SWITCHING EXTENSIBLE FIR FILTER BANK FOR ADAPTIVE HORIZON SIZE IN FIR FILTERING

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ABSTRACT

Horizon size is an important parameter that influences estimation performance of finite impulse response (FIR) filter. In this paper, we propose a novel method called switching extensible FIR filter bank (SEFFB) to adapt horizon size based on maximum likelihood strategy. We verify that the SEFFB achieves a significant performance improvement compared with an ordinary FIR filter which uses a fixed horizon size.

***Index Terms*—** switching extensible FIR filter bank (SEFFB), FIR filter, state estimation, horizon size

1. INTRODUCTION

Finite impulse response (FIR) filters [1–8] have been studied as alternatives to the Kalman filter (KF) with infinite impulse response (IIR) structure. Because FIR filters use recent finite measurements, they are more robust against accumulation of modeling and computational errors than the KF. Furthermore, the H_∞ FIR filter [1] and the l_∞ FIR filter [2] developed by Ahn are robust against external disturbances as well as error accumulation.

One of the most significant current issues in FIR filtering is to manage the horizon size (also known the memory size) because it is an important parameter that influences the FIR filter's estimation performance. Shmaliy [4–7] developed various methods to find a best constant (i.e., fixed) horizon size. These methods are very effective with linear time-invariant systems. However, when system characteristics vary owing to external disturbances or modeling uncertainties, it is more effective to adjust the horizon size constantly than to use a best constant horizon size.

In this paper, we propose a novel method for adapting the horizon size of the FIR filter. Our approach is to use the FIR filter bank called the switching extensible FIR filter bank (SEFFB). In the SEFFB, the horizon size is adjusted constantly by means of maximum likelihood strategy. We show that for system with time-varying characteristics, adapting the horizon size leads to better results than using a constant horizon size.

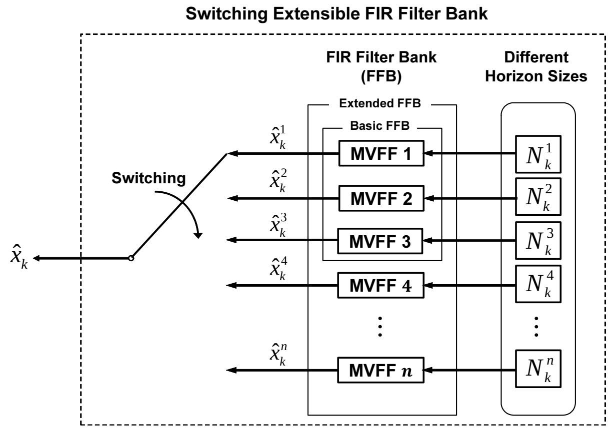

Fig. 1. Concept of SEFFB

2. SWITCHING EXTENSIBLE FIR FILTER BANK (SEFFB)

In this section, we propose the SEFFB to adapt the horizon size of the FIR filter. Figure 1 illustrates the concept of the SEFFB. The FIR filter bank (FFB) consists of n FIR filters, where n is the number of FIR filters. In this paper, we use the minimum variance FIR filter (MVFF) [9] as a component of the FFB. Each MVFF is matched to a particular horizon size $N_k^i \in S_k = \{N_k^1, N_k^2, \cdots, N_k^n\}$, where N_k^i is the i-th horizon size in S_k. S_k is the set of horizon sizes at time k. At time k, the FFB produces n state estimates $\hat{x}_k^1,\ \hat{x}_k^2,\ \cdots,\ \hat{x}_k^n$ from the n MVFFs. Among these multiple estimates, the most probable one is selected as $\hat{x}_k$ (i.e., the output of the SEFFB at time k). Filtering algorithm using the SEFFB will be presented in the following subsections.

2.1. Constructing the FFB

In this subsection, an efficient method to construct the FFB is proposed. The purpose of constructing the FFB instead of using a single FIR filter is to obtain multiple estimates from various horizon sizes. Thus, it is desirable that there are a large number of MVFFs using different horizon sizes in the FFB. However, if there are too many MVFFs in the FFB, the com-

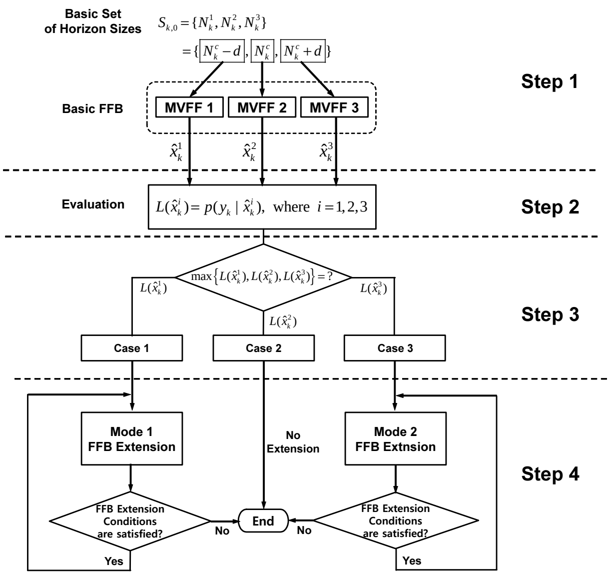

Fig. 2. Flow diagram of FFB extension

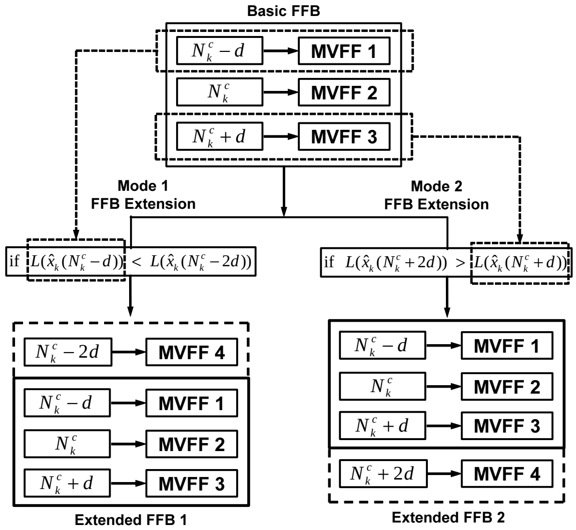

Fig. 3. Start of FFB extension

putational burden may become an issue. This type of problem, trade-off between the estimation performance and computational burden, can be found in the particle filtering. In the particle filtering, as the number of particles increases, the estimation accuracy increases but the computational burden also increases. To solve this problem, the KLD-sampling [10] was developed. In this method, particles are constantly generated until the error between the true posterior density and the sample-based approximation becomes less than the threshold value. In this way, the number of particles is automatically determined at each time step. The key idea of our approach is similar to the aforementioned adaptation methods in the particle filtering. In our method, a new MVFF is constantly added to the FFB as long as certain conditions (i.e., the FFB extension conditions) are satisfied. This process is called the FFB extension. Details of the FFB extension conditions and algorithm are described below.

2.1.1. Step 1. Constructing the Basic FFB and Obtaining State Estimates Using the Basic FFB:

At time k, the basic set of horizon sizes $S_{k,0}$ (subscript '0' means the 'basic' set) consists of three horizon sizes as follows:

$$\begin{aligned} S_{k,0} &= \{N_k^c - d,\ N_k^c,\ N_k^c + d\} \\ &= \{N_k^1,\ N_k^2,\ N_k^3\}, \end{aligned} \tag{1}$$

where $N_k^c - d$, N_k^c, and $N_k^c + d$ correspond to N_k^1, N_k^2, and N_k^3, respectively. N_k^c is the center horizon size (i.e., median value among the three horizon sizes). d is the equal distance between the horizon sizes and this is one of the design parameters of the SEFFB. It is recommended that the value of d is set to be 1 or 2. As shown in Figure 2, the basic FFB consists of three MVFFs using the horizon sizes in the basic set of horizon sizes (1). The basic FFB produces three state estimates $\hat{x}_k^1$, $\hat{x}_k^2$, and $\hat{x}_k^3$, which is obtained by using the horizon sizes $N_k^c - d$, N_k^c, and $N_k^c + d$, respectively.

2.1.2. Step 2. Evaluating the State Estimates:

The three estimates obtained from the basic FFB are evaluated to find the most probable one among them. Evaluation is performed by means of the likelihood function. The state estimate that produces the maximum likelihood is considered as the most probable state estimate. Consider the following linear measurement system with Gaussian noise:

$$y_k = Cx_k + v_k,\ \ v_k \sim (0, R), \tag{2}$$

where y_k is the measurement, and v_k is the Gaussian measurement noise with zero mean and the covariance R. Then, the likelihood function can be written as follows:

$$\begin{aligned} L(\hat{x}_k^i) &= p(y_k|\hat{x}_k^i) \\ &= \frac{1}{\sqrt{(2\pi)^q \det(R)}} \exp\Big\{-\frac{1}{2}(y_k - C\hat{x}_k^i)^T \times \\ &\quad R^{-1}(y_k - C\hat{x}_k^i)\Big\},\ \text{for}\ \ i = 1, 2, 3. \end{aligned} \tag{3}$$

2.1.3. Step 3. Determining the Mode of the FFB Extension:

The mode of the FFB extension is determined according to the result of the evaluation on the three estimates obtained from the basic FFB. If $\hat{x}_k^2$ obtained from the MVFF using the center horizon size N_k^c produces the maximum likelihood among the three estimates (Case 2 in Figure 2), we do not extend the FFB. If $\hat{x}_k^1$ obtained by using the horizon size $N_k^c -$

d produces the maximum likelihood (Case 1 in Figure 2), it means that small horizon size is more effective to produce state estimate with high likelihood. In this case, it is necessary to obtain an estimate one more time using the horizon size which is smaller than $N_k^c - d$. Thus, we add a new MVFF using the smaller horizon size $N_k^c - 2d$ to the FFB and obtain a state estimate using it. This process is called the mode 1 FFB extension. On the contrary, if $\hat{x}_k^3$ obtained by using the horizon size $N_k^c + d$ produces the maximum likelihood (Case 3 in Figure 2), we add a new MVFF which uses the larger horizon size $N_k^c + 2d$ to the FFB. This process is called the mode 2 FFB extension. Determining the mode of the FFB extension is depicted in Figure 2.

2.1.4. *Step 4. Performing the FFB Extension:*

Figure 3 shows how to start the FFB extension. In case of the mode 1 FFB extension, the new horizon size $N_k^c - 2d$ is tested for the FFB extension. Using the horizon size $N_k^c - 2d$, we obtain a state estimate from the MVFF and compute its likelihood. In the left side of Figure 3, $\hat{x}_k(N_k^c - 2d)$ and $L(\hat{x}_k(N_k^c - 2d))$ are the state estimate obtained by using the horizon sizes $N_k^c - 2d$ and its likelihood, respectively. If $L(\hat{x}_k(N_k^c - 2d))$ is larger than $L(\hat{x}_k(N_k^c - d))$, the MVFF with the new horizon size $N_k^c - 2d$ is approved to be a new member of the FFB. In other words, if the likelihood obtained from the new MVFF is larger than the maximum likelihood obtained from the MVFFs in the current FFB, the new MVFF is added to the FFB. In case of the mode 2 extension, the FFB extension is started in a similar manner. (See the right side of Figure 3). In Figure 3, the extended FFB 1 and 2 are the FFBs made by the mode 1 and the mode 2 FFB extensions, respectively. In this figure, MVFFs in the extended FFB are arranged according to the magnitude of the horizon sizes.

Once the FFB extension is started, it is constantly performed as long as the following conditions are satisfied:

1. The likelihood of the state estimate obtained from the newly added MVFF is higher than the maximum one among the likelihood values of the already obtained state estimates.
2. The number of execution of the FFB extension n_{ext} is lower than n_{limit}, where n_{limit} is the predetermined limitation on the number of execution of the FFB extension.
3. The newly generated horizon size N_{new} satisfies the horizon size conditions.

The horizon size should satisfy the following conditions:

1. At time k, the horizon size used for the MVFF should be smaller than k.
2. The horizon size used for the MVFF should be larger than or equal to the dimension of the state vector.
3. The horizon size used in the SEFFB should be within the interval $[N_{\min}, N_{\max}]$, where $N_{\min}$ and $N_{\max}$ are the predetermined lower bound and upper bound imposed on the horizon size.

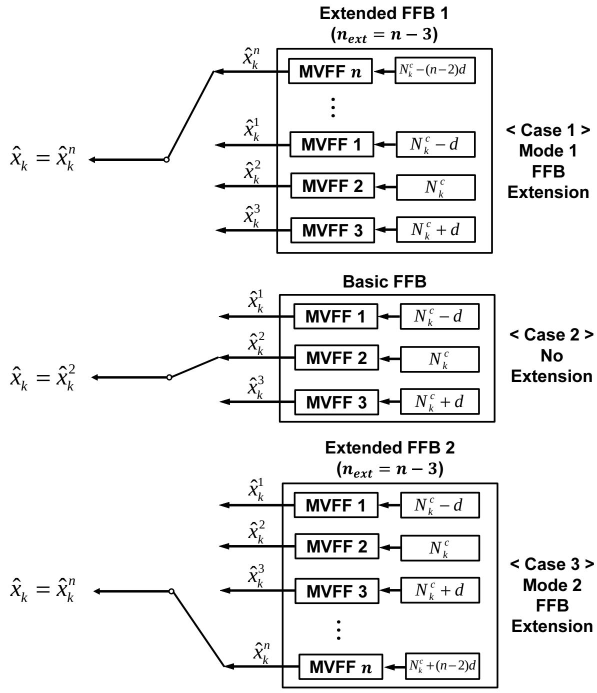

Fig. 4. Switching process of SEFFB

In the above conditions, n_{limit}, $N_{\min}$, and $N_{\max}$ are design parameters of the SEFFB. It is recommended that $N_{\min}$ is set to be the minimum available value of the horizon size (i.e., the same value as the dimension of the state vector). Since n_{limit} and $N_{\max}$ are the limits to prevent excessive computational burden, they can be set by considering the maximum operation time that is allowed in the specific system.

2.2. Switching Process: Selecting the Ultimate State Estimate

Suppose that the extended FFB has n MVFFs when the FFB extension terminates. If so, the n-th MVFF (i.e., the finally added MVFF) is the one that produces the maximum likelihood among all the MVFFs in the extended FFB. In the case that the FFB extension was not performed (i.e., Case 2 in Figure 2), $\hat{x}_k^2$ obtained from the horizon size N_k^c is the one that produces the maximum likelihood. The state estimate that produces the maximum likelihood is selected as the ultimate estimate $\hat{x}_k$ (i.e., output of the SEFFB at time k). The switching process to select the ultimate estimate is depicted in Figure 4. As shown in Figure 4, in Case 1 and Case 3, $\hat{x}_k^n$ obtained from the finally added MVFF is selected as the ultimate estimate $\hat{x}_k$ because it produces the maximum likelihood. In Case 2, $\hat{x}_k^2$ obtained from the MVFF 2 using the center horizon size is selected.

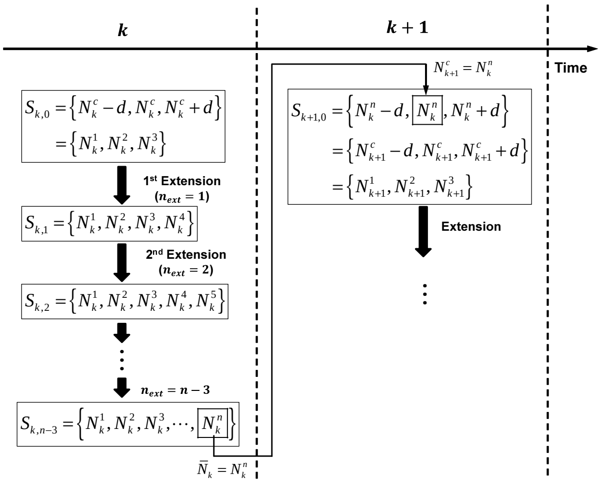

Fig. 5. Shifting basic set of horizon sizes

2.3. Shifting the Basic Set of Horizon Sizes

Now we explain the process of shifting the basic set of horizon sizes which is the key process of the SEFFB. The horizon size that produces the ultimate estimate at time k is utilized as the center horizon size of the basic set of horizon sizes at time $k+1$. Let $\bar{N}_k$ be the horizon size which is matched to the ultimate estimate with maximum likelihood at time k. Then, at time $k+1$, the basic set of horizon sizes $S_{k+1,0}$ is constructed by using $\bar{N}_k$ as follows:

$$S_{k+1,0} = \{\bar{N}_k - d,\ \bar{N}_k,\ \bar{N}_k + d\} \quad (4)$$
$$= \{N^c_{k+1} - d,\ N^c_{k+1},\ N^c_{k+1} + d\}, \quad (5)$$

where $\bar{N}_k - d$, $\bar{N}_k$, and $\bar{N}_k + d$ correspond to $N^c_{k+1} - d$, N^c_{k+1}, and $N^c_{k+1} + d$, respectively. In (4), we see that the newly generated horizon sizes in $S_{k+1,0}$ are distributed around $\bar{N}_k$. In other words, the basic set of horizon sizes are shifted so that the horizon sizes are located in the vicinity ($+d$ and $-d$) of $\bar{N}_k$. Thus, the FFB at time $k+1$ can utilize the horizon sizes that can produce better performance than the FFB at time k. In this way, the horizon sizes for the FFB are constantly adjusted so that they produces the higher likelihood. The process of shifting the basic set of horizon sizes is depicted in Figure 5.

3. NUMERICAL EXAMPLE

In this section, we test the SEFFB with the sinusoid signal model [7, 8], which is given as follows:

$$x_{k+1} = \begin{bmatrix} cos(\frac{\pi}{32}) + \delta_k & sin(\frac{\pi}{32}) \\ -sin(\frac{\pi}{32}) & cos(\frac{\pi}{32}) + \delta_k \end{bmatrix} x_k + w_k, \quad (6)$$
$$y_k = \begin{bmatrix} 1 + 0.25\delta_k & +0.25\delta_k \end{bmatrix} x_k + v_k, \quad (7)$$

where w_k and v_k are zero-mean white Gaussian noises with covariances $Q = diag(0.1^2 \quad 0.1^2)$ and $R = 0.2^2$, respectively. δ_k is a variable for injection of temporary model uncertainty. The simulation is performed for the interval $0 < k \leq 400$, where k is the time index. Design parameters of the SEFFB are taken as $d = 1, n_{\text{limit}} = 10, N_{\min} = 2, N_{\max} = 50$. To make the comparison fair, the minimum and the maximum horizon sizes of the MVFF are set to be the same as $N_{\min}$ and $N_{\max}$ of the SEFFB, respectively.

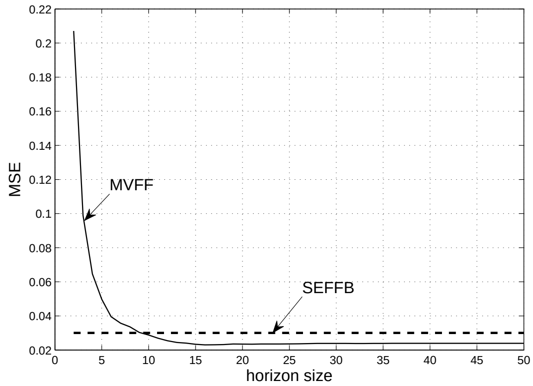

Fig. 6. MSEs of both MVFF and SEFFB for sinusoid signal model in case that model uncertainty does not exist

To begin with, we consider the case that model uncertainty does not exist (i.e., δ_k is zero at all times). Figure 6 represents the mean squared errors (MSEs) of the MVFF for diverse horizon sizes in this case (solid line). Figure 6 also represents the MSE of the SEFFB (dashed line). Note that the MSE of the SEFFB does not change according to the horizon size. In Figure 6, the best (i.e., minimum MSE) horizon size of the MVFF, N_{best}, is 16 and its MSE is 2.305×10^{-2}. The worst (i.e., maximum MSE) horizon size of the MVFF, N_{worst}, is 2 and its MSE is 2.071×10^{-1}. The MSE of the SEFFB is 3.003×10^{-2}. If we define the MSE of the best and worst horizon sizes as 100% and 0% of the best performance, respectively, it can be posited that the SEFFB achieves 96% of the best performance. Thus, it is verified that the SEFFB shows quite good performance when model uncertainty does not exist.

Now we consider the case that temporary model uncertainty exists. In this paper, two different levels of temporary model uncertainty ($\delta_k = 0.1$ and $\delta_k = 0.05$) are considered. The first case of temporary model uncertainty is defined as follows:

$$\delta_k = \begin{cases} 0.1, & 100 \leq k \leq 150, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Figure 7 represents MSEs of both the MVFF and the SEFFB in this case (i.e., $\delta_k = 0.1$). The best horizon size of the MVFF, N_{best}, is 2 and its MSE is 1.699. The MSE of the SEFFB is 1.144×10^{-1}, which is smaller than than of the MVFF using the best horizon size.

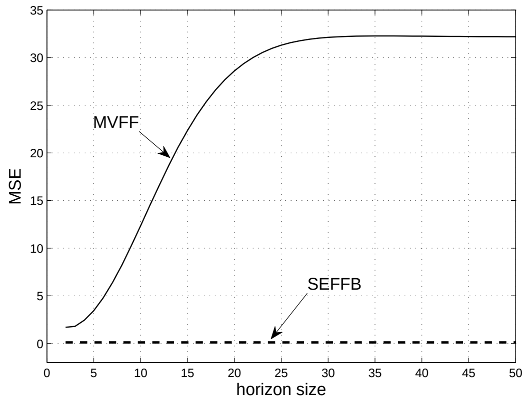

Fig. 7. MSEs of both MVFF and SEFFB for sinusoid signal model in case that $\delta_k = 0.1$

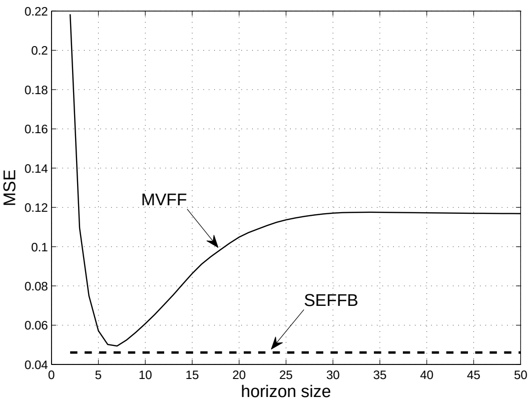

Fig. 8. MSEs of both MVFF and SEFFB for sinusoid signal model in case that $\delta_k = 0.05$

The second case of temporary model uncertainty is defined as follows:

$$\delta_k = \begin{cases} 0.05, & 100 \le k \le 150, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Figure 8 represents MSEs of both the MVFF and the SEFFB in this case (i.e., $\delta_k = 0.05$). In this case, the best horizon size of the MVFF, N_{best}, is 7 and its MSE is 4.938×10^{-2}. The MSE of the SEFFB is 4.605×10^{-2}, which is smaller than than of the MVFF using the best horizon size. In the both cases of temporary model uncertainty, the MSEs of the SEFFB are smaller than those of the MVFF which uses the best constant horizon size. Therefore, it is verified that the SEFFB shows better performance than the single FIR filter using the best constant horizon size when temporary model uncertainty exists.

4. CONCLUSION

In this paper, we have proposed the SEFFB to adapt the horizon size. In the case that temporary model uncertainty exists, the SEFFB showed better performance than the MVFF using the best constant horizon size. The SEFFB also showed quite good performance in the case that temporary model uncertainty does not exist. Therefore, it is verified that the SEFFB achieves significant improvement in the performance compared with the single FIR filter which uses the best constant horizon size. Since various FIR filters other than the MVFF can be used as a component of the SEFFB, it is expected that the SEFFB can be used widely in FIR filtering.

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