COMPRESSED SENSING UNDER STRONG NOISE. APPLICATION TO IMAGING THROUGH MULTIPLY SCATTERING MEDIA

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ABSTRACT

Compressive sensing exploits the structure of signals to acquire them with fewer measurements than required by the Nyquist-Shannon theory. However, the design of practical compressive sensing hardware raises several issues. First, one has to elicit a measurement mechanism that exhibits adequate incoherence properties. Second, the system should be robust to noise, whether it be measurement noise, or calibration noise, i.e. discrepancies between theoretical and actual measurement matrices. Third, to improve performance in the case of strong noise, it is not clear whether one should increase the number of sensors, or rather take several measurements, thus settling in the multiple measurement vector scenario (MMV). Here, we first show how measurement matrices may be estimated by calibration instead of being assumed perfectly known, and second that if the noise level reaches a few percents of the signal level, MMV is the only way to sample sparse signals at sub-Nyquist sampling rates.

***Index Terms*—compressive sensing, calibration, MMV, experimental study, optical imaging, scattering media**

I. INTRODUCTION

In their groundbreaking studies [4], [8], [3], CANDÈS, DONOHO, TAO and ROMBERG demonstrated that the sparsity of a signal [10] can be exploited so as to dramatically reduce the number of sensors and measurements required for its acquisition, without loss of quality as compared to traditional Shannon-Nyquist sampling. Compressed sensing, or compressive sensing, CS in short, is a field of research that bloomed since then (see [12] for a recent review).

However, there are still only few actual sensing devices that implement CS in hardware. The design of such devices must take into account a number of important aspects. The most documented one is the choice and design of the sampling mechanism, i.e. the choice of a good *measurement matrix*. Several specific technical conditions were proposed such as the restricted isometry [2] that are sufficient to guarantee good performance. Unfortunately they are also difficult to translate into practical design guidelines. In this respect, the most interesting argument featured very early on in [4], [8], [3] is that a randomized sensing mechanism yields perfect reconstruction with high probability. In the past few years, several hardware implementations capable of performing such random compressive sampling were introduced [9], [6], [20], [28], that emulate random measurements through the use of multiplexing devices such as arrays of digital micromirrors (abbreviated DMD). More recently, some studies focused on compressive hardware imagers that replace such multiplexers by the natural disorder occurring in complex materials [23], [13], [19], [22].

Beyond the choice of the sensing mechanism, other practical concerns have to be addressed to build a CS hardware. First, the sampling mechanism may be unknown [22] or not perfectly controlled and may thus require calibration [16], [1]. The second practical question when building a CS imager is the choice of the number M of sensors and of the number P of measurements to take for the reconstruction of one single object. It has indeed been shown that using Multiple Measurement Vectors (MMV, $P > 1$) yields better performance than a Single Measurement Vector (SMV) in the noisy case [7], [12].

Our aim in this work is to provide guidelines concerning the choice of the number M of sensors and the number P of measurements to use in the presence of noise, both when the measurement matrix is perfectly known or experimentally measured. If many results were already obtained concerning the design of sparse reconstruction algorithms that are robust to noise (see [10], [12] for a review and, e.g. [21] for some recent trends), few empirical results have yet been given that may easily translate into design guidelines concerning the choice of MMV over SMV. Here, we show experimentally that, depending on the amount of noise in the observations, taking several measurements ($P > 1$) may be the only way to reach a working regime in practice, which has already been noticed, e.g. in [5]. However, since increasing P also dramatically increases the number of samples, a practitioner may want to choose the smallest acceptable P for a particular level of noise. Here, we go further in the investigation by providing a thorough simulation study that compares

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performance of many (M, P) configurations with varying levels of noise. Making use of the multiple scattering imager recently presented in [22], we experimental demonstrate that choosing MMV instead of SMV may be much more important than perfectly knowing the measurement matrix in the design of a CS hardware that actually reaches sub-Nyquist sampling rates.

II. SMV WITH NOISE

Assume that the signal we want to acquire is a vector $x \in \mathbb{R}^N$. The measurements are given as a vector $y \in \mathbb{R}^M$, given by $y = Hx$, where the $M \times N$ matrix H is called the measurement matrix (MM), with $M < N$. If x is known to be *sparse*, i.e. with only a few nonzero coefficients, it can be recovered through sparsity promoting solving algorithms that are able to recover a $K-$sparse x with $N \sim \mathcal{O}(K \log N/K)$ measurements, under some requirements over the MM (see [10], [12] for a review). If the signal x is not sparse, it is often assumed sparse in some given basis B, so that $x = Bs$, where s is sparse. In that case, estimation is performed by solving for s with HB as a MM.

II-A. Calibration: measuring the measurement matrix

When the MM is not perfectly known, several studies have focused on *calibration*, that permits to estimate it [16], [1] during a preliminary stage where both x and y are known.

When the sensing mechanism involves some complex physical diffusion as is the case in multiple scattering [27], [24], the MM may be totally unknown, even if it may be assumed deterministic. In that case, previous studies in optics concentrated on the estimation of that matrix [24], [25].

Here, we generalize those methods and assume that during a calibration stage, we can control the input x and measure the corresponding output y. Doing this L times and stacking the inputs and the outputs into matrices X and Y respectively, we have $Y = HX$. Assume these measurements come with additive white Gaussian noise of variance σ^2, H is estimated as:

$$\hat{H} = YX^H \left(XX^H + \sigma^2 I\right)^{-1}, \qquad (1)$$

where I is the $N \times N$ identity matrix. A thorough Bayesian treatment can take correlated measurement into account, as well as a prior distribution over H [21]. The a posteriori variance of H can be exploited as the power of a multiplicative noise [18].

II-B. Experimental phase transitions under noise

All CS reconstruction algorithms exhibit some level of robustness to noise. However, when noise becomes prominent, performance of CS eventually drops. In figure 1, we show how the performance of SMV drops when the amplitude of noise reaches about 3% of the amplitude of y. These figures give the 50% phase transition between success (below each curve) and failure (above). Success is understood as a good recovery of more than 90% of the support of x. The algorithm considered is the standard OMP algorithm. Remarkably, CS with SMV behaves the same when the MM is perfectly known or estimated through (1) with $L = 6N$ calibration measurements.

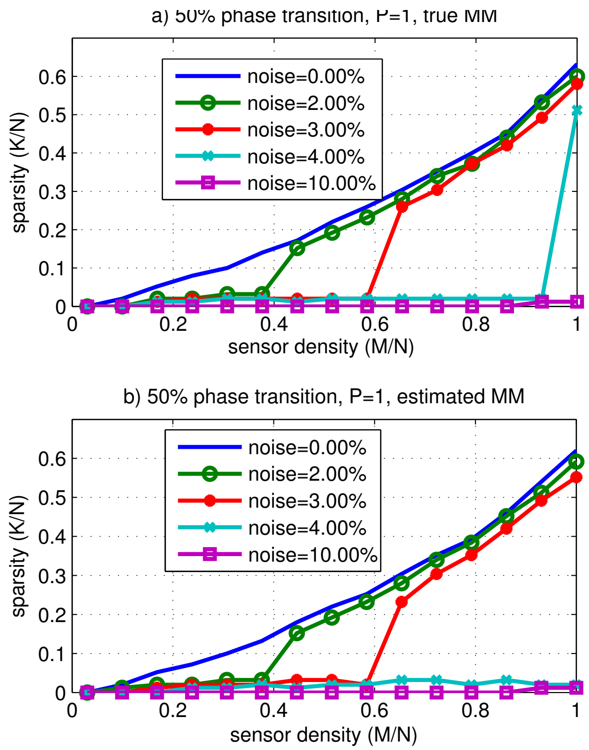

Fig. 1. Experimental 50% phase transitions of OMP-SMV with a) True MM and b) Estimated MM, at different levels of noise. Performance drop at 3% of noise. The (M, N) configurations below (above) each curve correspond to success (failure), respectively.

When noise is present at level above a few percents, which occurs in a number of practical scenarios, reconstruction using SMV-CS becomes impossible.

III. MMV WITH NOISE

Suppose that we can take P different measurements of our signal. The observations are given as a $M \times P$ matrix $Y = HX$, where X is the unknown $N \times P$ signal. Each column of X corresponds to a different "version" of the input. We assume that they all share the same support, with different nonzero values, which can be achieved in optics through different phase illuminations of the same object [22], [5].

Many algorithms are available today to estimate the support of X in the case of MMV [7], including the classical multichannel OMP [17].

III-A. Experimental phase transitions under noise

In figure 2, we give the 50% phase transitions observed using MMV with different numbers P of observations and

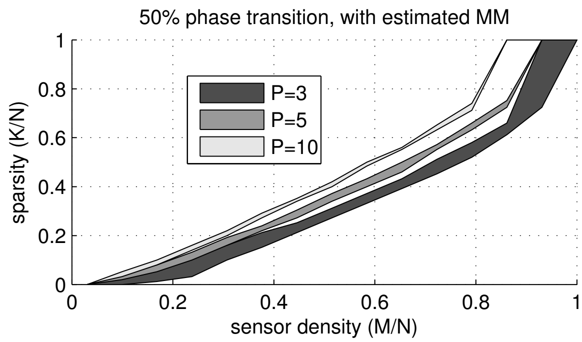

Fig. 2. Experimental performance of MMV with estimated MM. Each shaded area comprises all the 50% phase transitions for noise levels ranging from 0% to 10%. MMV is seen to be much robust to the presence of noise.

noise levels from 0% to 10% of the average amplitude of Y. For each P, the shaded area on the figure includes all 50% phase-transitions between these two noise levels. Due to space constraints, we only give the results for CS using a MM that is estimated through (1). Results using the true MM are similar.

Figure 2 demonstrates that MMV significantly improves robustness to noise. Its 50% phase transition is seen to be almost the same for noiseless observations or when the noise reaches 10% of the amplitude of Y. These simulations confirm the theoretical results given in [11] and the empirical findings in [22], [5] on that matter.

III-B. MMV sub-Nyquist CS under noise

When MMV is chosen, each one of the M sensors performs P measurements, yielding MP samples. When the objective is *compression*, i.e. storing x with only $MP < N$ samples, one may wonder whether increasing P may be interesting over the classical SMV case.

In figure 3, we give the results of a comparative study of MMV and SMV with respect to the sampling rate MP/N. As expected, SMV is best in the noiseless case. However, as soon as the noise level reaches a few percents, only MMV permits to reach sub-Nyquist sampling. In figure 4, we display the maximal sparsity K/N allowed for good reconstruction as a function of the noise level when the sampling rate $MP/N = 0.6$ is fixed. When the noise level reaches 4% here, having $P > 1$ is necessary.

Notwithstanding those results, we highlight that the main issue in the design of an imager may lie in the crafting of each one of its sensors, rather than in storage capacity. Hence, if the choice of MMV with a large P leads to a number MP of samples possibly higher than N, it produces good reconstruction with only a few sensors. In figure 5, we fix the sensor density M/N and display the smallest P required for good reconstruction as a function of the sparsity K/N and noise level.

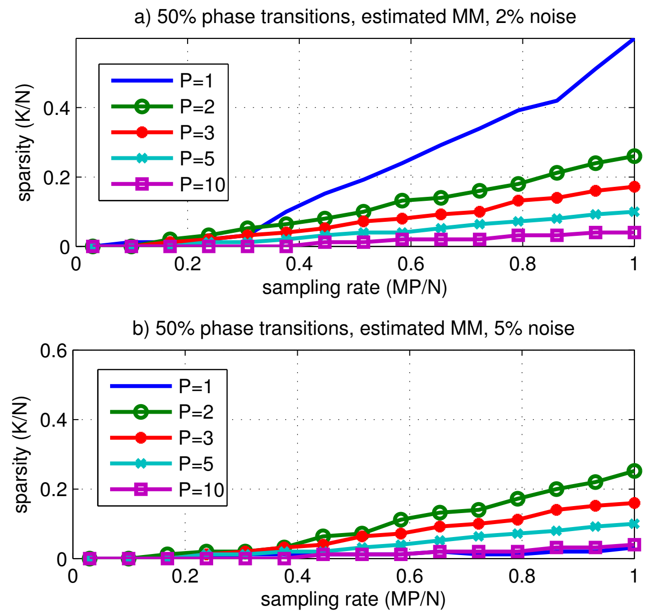

Fig. 3. Experimental 50% phase transitions of MMV for (a) 2% noise and b) 5% noise, with different P.

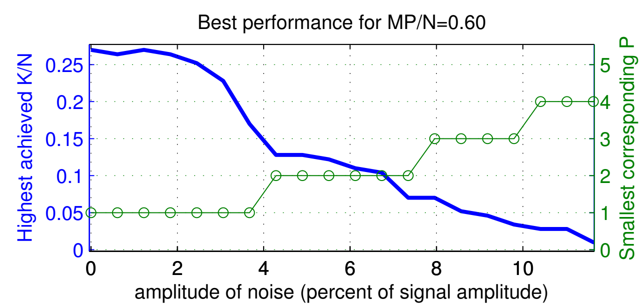

Fig. 4. Best observed performance for a fixed sampling rate $MP/N = 0.6$.

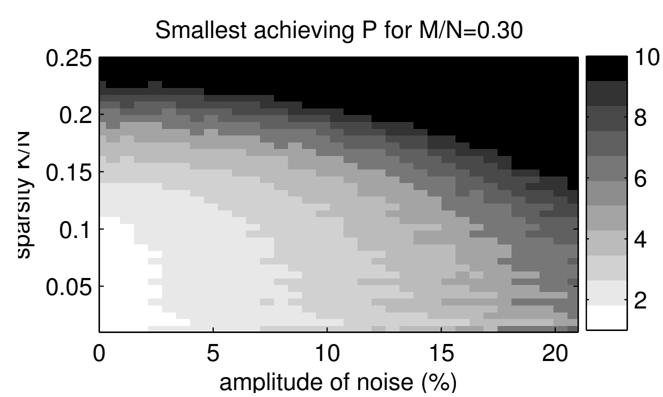

Fig. 5. Smallest achieving P for successful reconstruction with a fixed sampling rate $M/N = 0.3$ as a function of the noise level and the sparsity K/N.

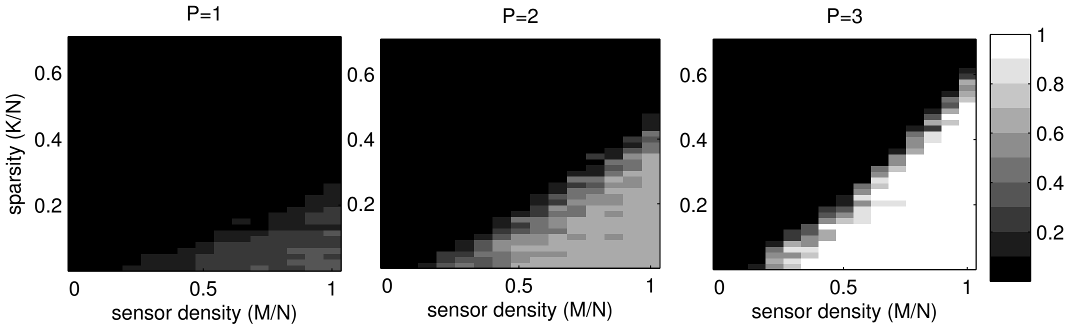

Fig. 6. Performance of CS reconstruction of sparse inputs imaged experimentally through our optical setup. Due to the presence of strong noise, only MMV at $P = 3$ permits reconstruction with high probability.

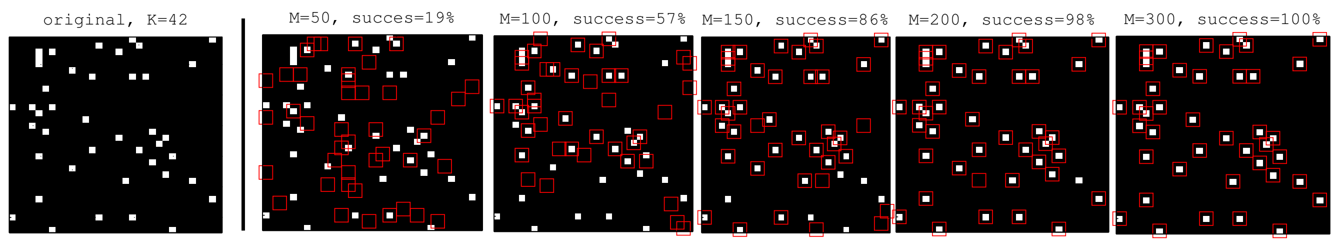

Fig. 7. Reconstruction of a sparse input with a real hardware compressive sampler using scattering material and $P = 3$. Red squares indicate the pixels identified as active, the original image is shown in the background.

IV. APPLICATION TO OPTICAL IMAGING WITH SCATTERING MEDIA

IV-A. Background

The diffusion of light through highly complex material has been the topic of much research in the recent optical literature [27], [26]. Light entering a highly scattering material undergoes many scattering events at a nanoscopic level [15], which were recently shown to be amenable to a macroscopic linear modelling $y = Hx$, where H is called a transmission matrix [24], [25]. In short, if a thin layer of white paint is placed between the object and the sensors, the transmission matrix H is random [14] and was recently shown to be a very good candidate for CS [22]. Whereas recent experimental studies [22], [5] consider MMV reconstruction for CS, they do not provide evidence that doing so is mandatory to reach good performance. We here provide an experimental validation of the superiority of MMV over SMV in the presence of strong noise using a new optical imaging system.

IV-B. Results

We refer the interested reader to [22] for details on the scattering optical imager. We tested for the MMV reconstruction performance of the system with different numbers P of measurements. The results are given in figure 6 as a complete exploration of the proportion of good reconstruction over all (K, M) configurations. As can be seen, due to the presence of noise, SMV does not permit good reconstruction, whereas $P = 2$ and $P = 3$ show better performance. An example of actual reconstruction with $K = 42$, $P = 3$ and a varying number M of sensors is given in figure 7.

V. CONCLUSION

In this paper, we have recalled some important challenges for the design of hardware implementing compressed sensing ideas. Independently of the reconstruction algorithm considered, one has to elicit the measurement matrix, which may need to be experimentally measured, and one needs to choose the number of sensors to embed in the imager, as well as the number of measurements to take for one single object.

We have provided a simple way to measure the measurement matrix, and shown both by a thorough simulation and a real-world optical experiment that whenever noise is present, exploiting several measurements of the same object under different illuminations may give far better results than an increase in the number of sensors or a better calibration. Since the crafting of each sensor may be a delicate matter by itself and calibration a long process, this result has important consequences in practice.

VI. ACKNOWLEDGEMENTS

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VII. REFERENCES

[1] C. Bilen, G. Puy, R. Gribonval, and L. Daudet. Blind sensor calibration in sparse recovery. In *international biomedical and astronomical signal processing (BASP) Frontiers workshop*, 2013.

[2] E. Candès. The restricted isometry property and its implications for compressed sensing. *Comptes Rendus Mathematique*, 346(9):589–592, 2008.

[3] E.J. Candès. Compressive sampling. In *Proceedings oh the International Congress of Mathematicians: Madrid, August 22-30, 2006: invited lectures*, pages 1433–1452, 2006.

[4] E.J. Candès, J. Romberg, and T. Tao. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *Information Theory, IEEE Transactions on*, 52(2):489–509, 2006.

[5] A. Chai, M. Moscoso, and G. Papanicolaou. Imaging strong localized scatterers with sparsity promoting optimization. *arXiv preprint arXiv:1309.7664*, 2013.

[6] W.L. Chan, K. Charan, D. Takhar, K. Kelly, R. Baraniuk, and D. Mittleman. A single-pixel terahertz imaging system based on compressed sensing. *Applied Physics Letters*, 93(12):121105–121105, 2008.

[7] S. Cotter, B. Rao, K. Engan, and K. Kreutz-Delgado. Sparse solutions to linear inverse problems with multiple measurement vectors. *Signal Processing, IEEE Transactions on*, 53(7):2477–2488, 2005.

[8] D.L. Donoho. Compressed sensing. *Information Theory, IEEE Transactions on*, 52(4):1289–1306, 2006.

[9] M. Duarte, M. Davenport, D. Takhar, J. N Laska, T. Sun, K. F Kelly, and R. Baraniuk. Single-pixel imaging via compressive sampling. *Signal Processing Magazine, IEEE*, 25(2):83–91, 2008.

[10] M. Elad. *Sparse and redundant representations: from theory to applications in signal and image processing*. Springer, 2010.

[11] Y. Eldar and H. Rauhut. Average case analysis of multichannel sparse recovery using convex relaxation. *Information Theory, IEEE Transactions on*, 56(1):505–519, 2010.

[12] Y. C Eldar and G. Kutyniok. *Compressed sensing: theory and applications*. Cambridge University Press, 2012.

[13] Rob Fergus, Antonio Torralba, and William T Freeman. Random lens imaging. Technical report, Massachusetts Institute of Technology, 2006.

[14] A Goetschy and A.D. Stone. Filtering random matrices: The effect of incomplete channel control in multiple scattering. *Physical review letters*, 111(6):063901, 2013.

[15] J. Goodman and S. Gustafson. Introduction to fourier optics. *Optical Engineering*, 35(5):1513–1513, 1996.

[16] R. Gribonval, G. Chardon, and L. Daudet. Blind calibration for compressed sensing by convex optimization. In *Acoustics, Speech and Signal Processing (ICASSP), 2012 IEEE International Conference on*, pages 2713–2716. IEEE, 2012.

[17] R. Gribonval, H. Rauhut, K. Schnass, and P. Vandergheynst. Atoms of all channels, unite! average case analysis of multi-channel sparse recovery using greedy algorithms. *Journal of Fourier analysis and Applications*, 14(5-6):655–687, 2008.

[18] M. Herman and T. Strohmer. General deviants: An analysis of perturbations in compressed sensing. *Selected Topics in Signal Processing, IEEE Journal of*, 4(2):342–349, 2010.

[19] J. Hunt, T. Driscoll, A. Mrozack, G. Lipworth, M. Reynolds, D. Brady, and D. Smith. Metamaterial apertures for computational imaging. *Science*, 339(6117):310–313, 2013.

[20] O. Katz, Y. Bromberg, and Y. Silberberg. Compressive ghost imaging. *Applied Physics Letters*, 95(13):131110–131110, 2009.

[21] F. Krzakala, M. Mézard, F. Sausset, Y. Sun, and L. Zdeborová. Probabilistic reconstruction in compressed sensing: algorithms, phase diagrams, and threshold achieving matrices. *Journal of Statistical Mechanics: Theory and Experiment*, 2012(08):P08009, 2012.

[22] A. Liutkus, D. Martina, S. Popoff, G. Chardon, O. Katz, G. Lerosey, S. Gigan, L. Daudet, and I. Carron. Imaging with nature: A universal analog compressive imager using a multiply scattering medium. *arXiv preprint arXiv:1309.0425*, 2013.

[23] R. Pappu, B. Recht, J. Taylor, and N. Gershenfeld. Physical one-way functions. *Science*, 297(5589):2026–2030, 2002.

[24] S. Popoff, G. Lerosey, R. Carminati, M. Fink, A. Boccara, and S. Gigan. Measuring the transmission matrix in optics: an approach to the study and control of light propagation in disordered media. *Physical review letters*, 104(10):100601, 2010.

[25] S. Popoff, G. Lerosey, M. Fink, A. Boccara, and S. Gigan. Controlling light through optical disordered media: transmission matrix approach. *New Journal of Physics*, 13(12):123021, 2011.

[26] I. Vellekoop, A. Lagendijk, and A. Mosk. Exploiting disorder for perfect focusing. *Nature Photonics*, 4(5):320–322, 2010.

[27] I. Vellekoop and A. Mosk. Focusing coherent light through opaque strongly scattering media. *Optics letters*, 32(16):2309–2311, 2007.

[28] C. Zhao, W. Gong, M. Chen, E. Li, H. Wang, W. Xu, and S. Han. Ghost imaging lidar via sparsity constraints. *Applied Physics Letters*, 101(14):141123–141123, 2012.