A Multivariate Singular Spectrum Analysis Approach to Clinically-Motivated Movement Biometrics

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ABSTRACT

Biometrics are quantities obtained from analyses of biological measurements. For human based biometrics, the two main types are clinical and authentication. This paper presents a brief comparison between the two, showing that on many occasions clinical biometrics can motivate for its use in authentication applications. Since several clinical biometrics deal with temporal data and also involve several dimensions of movement, we also present a new application of Singular Spectrum Analysis, in particular its multivariate version, to obtain significant frequency information across these dimensions. We use the most significant frequency component as a biometric to distinguish between various types of human movements. The signals were collected from triaxial accelerometers mounted in an object that is handled by a user. Although this biometric was obtained in a clinical setting, it shows promise for authentication.

Index terms - *Multivariate singular spectrum analysis, accelerometer, biometrics, instrumented objects, eigenvalues*

1. INTRODUCTION

A cursory observation of current consumer gaming electronics show the ubiquity of spatial three dimensional (3D) vision systems that analyse human motion to bring added interactivity in computer games. Indeed, such human motion systems are being pressed into service for clinical use. The analysis of motion implies a temporal dimension to the 3D data. Now biometrics is the measurement and analysis of biological data and when human data is used, the term is popularly used for identification of persons. However in this context the term used should be biometric authentication as compared to say, clinical biometrics. From these considerations, we can say that biometrics are used in these consumer gaming systems. Together with the widespread deployment of video cameras for surveillance purposes, we see that it is easy to obtain temporal 3D biometrics.

Often there is a need to combine and analyze 3D data originating from a common source like the movement of a person. In including the dimension of time, frequency based biometrics may be obtained as well. In this paper we show the limitations of traditional fixed basis methods of frequency analysis and show a novel application of a multidimensional data analysis approach which combines the data in a mathematically principled way and also provides easily interpretable frequency information which we will use for detection of a feature in this case, tremor.

We apply this method in a clinical application noting that the use of clinical biometrics as a precursor to authentication biometrics has several antecedents for example the use of DeoxyNucleuic Acid (DNA) chains from medical and now, used for identification as well. One of the more recent time based authentication biometric has been gait which was used in originally in a clinical setting as discussed in [1]. Further more we use the standardized patient methodology as a precursor to full scale medical trials. So as compared to authentication biometrics, for this paper we do not use large scale tests or attempt to perform classification but rather detect a feature.

In this paragraph we summarize the use of the temporal dimension in biometrics - in particular gait and for the sake of space, the primary reference will be [1] where more specific information can be found. Time varying features of a motion can be weighted and averaged into an image as in the Motion History Image methods. Variations of movements in time can also be summarized in the form of frequencies, or other nonlinear measures such as the Lyapunov exponent. These methods average time in an elementary way. In contrast, we show how common frequency information across 3D data streams can be obtained using a Singular Value Decomposition type of analysis which is has a better mathematical basis.

Often the signals produced by human movements are processed in the frequency domain. This is true of most current methods of analysing biomedical signals which use standard time or frequency measurements. However biological signals are never so well behaved, leading to the search for newer types of analyses which we feature in this paper.

In Section 2 we outline the motivation for our approach and cover background material. Section 3 describes our experimental setup. The theory we use for our signal analyses is covered in Section 4. Then we report the results of our experiments in Section 5 before we conclude in Section 6.

2. ASSESSING LIMB FUNCTION AND MOTION

In this section we present the clinical motivation for our work, presenting the case for using instrumented objects used in standardised clinical tests.

In formulating tests of limb function and movement, enforcing a protocol for their administration provides for objective and quantitative measurements. By performing statistical analysis of the data, various biometrics can be derived.

Currently, several of these tests use visual based scoring which introduces a degree of subjectivity and an inability to perceive subtle motions. This motivates for automating and monitoring these tests through electronic means by instrumenting the objects used in these tests, which is still a new field of research.

We seek to use tests that are widely accepted by the industry as they have been ratified through years of deployment. This provides a point of focus and discussion with clinicians who would be familiar with the methodology.

One such test is the Action Research Arm Test (ARAT) formulated by Lyle [2]. These can help assess two general types of movement disorders. First are involuntary periodic-

like muscle movements which come under several categories depending on the intensity of motion, its rapidity and the underlying causes. In this paper we use the generic term *tremor*, which is well understood although the word is more commonly associated with neurological disorders. Second are distorted static hand postures which may be a result of dystonia, caused by the inability to control the muscle tone. Then there are various degrees and combinations of these movements. Gross tremor frequencies occurring in the movements of the hand were around 1-4 Hz and 6-11 Hz. However, if the hand is weighed down, these frequencies are reduced [3]. A higher band of frequencies at 15-30 Hz were attributed to finger tremor.

Lee et al. [4] built an earlier prototype of the instrumented device described in this paper which incorporates accelerometers. Portions of their paper have been reproduced here for the sake of continuity in discussion.

The analysis of biomedical signals benefit from decomposition into constituent parts to identify features of interest. Frequently such signals are nonlinear and nonstationary so that applying Fourier-based signal decomposition produce mathematically correct functions, but these may not have any physical meaning at all. These signal constituents serve only to accommodate the lack of linearity and stationarity as shown in [5]. Decomposition methods using basis functions decided *a priori* have this problem.

To overcome this, recent frequency analyses using data driven decomposition processes have been introduced. Singular Spectrum Analysis (SSA) has been used to analyse naturally occurring physical phenomena and only recently it has been applied to biological signals. The forms of the constituent signals it produces are not constrained to sinusoids, for example. SSA produces readily interpretable constituent signals such as trends, periodic data and noise from short noisy signals.

In our work we combine two types of sensors not often used together, namely accelerometers and force sensors. This has the following benefits: i) it is capable of sensing fine motion and pressure exerted by a person and ii) there is no need to mount sensors on the body of a person. The next section describes our setup.

3. EXPERIMENTAL SETUP AND EARLY RESULTS

In this section we describe how we implement the ARAT and show some preliminary results. In our work, we focus on Test1 of the ARAT Grasp Subtest which involves the grasping of a wooden block 10 cm^3 in size. This object which we will call the Cube, is moved from a specified point directly to a target. The three main components of our instrumented object system are:

i) A set of resistive sensors used for measuring forces exerted on the faces of the Cube.
ii) A tri-axial accelerometer for acceleration measurements.
iii) A microcontroller converting the force sensor and accelerometer readings, sending the data to a workstation.

The sensor readings are taken at a rate of 30 samples/sec so that a maximum frequency of 15 Hz can be reliably recorded which is sufficient for hand tremor, as discussed in Section 2. We are in the process of collecting data with actual patients who have had a history of stroke and undergone rehabilitation. We perform a preliminary study on two of them. As part of our analyses, we filter the raw data with a noncausal high pass filter, leaving out frequencies of 4 Hz or lower as these would be due to non-tremor causes.

3.1 Test subjects and patients

We use the standardized patient (SP) methodology to characterise normal movements and also to *simulate* movement disorders caused for example by stroke, which are tremor and dystonia as described by Barrows [6]. While the main use of SP is for training, we extend its use to characterize movements. While in no way can this replace data from actual patients, the use of SP has its advantages. First is that it serves to highlight data capture oversights before actual trials begin. In practice, signal processing and analysis occurs quite some time after data capture and deficiencies in the capture process cannot be easily compensated for. Second is that the SP provides a baseline for comparison as it is quite to difficult to obtain *normal* movement data from patients that which are considered to be fully recovered. So we had five healthy subjects and assessed their movements, which will be classified and discussed in the next sub-sections. The four sets of movements are repeated for five times for each person, giving a total of 100 sets of data. The test subjects were briefed as to what constitutes dystonia and tremor and to reproduce them to the best of their ability. Dystonia was simulated by not attempting to keep the Cube upright during movements. Tremors were simulated by stiffening the forearm muscles and attempting to shake the hand, which causes involuntary movement in the hands.

3.2 Types of movements

Here we describe the possible types of movements that are executed in order to simulate movement disorders.

Normal grasp and move
In Fig. 2 we see the Cube being grasped by a right handed person moving it from the lower, hand silhouette to the higher black target, the trajectory shown by a broken line. We would expect this task to be completed smoothly, with a minimum of energy.

Skewed grasp and move
In this type of grasp, we consider loss of muscle function that prevents the Cube from being held upright. Rather than use the term tilt, we use SKEW as this denotes a sense of imbalance when executing this type of movement.

Grasp and move with tremor and/or skew
When the muscles are struggling to keep the Cube in the air, the muscles may tense up and voluntary control is diminished, resulting in tremor. Depending on the nature of the disorder there may also be a skewed grasp on the Cube as well. Thus we have for a subject, four sets of data representing the presence or absence of tremor and/or skewed grasp.

3.3 Qualitative results

In Fig. 1. we show the signals obtained for a SKEW movement reproduced from our earlier work.

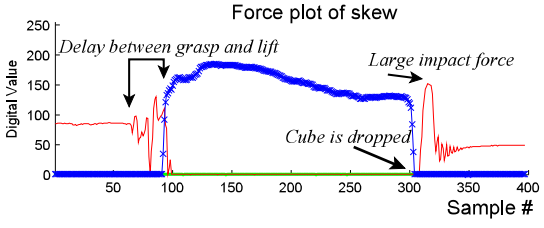

Fig. 1. Skew motion - note the force plot where the cube is dropped rather than placed in the top plot.

The line with 'x' markers have values that are close to zero initially. These denote the surface for the hand to grasp. The non-marker line denotes the force on the bottom sensor exerted by the mass of the Cube when it rests on a surface. It goes to zero when the Cube is lifted and this acts as a cue to indicate the start and *end* of a movement. This allows automatic segmenting of signals yielding an accurate measure of the duration of the movement. From this signal, another observation from Fig. 1 is that the subject may incorrectly *drop* the Cube rather than placing it on the table.

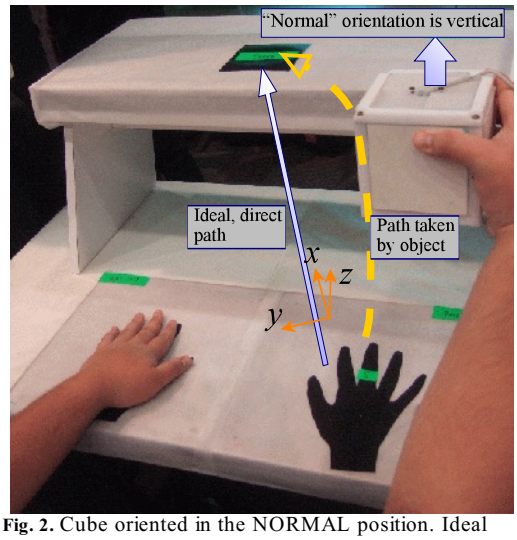

Fig. 2. Cube oriented in the NORMAL position. Ideal path of object compared to actual path taken

The accelerometer plots are in the left column of Fig 3. In the right column are the frequency plots where the signals have been filtered to pass signals greater than 4 Hz. Significantly, it can be seen that there is *no* dominant (amplitude wise) frequency for the 3 axes of movement.

4. THEORY OF ANALYSIS

In this section we consider the theory of SSA and the determination of the important eigenvalues of the system. In keeping with the relevant literature, we will substitute the term time series for a signal in this section.

4.1 Multivariate Singular Spectrum Analysis

SSA is a subspace analysis method originally developed for single time variable analysis and extended to multiple variables describing a common phenomenon. For Multivariate SSA (MSSA), an important difference is that the analysis is done across *all* relevant variables. In this section we describe the process, a clear explanation which can be found in [7]. For a set of D time series, at each time instant t the data is represented by a vector $\boldsymbol{x}(t) = \{x_d(t)\text{: } d=1...D,\ t=1...N\}$ with N sample points.

First for the d^{th} series, a window of length $M < N$ is used to embed this series into a matrix $\mathbf{Y}_d$, of size $N\times M$ where for the first and second columns:

$$\mathbf{y}_d^1 = [x_d(1),\ x_d(2),...,\ x_d(N)]^T$$
$$\mathbf{y}_d^2 = [x_d(2),\ x_d(3),...,\ x_i(N)\}\{0,1\}]^T \text{ and for column } c$$

$$\mathbf{y}_d^c = [x_d(c),\ x_d(c+1),...,\ x_i(N)\}\{0,c-1\}]^T$$

for $c = \{1,2...M\}$ and ^{T} denoting the transpose operator. By concatenating the embedding matrices, we have the grand embedding matrix of size $N\times DM$:

$$\mathbf{Y} = [\mathbf{Y}_1\ \mathbf{Y}_2\ ...\mathbf{Y}_d]$$

In contrast with SSA, for MSSA all time series need to be normalized to zero mean and unit standard deviation. The grand covariance matrix $\mathbf{C}$ for the system is given by:

$$\mathbf{C} = \mathbf{Y}^T\mathbf{Y} / N$$

which is $DM\times DM$ in size. Using SVD on $\mathbf{C}$ produces the sorted scalar eigenvalues λ and the eigenvectors ρ which are of length DM. Each eigenvector consists of D segments of length M. For the j^{th} eigenvalue and d^{th} time series, the corresponding sub-eigenvector is ρ_{jd} which make up the eigenvector ρ.

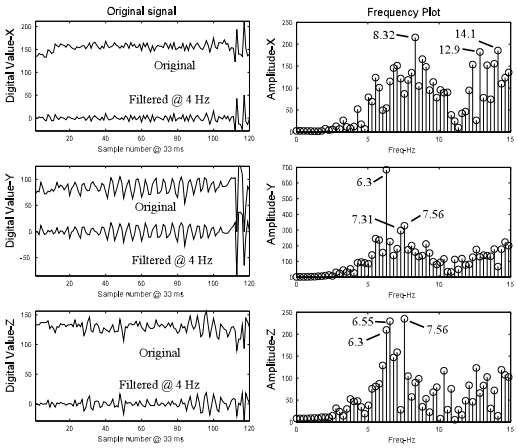

Fig. 3. Plots of accelerometer outputs of healthy subject simulating movement problems in the *xyz* directions, left column. Right column shows frequency plots for the respective axis. The y-axis is vertical.

The eigenvectors can be concatenated columnwise into a matrix $\mathbf{P}$ also of size $DM\times DM$. The principal components (PC) of $\mathbf{C}$ are given by the matrix:

$$\mathbf{E} = \mathbf{YP} \qquad (1)$$

Each column of $\mathbf{E}$ is a PC which is a vector whose elements are formed by a sum of all the time series weighted by their eigenvector elements.

A useful step is to reconstruct a signal component corresponding to the j^{th} eigenvalue. This is done with the corresponding PC_j with its eigenvector $\boldsymbol{e}_j$ taken from $\mathbf{E}$. A series of vectors $\boldsymbol{z}$ are formed:

$$\boldsymbol{z}_j^1 = [e_j(1),\ e_j(2),...,\ e_j(DM)]^T$$
$$\boldsymbol{z}_j^2 = [\{0,1\}\ e_j(2),\ e_j(3),...,\ e_{ij}(DM\text{-}1)\}]^T \text{ and for column } c$$

$$\boldsymbol{z}_j^c = [\{0,c\text{-}1\}\ e_j(c),\ e_j(c+1),...,\ e_j(DM\text{-}c)\}]^T$$

These are concatenated into an $N\times M$ matrix:

$$\mathbf{Z}_j = [\boldsymbol{z}_1\ \boldsymbol{z}_2\ ...\boldsymbol{z}_M] \qquad (2)$$

The reconstructed signal or component (RC) $\hat{\boldsymbol{x}}_{jd}$ for the d^{th} channel corresponding to the j^{th} eigenvalue is:

$$\hat{\boldsymbol{x}}_{jd} = \mathbf{Z}_j\,\rho_{jd} \qquad (3)$$

4.2 Significant eigenvalues

Performing Singular Value Decomposition (SVD) on a data set produces a set of principal components which explain the

variability in the data. The method is well covered in the literature and normally consists of eigenvector decomposition of the covariance matrix of the data. This returns a set of sorted λs in descending order with their corresponding eigenvectors which are the PCs. The rank of the eigenvalue indicates the amount of contribution of that component to the variability of the data. By plotting the λs in terms of their size, the scree plot (introduced by Cattell [9]) resembles the scree which is the rubble at the foot of a mountain as seen in Fig. 4.

It can be seen that the slope of the scree plot changes drastically in the first few λs, and settles on a gentle slope for the rest. This allows identification of the significant λs and eigenvectors ignoring the rest which can be attributable to noise. This reduces the data dimensionality.

The threshold between significant λs and the other λs can be determined objectively, where regression lines are drawn through sets of consecutive points on the scree plot. The point j_{sig} at which the largest *change* between the slopes of consecutive regression lines is taken as the threshold. For a set of λ indicated by the indices j, the slope b of a regression line is given by:

$$b_j = \frac{N\sum_{i=k}^{l} \lambda_i \, i - \sum_{i=k}^{l} i \sum_{i=k}^{l} \lambda_i}{N\sum_{i=k}^{l} i^2 - (\sum_{i=k}^{l} i)^2} \quad \text{where } k = f(j) \qquad (4)$$

Raîche et al. [8] recently analysed a variety of methods to determine this threshold. A moderately useful method was developed by Gorsuch and Nelson [9] and is denoted by *CNG*. For an index j, the slope b_j for points $k = j\text{-}2$ to $l = j$ for $j = \{3...N\text{-}3\}$ are computed and the point j_{sig} is where the largest b_j occurs. This is shown in Fig. 3 for lines RL1 and RL2, for points $j = 3$ and 4.

However Raiche's paper concluded that the best method to determine j_{sig} was that of Zoski and Jurs [10] in terms of accuracy and parsimonious use of λs. Their test is designated as *ZJb*. As compared to *CNG*, for a point j an additional slope b_j' for $k' = j+1$ to $l=N$ is computed. This is indicated as lines RL1 and RL3 for point $j = 3$. We follow the derivation in [11]where the slopes are compared by way of a t-statistic and the largest value for $j = \{3...N\text{-}3\}$ will be the threshold point j_{sig}. The t-statistic is given by:

$$t = \frac{b_j - b_j'}{s_{b_j - b_j'}} \quad \text{where} \quad s_{b_j - b_j'} = \sqrt{\frac{s_{\lambda j}}{s_j^2 (N_j - 1)} + \frac{s_{\lambda j}}{s_j'^2 (N_j' - 1)}} \qquad (5)$$

and

$$s_{\lambda j}^2 = \frac{(N_j - 1) s_{\lambda j}^2 + (N_j' - 1) s_{\lambda j'}^2}{N_j + N_j' - 4}$$

where the unprimed variables refer to the first set of indices $\{j\text{-}2...j\}$ and primed, the second set $\{j+1...N\}$ with N_j the number of elements in each set. The $s_{.}$ are the standard error of estimation between the variables involved and $s_{.}$ the standard deviation.

5. RESULTS

Here we discuss some initial results and show how the design of the sensors help us to interpret the obtained readings. This is followed by an analysis of the smoothness of movement.

5.1 Computational results with MSSA

In the course of our earlier work [4] we resorted to using frequency domain measures with a measure of success in an early prototype of the Cube in a limited experiment. However, with more data, this approach did not yield good results with the frequency spectrum becoming very cluttered.

When an object is moving with a periodic motion, the *projected* movements in an Euclidean three dimensional space should exhibit the same frequencies of movement. So for a triaxial accelerometer and the output of each of the axes, we expect the frequencies corresponding to the dominant amplitudes to be similar or the same.

However examining the signal from trial *H01_TS_4* (notation explained later) in Fig 3, when we identify the dominant three frequencies, they do not correspond well. For example, the frequency 6.3 Hz is dominant in the y and z axes, but not at all for the x axis. This was true for most of our data.

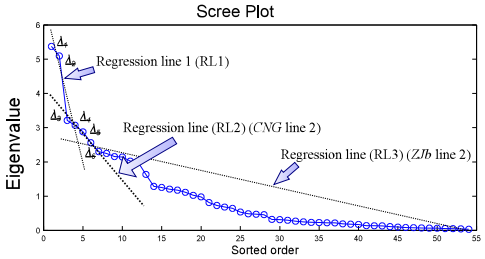

Fig. 4. Scree plot - larger version of that in Fig. 5.

Thus we consider MSSA, which separates out dominant harmonic components of a set of signals. The results are shown in Fig. 5 where the eigenvalue scree diagram is shown in the top right subplot. The lower right subplot show the RCs for the first ten λs.

It is observed that for pairs of *consecutive* λs that have similar values they may represent actual oscillations [7] in the original signal. So for RCs corresponding to those pairs of λs we perform a spectral analysis and select the frequency corresponding to the largest amplitude and denote it by F_{RCmax}. If the F_{RCmax} of the *largest* pairs of λs are the same, this would be the most significant frequency (MSF) in the signal.

Table 2 Frequencies with largest amplitude for the Reconstructed Components corresponding to eigenvalues of signal in Fig.5.

Rank	Eigenvalue λ	% Δ to next λ	Amplitude	Freq (Hz)
1	5.37	5.1	3.37	6.3
2	5.09	37	21.02	6.3
3	3.21	4.3	13.69	6.3
4	3.07	6.2	2.91	6.3
5	2.88	11.1	16.15	6.3
6	2.56	10.1	10.62	6.3
7	2.3	2.3	15.18	0.5
8	2.25	4.1	5.23	0.5
9	2.15	0.5	15.73	0.5
10	2.14	5.6	5.55	7.56

Consider the signal in Fig 3 for which the threshold for significant λs is $j_{sig} = 19$. In Table 2 we consider the first ten λs and see that the first two largest values differ by only 5.1%. For the corresponding Rcs the F_{RCmax} is the same, 6.30 Hz. In fact, this is repeated for the 3rd and 4th λ. For the 7th and 8th λ and these smaller λs, the F_{RCmax} is 0.5 Hz which we may interpret as trend. We conclude that these set of signals have a common dominant frequency of 6.30 Hz which is a sign of tremor. Referring to the spectrum plot in Fig 3, we see this in the second row and less so in the first and third rows. Subsequent tests show that when there is a large reduction of values between the first and second λs, the threshold value j_{sig} is greater than half of the λs which detects tremor frequencies at insignificant amplitudes. In this case we use *CNG*.

To summarize then, the steps taken to identify tremor in a set of triaxial accelerometer signals are:

i) normalize signals to zero mean, standard deviation of 1.
ii) perform an MSSA, obtain the λs.
iii) perform a multiple regression analysis.
iv) note the threshold for significant λs.
v) if threshold > 1/2 of all available λs, use *CNG* method.
vi) for significant λs, look for consecutive ones that differ by less than 10%.
vii) compute their Reconstructed Components (RC).
viii) for each of the RCs compute its frequency spectrum.
ix) note the dominant frequency corresponding to the largest amplitude, F_{RCmax}
x) if the F_{RCmax} of consecutive λs are the same, note the frequency which is the most significant frequency (MSF).
xi) if 4kHz <MSF< 15 kHz this indicates tremor.

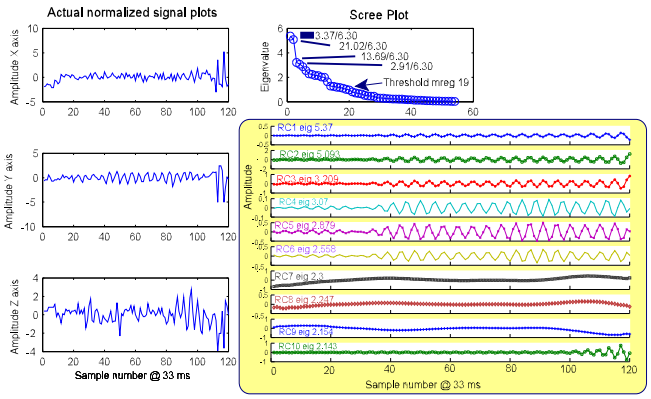

Fig. 5. Normalized waveforms on the left for a healthy subject performing a tremor and skew movement with the scree plot. The reconstructed waveforms according to first ten eigenvalues on the lower right subplot.

We finally perform this detection step on our data. In our data set, each trial has an identification (ID) code formulated as *SCC_MM_T* where *S* is H/P for healthy subject and patient respectively, *CC* the subject code, *MM* the movement type, NM for Normal, TR for tremor, SK for skew and TS for Tremor and Skew with *T* being the trial number, 1 to 5. The results are shown in Table 3. Some error is to be expected because of variation in the execution of the simulated moves.

Table 3 Summary results of the tremor test. False positive and false negative percentages with actual patient tests.

Trial type	False Positive	False Negative
all NM (non tremor)	13%	
all SK (non tremor)	23%	
all TR		20%
all TS		23%
P01_TS		30%
P02_TS		100%

False negatives are interpreted as tremor supposedly present but not detected due to healthy subjects being unable to properly execute the required simulated movements. False positives are detecting tremor when none is supposed to be present. For the actual patients, *P01* has a trial which shows tremor, but the other none. *P02* did not show any tremor at all but in actuality, two hands were used which is an invalid result, but it was a smooth move.

6. CONCLUSIONS

In summary, we have put forward a mathematically sound way to combine data from multiple related sensor data to yield frequency based features of interest.

In highlighting the pitfalls of using data decomposition methods using predetermined basis functions, we saw that traditional frequency analyses produces results that may be conflicting, in this case the dominant frequency of a tremor. The use of MSSA and the *ZJb* / *CNG* significant eigenvalue identification procedure allowed us to identify the Most Significant Frequency of movement confidently.

Future work will involve the analyses of other accelerometer signals, other types of eigenvalue analyses for a more robust determination and characterisation of movement disorders and determination of other biometrics. This would also include the use of MSSA to analyse characteristic human movements in 3D such as gait, to establish its use for providing authentication biometrics.

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