}}$ instant.

III. SUM RATE MAXIMIZING SCHEDULING

This section discusses the existing and the proposed user scheduling algorithms for the MU-MIMO transmission.

III-A. Existing Scheduling Algorithms

Scheduling and precoding should be jointly optimized in order to maximize the WSRM objective. Due to the complexity involved in the joint design, a two step approach if followed by performing scheduling first, which selects the users for the transmission set U_t, then the precoders are designed for the users only in the set U_t. Selecting a subset $\mathcal{U}_t$ from the set $\mathcal{U}$ is a combinatorial problem in general requiring an exhaustive search of $O(K^{N_T})$ complexity. The scheduler should select users with the channel vectors as linearly independent as possible in order to maximize the sum rate performance with the MU-MIMO transmission using the linear precoders.

One such scheduling is the SP scheme, which selects the user by projecting the channel vectors on to the null space of the existing user channel vectors as in [3], [4]. Initially, the user with higher channel gain from the set $\mathcal{U}$ is selected for the transmission set $\mathcal{U}_t$. Then, the remaining users are selected from $\mathcal{U} \setminus \mathcal{U}_t$ by projecting the channel vector on to the null space of the user channel vectors in $\mathcal{U}_t$. The selection metric of a user is given as

$$\mathbf{H} = \left[\mathbf{h}_{\mathcal{U}_t(1)}^T, \ldots, \mathbf{h}_{\mathcal{U}_t(|\mathcal{U}_t|)}^T\right] \tag{4a}$$

$$\mathbf{N} = \mathbf{I} - \mathbf{H}\left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H \tag{4b}$$

$$\zeta_i = \|\mathbf{N}\, \mathbf{h}_i^T\|^2, \; \forall\, i \in \mathcal{U} \setminus \mathcal{U}_t \tag{4c}$$

$$k = \arg\max_i \; \zeta_i, \quad \mathcal{U}_t = \mathcal{U}_t \cup \{k\}\,. \tag{4d}$$

The complexity is measured by the number of complex multiplications involved in an algorithm. The number of complex multiplications involved in selecting the initial user is $\delta = K\, N_T$, owing to the norm evaluations. The remaining users are selected by projecting on the null space $\mathbf{N}$ as in (4b). The complex multiplications required for (4b) is

$$\beta(\nu) = \nu^2\, N_T + O(\nu^3) + \nu\, N_T\, (N_T + \nu), \tag{5}$$

where $\mathbf{N} \in \mathbb{C}^{N_T \times i}$ and $\nu = |\mathcal{U}| - i$ denotes the number of users remaining in the set at i^{th} iteration. The overall complexity in the selection procedure is given as

$$\chi = O\left(\delta + \sum_{i=2}^{N_T} \underbrace{N_T(1+N_T)\,(K-i+1)}_{\text{projection and norm calculations}} + \beta(i-1)\right) \tag{6}$$

III-B. Product of Independent Projection Displacements (PIPD) Scheme

In this section, we propose an algorithm requiring minimal computational complexity compared to the scheduling scheme discussed in Section III-A. The complexity involved in selecting the first user remains the same, since the user with the highest channel norm is considered for the set $\mathcal{U}_t$ in both schemes. The complexity of selecting the remaining users are significantly reduced by the proposed PIPD scheduling scheme using the product of independent vector projection displacements metric.

Since the norm of the null space projections are used as the metric in the SP procedure, the complexity required for the null space projection (6) scales exponentially to the size of the null space matrix $\mathbf{N}$ in (4b). The proposed PIPD scheme provides an alternative way to evaluate a metric equivalent to the norm of the null space projection.

Let $\hat{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ be the normalized channel vector of the user k, $\forall k \in \mathcal{U}_t$. The absolute value of the null space projection of any channel vector $\mathbf{h}_i$ on to the vector $\mathbf{h}_k$ is comparable to the difference between the norm of the vector $\mathbf{h}_k$ and its projection on to the user channel vector as

$$x_{k,i} = \|\, \mathbf{h}_i \,\|_2 \; - \; |\, \hat{\mathbf{h}}_k \, \mathbf{h}_i^H \,|, \quad \forall i \in \mathcal{U} \setminus \mathcal{U}_t, \; \forall\, k \in \mathcal{U}_t, \tag{7}$$

where $x_{k,i}$ is the metric which quantifies the minimum distance between the vectors $\mathbf{h}_k$ and $\mathbf{h}_i$ respectively.

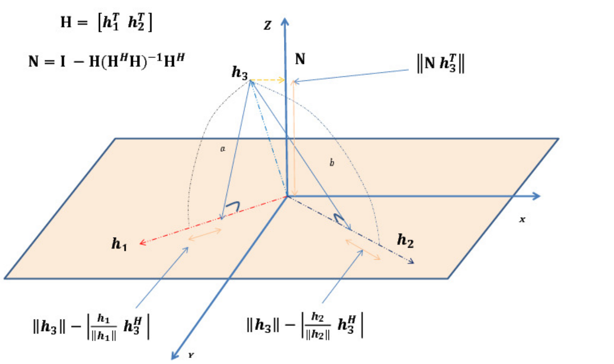

Fig. 1. Interpretation of PIPD metric and SP metric evaluation

Once $x_{k,i}$ is evaluated for all vectors corresponding to the user indices in $\mathcal{U}_t$ for the user i, the equivalent PIPD metric for the user i is given by

$$\zeta_i = \prod_{k \in \mathcal{U}_t} x_{k,i}, \quad k = \arg\max_i \ \zeta_i, \ \ \forall i \in \mathcal{U} \backslash \mathcal{U}_t \qquad (8)$$

which is the product of vertical displacements between the channel vector of the user i and the normalized vectors of the users in the set $\mathcal{U}_t$.

Fig. 1 illustrates the metric calculation for the successive projections and the PIPD algorithm for a $N_T = 3$ scenario, allowing three users to be multiplexed spatially. A three user scenario with two users $\mathcal{U}_t = \{1, 2\}$ already selected and the third user is being discussed in Fig. 1. The sub space spanned by the channel vectors of the users in $\mathcal{U}_t$ is shown by the shaded plane. The null space of the plane having vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ is the vertical axis denoted by $\mathbf{N}$ in Fig. 1. The projection length of the vector $\mathbf{h}_3$ on $\mathbf{N}$ is used as the decision metric for the SP scheme.

On contrary, the PIPD scheme uses the product of the difference metrics as in (8), where the minimum displacement in (7) is bounded by the vertical displacement as $x_{1,3} \leq a$ and $x_{2,3} \leq b$, where a and b are the vertical displacements of the vector $\mathbf{h}_3$ from vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ respectively. From (8), the PIPD metric will be zero when a vector $\mathbf{h}_i$ is collinear to any normalized vectors $\hat{\mathbf{h}}_k \ \forall \, k \in \mathcal{U}_t$ and it will be maximum when the vector is orthogonal to all the vectors defined by the set $\mathcal{U}_t$ thereby having $x_{1,3} = a$ and $x_{2,3} = b$.

The users chosen by the PIPD and by the SP scheme are same until $|\mathcal{U}_t| \leq 2$. The PIPD metric evaluation for the second user, after having the first user as k, is given by

$$x_{k,i} = \|\mathbf{h}_i\| - \left| \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|} \mathbf{h}_i^H \right|, \ \forall \, i \in \mathcal{U} \backslash \mathcal{U}_t, \qquad (9)$$

and for the SP scheme is

$$\mathbf{N} = \mathbf{I} - \frac{\mathbf{h}_k^T \mathbf{h}_k^*}{\|\mathbf{h}_k\|^2}, \quad y_i = \left\| \mathbf{h}_i^T - \frac{\mathbf{h}_k^T \mathbf{h}_k^*}{\|\mathbf{h}_k\|^2} \mathbf{h}_i^T \right\|. \qquad (10)$$

Using the triangular inequality $\|\mathbf{p} - \mathbf{q}\| \geq \|\mathbf{p}\| - \|\mathbf{q}\|$, the metrics in (9) and (10) can be related as $x_{k,i} \leq y_i$, where $y_i = \|\mathbf{N}\mathbf{h}_i^T\|$. With $\mathbf{h}_k^*$ denoting the complex conjugate of the vector $\mathbf{h}_k$, note that

$$\left| \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|} \mathbf{h}_i^H \right| = \left\| \frac{\mathbf{h}_k^T \mathbf{h}_k^*}{\|\mathbf{h}_k\|^2} \mathbf{h}_i^T \right\|, \qquad (11)$$

Since the selection is performed using relative metric, (9) and (10) identifies the same user for the set $\mathcal{U}_t$.

Beyond the second iteration, the performance of the PIPD metric is different from the SP metric which involves the matrix inversion to evaluate the null space (4b). At each iteration, the SP scheme projects the user channel vectors on to the null space spanning $N_T - |\mathcal{U}_t|$ dimension to find a user with maximum gain on the orthogonal subspace. Instead, the PIPD scheme evaluates the product of differences between the norm and the projection length, which is clearly a suboptimal method but does not require matrix inversion.

It is highly unlikely to select a user with the channel vector lying inside the space spanned by the channel vectors of users in $\mathcal{U}_t$, since a vector lying outside the subspace would have higher metric as given by (8). The complexity can further be reduced by saving the difference and product computed in (7) and (8) from the previous iteration, thereby requiring only a difference and a multiplication needed for user $i \in \mathcal{U} \backslash \mathcal{U}_t$ in the current iteration to evaluate the updated metric ζ_i. The number of complex operations involved in evaluating the PIPD metric is given by

$$\chi = O\left(\delta + \sum_{i=2}^{N_T} (K - i + 1) \left(i \left(N_T + 1 \right) - 1 \right) \right), \qquad (12)$$

where $\delta = K \, N_T$ denote the complex operations involved in the norm calculation and i representing the iteration index.

IV. WEIGHTED SCHEDULING SCHEME

In this section, we discuss an alternative metric for the user selection, which performs scheduling for the WSRM objective. We propose a modified metric for (4c) and (8), which select users for the WSRM instead of the sum rate maximization objective. We also propose a heuristic algorithm, which provides an alternative to select users for maximizing the WSRM objective.

In the spatially overloaded weighted minimum mean squared error (WMMSE) formulation [2], where all users are assumed to be in the transmission set $\mathcal{U}_t = \mathcal{U}$, scheduling is performed implicitly by forcing the precoder powers to zero for all users in the system except for $\leq N_T$ users. Even though the spatially overloaded WSRM precoder design is optimal, the computational complexity increases with the available users in the system. Alternatively, if the scheduling algorithm can identify the user sub set $\mathcal{U}_t$ based on the weighted sum rate maximization objective, the complexity can significantly be reduced.

Here we aim to achieve the same target with the separate user selection and the precoding optimization procedure for the WSRM objective. In order to achieve that, scheduling schemes should consider the users weights in addition to the sum rate maximization objective. The general WSRM formulation for the overloaded case is written as

$$\underset{\mathbf{m}_k\,\forall\,k\in\mathcal{U}}{\text{maximize}} \quad \sum_{k\in\mathcal{U}} \alpha_k \, \log(1+\gamma_k) \tag{13a}$$

$$\text{subject to} \quad \gamma_k = \frac{|\mathbf{h}_k \mathbf{m}_k|^2}{N_0 + \sum_{i\in\mathcal{U}\setminus\{k\}} |\mathbf{h}_k \mathbf{m}_i|^2} \tag{13b}$$

$$\textstyle\sum_{k\in\mathcal{U}} \|\mathbf{m}_k\|^2 \leq P_{\max}, \tag{13c}$$

where γ_k is the signal-to-interference-plus-noise ratio (SINR) seen by the user k using $\mathbf{m}_k$ as the transmit precoders and α_k is the weighing factor assuming the precoders are designed in a successive manner as in (4c) $\mathbf{N}\,\mathbf{h}_k^T$.

The scheduling decision metric is evaluated for all users in the set $\mathcal{U}\backslash\mathcal{U}_t$ to identify a user, whose inclusion will maximize the weighted sum rate. The original WSRM objective is replaced with the interference-free signal-to-noise ratio (SNR) expression $\gamma_k = \frac{\|\mathbf{N}\,\mathbf{h}_k^T\|^2}{N_0}$, where the null space matrix $\mathbf{N}$ is formed as in (4b). Considering the high SNR scenario, the $\log(1+\gamma_k)$ can be approximated to $\log(\gamma_k)$. Assuming equal share in the sum power $P_{\max}$ for the transmission user set, the weighted rate objective of each user at high SNR is given by

$$\alpha_k \, \log\left(\frac{P_{\max}}{N_T\,N_0}\,\|\mathbf{N}\,\mathbf{h}_k^T\|^2\right) \tag{14a}$$

$$\alpha_k \, \log\left(\frac{\gamma_{\text{avg},k}}{N_T}\right) + \alpha_k \, \log\left(\|\mathbf{N}\,\mathbf{h}_k^T\|^2\right), \tag{14b}$$

where $\gamma_{\text{avg},k}$ denotes the average SNR and (14b) represents the metric used for the user selection at each iteration. The users are selected in sequence to maximize the modified metric (14b). To begin with, $\mathbf{N} = \mathbf{I}$ and $\mathcal{U}_t = \{\emptyset\}$ are initialized and the user selection at each iteration is carried out by using the metric in (14b). After selecting the first user, the stacked channel matrix $\mathbf{H}$ and the null space matrix $\mathbf{N}$ are updated using (4a) and (4b) respectively. The metric to perform WSRM scheduling can, in general, be given as

$$\alpha_k \, \log\left(\frac{\gamma_{\text{avg},k}}{N_T}\right) + \alpha_k \, \log\left(\zeta_k\right), \tag{15}$$

where ζ_k is the metric used for the scheduling decisions, for e.g, SP and PIPD schemes as given in (4c) and (8).

We found that we can replace the log weighted metric with linear scaling without any significant change in performance. Instead of using the metric in (14b), we can use (8) as the selection metric with the channel vectors replaced with the equivalent channels given by $\widetilde{\mathbf{h}}_k = \alpha_k\,\mathbf{h}_k$. The performance of the heuristic algorithm is similar to the proposed metric in (14b) or in general (15).

In order to show the behavior of the scheduler with the WSRM objective, we consider the objective of queue minimization, which requires queued packets to be the corresponding user weights as discussed in [8]. Using queued

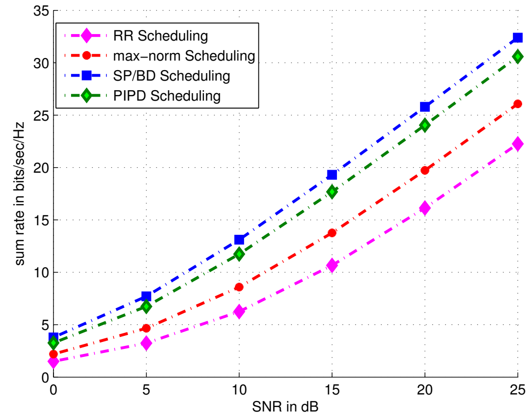

Fig. 2. Sum rate for $K = 20$ with $N_T = 4$ system

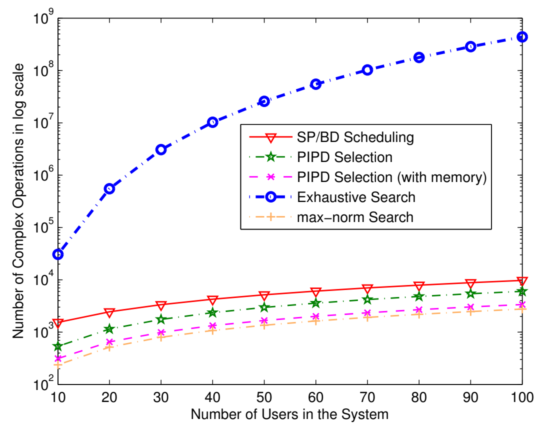

Fig. 3. Complexity plot for $N_T = 4$ system

packet sizes as the user weights, the overloaded WMMSE scheme with (13) is compared with the weighted scheduling schemes, which finds the subset $\mathcal{U}_t$, for which the precoders are designed using WMMSE scheme with $\alpha_k = Q_k$.

V. SIMULATION RESULTS

In this section, we discuss the performance of the above algorithms in a MU-MIMO framework. The comparisons are drawn between the proposed scheduler to the existing algorithms using $\mathbf{N}\mathbf{h}_k^T$ as the transmit precoders or by ZFBF precoder design for the users in $k \in \mathcal{U}_t$. In order to analyze the sum rate performance, the users are assumed have equal path loss so as to compare the scheduler algorithms for the objective of sum rate maximization.

Fig. 2 compares the performance of the proposed scheme with the existing SP scheme with $N_T = 4$ and $K = 20$ users. The sum rate of the proposed PIPD scheme performs close to the SP scheme with the complexity comparable to the max-norm scheduling scheme, which selects the users based on the channel norm only.

Fig. 3 compares the complexity involved in the metric calculation of various scheduling schemes plotted against the number of users. The complexity of the PIPD scheme

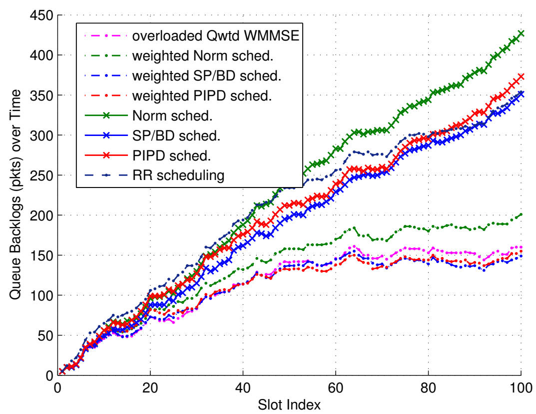

Fig. 4. Sum queued packets for $K = 40$ users

is greatly reduced by the use of independent projections to the null space used in the SP scheme. Even though the performance of the PIPD scheme is slightly inferior to the SP scheme, the relative benefit of the complexity reduction to the performance loss is high in practice. By storing and reusing the values $x_{k,i}$ (9) at each iteration, significant reduction can be achieved.

Fig. 4 plots the total backlogged packets of different selection schemes after each scheduling instant. The scenario considered $N_T = 4$ antenna base station (BS) with $K = 40$ single antenna receivers with the path-loss drawn uniformly from $[-10, 0]$ dB. The arrival process is assumed to follow Poisson distribution with the mean packet arrival rate of 0.25 bits per user. Fig. 4 shows the performance of the non-weighted scheduling schemes and the weighted counterparts. In the overloaded WMMSE scenario, precoders are designed for all users by considering $\mathcal{U}_t = \mathcal{U}$.

In contrast to the overloaded case, the scheduler provides the set $\mathcal{U}_t$, for which the precoders are designed using queue weighted sum rate maximization (Q-WSRM) schemes. The performance of the queue minimizing schemes are significantly better than the non-weighted schemes with the Q-WSRM based precoder design as shown in Fig. 4. The queue minimizing behavior of the low complex PIPD scheme performs similar to the SP scheduler design in both weighted and non-weighted scenario. The queue stability is said to be achieved when the average incoming and outgoing packets are equal as achieved by the WSRM schemes.

VI. CONCLUSIONS

The paper considered scheduling for a single cell downlink multi-user multiple-input multiple-output (MU-MIMO) system. We proposed a low complexity product of independent projection displacements (PIPD) scheduler algorithm, requiring significantly less computations than the existing successive projections (SP) scheme without compromising much on the sum rate performance. In addition, we proposed a heuristic scheme, which minimizes the queued packets using the queue weighted channel vectors. The proposed queue weighted metric can be used with the existing sum rate maximizing designs to achieve the queue minimization objective. The total number of backlogged packets with the weighted scheduling is significantly reduced as compared to the non-weighted scheduler designs.

REFERENCES

[1] Q. H Spencer, A. L Swindlehurst, and M. Haardt, “Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” in *IEEE Transactions on Signal Processing*. 2004, vol. 52, pp. 461–471, IEEE.

[2] Q. Shi, M. Razaviyayn, Z. Q. Luo, and C. He, “An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel,” in *IEEE Transactions on Signal Processing*. 2011, vol. 59, pp. 4331–4340, IEEE.

[3] T. Yoo and A. Goldsmith, “On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming,” in *IEEE Journal on Selected Areas in Communications*. march 2006, vol. 24, pp. 528 – 541, IEEE.

[4] A. Tolli and M. Juntti, “Scheduling for Multiuser MIMO Downlink with Linear Processing,” in *IEEE 16th International Symposium on Personal, Indoor and Mobile Radio Communications, (PIMRC)*. sept. 2005, vol. 1, pp. 156 –160, IEEE.

[5] Z. Youtuan, T. Zhihua, and Z. Jinkang, “An improved norm-based user selection algorithm for multiuser MIMO systems with block diagonalization,” in *IEEE 66th Vehicular Technology Conference, VTC-Fall*. 2007, pp. 601–605, IEEE.

[6] L. Jin, X. Gu, and Z. Hu, “A novel volume-based scheduling scheme for multiuser multiple-input multiple-output downlink system,” in *IEEE Conference on Radio and Wireless Symposium (RWS)*. 2010, pp. 448–451, IEEE.

[7] X. Zhang, E.A. Jorswieck, B. Ottersten, and A. Paulraj, “User selection schemes in multiple antenna broadcast channels with guaranteed performance,” in *8th Workshop on Signal Processing Advances in Wireless Communications SPAWC*. 2007, pp. 1–5, IEEE.

[8] Michael J Neely, *Stochastic network optimization with application to communication and queueing systems*, vol. 3 of *Synthesis Lectures on Communication Networks*, Morgan & Claypool Publishers, 2010.

[9] Kibeom Seong, R. Narasimhan, and J.M. Cioffi, “Queue proportional scheduling via geometric programming in fading broadcast channels,” in *IEEE Journal on Selected Areas in Communications*. 2006, vol. 24, pp. 1593–1602, IEEE.