MOVING TARGET DETECTION IN AIRBORNE MIMO RADAR FOR FLUCTUATING TARGET RCS MODEL

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ABSTRACT

This paper considers the problem of target detection for multiple-input multiple- output with colocated antennas on a moving airborne platform. The target's radar cross section fluctuations degrade the detection performance of the radar. In this paper, first, we introduce a spatiotemporal signal model for airborne colocated MIMO radar which handles arbitrary transmit and receive antennas placement. Then, we employ the likelihood ratio test to derive the decision rules for fluctuating and nonfluctuating targets. In the case offull knowledge of target and interference statistic characteristics, we propose two detectors for fluctuating and nonfluctuating targets. The proposed detector can be used to evaluate adaptive detectors such as Kelly detector where the interference covariance matrix is estimated using training data.Simulation results have been provided to evaluate the detection performance of the proposed detectors.

Index Terms— Airborne MIMO radar, fluctuating target, likelihood ratio test (LRT), signal detection

1. INTRODUCTION

The detection of targets is one of the basic and most important radar functions [1]-[3]. In recent years, Multiple-Input Multiple-Output (MIMO) radar which employs multiple transmit and multiple receive antennas has become an active area of radar research and application [4]-[20]. Compare with the conventional phased-array radar using a single probing waveform, the MIMO radar makes use of multiple orthogonal probing waveforms, resulting in the waveform diversity which improves the detection performance [4], [5] and provides a higher spatial resolution [6], [7], more degrees of freedom (DOFs) [8], enhanced parameter identifiability [9] and possibility of direct application of adaptive array techniques [10]. Based on the array configurations used, the MIMO radars fall into statistical and coherent categories which use widely separated and colocated antennas respectively. In this paper, coherent MIMO radar on an airborne platform has been focused and thus the detection of fluctuating and nonfluctuating target detection has been investigated.

In [21], [22] and [23] the theory of space time adaptive processing (STAP) for traditional phased array airborne radar has been considered. The aim of the STAP is maximizing signal to interference ratio (SIR) which is needed for accurate parameter estimation. In target detection, the goal is to obtain the best possible probability of detection while guaranteeing that the false alarm probability does not exceed some tolerable value. Most of the recently proposed adaptive detection researches follow the lead of the seminal work by Kelly [24] where the generalized likelihood ratio test (GLRT) is used to obtain the detection rule in Gaussian interference with unknown covariance matrix. In [4], [6] and [8], the detection performance for widely separated MIMO radar has been considered while in [9] and [11] the authors have considered the colocated MIMO radar detection performance. The detection performance of the combined case which uses some widely separated subsystems where each subsystem contains colocated antennas, is considered in [12]. In [13] the linear quadratic GLRT has been extended to MIMO case in non-Gaussian interference.

In this paper, the detection rules for fluctuating and nonfluctuating targets are derived while we have the full knowledge of target and interference statistic characteristics in an airborne colocated MIMO radar system. Here we consider the case where the interference is Gaussian. The proposed signal model for airborne colocated MIMO radar is a general model for arbitrary transmit and receive antennas placement. The proposed detectors can be used to evaluate adaptive detectors performances.

.The remainder of this paper is organized as follows. Firstly, we present the signal model for MIMO radar on a moving airborne platform (Section 2). Using the signal model and likelihood ratio test, the proposed detectors for fluctuating and nonfluctuating targets are derived in Section 3. Section 4 contains simulation results for evaluating the performance of the proposed .detection rules. Finally, conclusions are drawn in Section 5.

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2. SIGNAL MODEL

Consider a colocated MIMO radar system (see Fig. 1) in which N receive antennas collect a finite train of L pulses sent by M transmit antennas and returned from a moving target. The transmitters and receivers are on moving airborne platform. The m-th transmit antenna and n-th receive antenna are at positions $\mathbf{r}_{TX,m}^T = [x_{TX,m}, y_{TX,m}, z_{TX,m}]$, $m = 1, \ldots, M$ and $\mathbf{r}_{RX,n}^T = [x_{RX,n}, y_{RX,n}, z_{RX,n}]$, $n = 1, \ldots, N$ in the Cartesian three-dimensional space. The airborne platform is located over the $x - y$ plane and flies along positive y −direction at a speed of v. We assume that the transmitters probe a common area of interest using M orthogonal waveforms. The orthogonality condition guarantees that each receive antenna extracts waveforms components employing a bank of M matched filters corresponding to the M orthogonal waveforms. The transmit and receive antennas are parallel, therefore they share the same azimuth θ and elevation angle ϕ. It is easy to verify that, for a point-like target located in the under test range cell corresponding to range R, the received signal vector can be expressed as

$$\mathbf{x} = \gamma b \mathbf{v}(f_0, \theta_0, \phi_0) + \mathbf{c} + \mathbf{n}. \tag{1}$$

where $\mathbf{c}$ denotes the clutter, $\mathbf{n}$ denotes the noise and b is the channel parameter, which is determined by the radar cross section of the target and R^{-4}. γ is equal to one for nonfluctuating target and γ is zero mean complex Gaussian variable with unit variance for fluctuating target. $\mathbf{v}(f_0, \theta_0, \phi_0) = \mathbf{a}_R(\theta_0, \phi_0) \otimes \mathbf{a}_T(\theta_0, \phi_0) \otimes \mathbf{a}_D(f_0, \theta_0, \phi_0)$ is the overall space-time steering vector to the target Doppler frequency for the stationary platform radar f_0, target azimuth angle θ_0 and target elevation angle ϕ_0 and

$$\mathbf{a}_R(\theta, \phi) = \begin{bmatrix} \exp(j\mathbf{k}^T \mathbf{r}_{R_1}) \\ \exp(j\mathbf{k}^T \mathbf{r}_{R_2}) \\ \vdots \\ \exp(j\mathbf{k}^T \mathbf{r}_{R_N}) \end{bmatrix}. \tag{2}$$

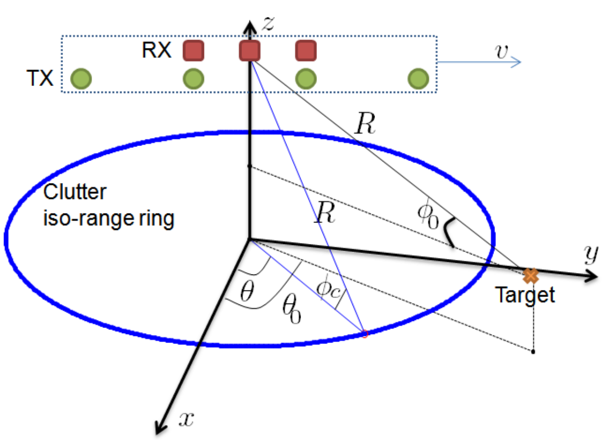

Fig. 1. The geometry of airborne MIMO radar system.

is the receive spatial steering vector and

$$\mathbf{a}_T(\theta, \phi) = \begin{bmatrix} \exp(j\mathbf{k}^T \mathbf{r}_{T_1}) \\ \exp(j\mathbf{k}^T \mathbf{r}_{T_2}) \\ \vdots \\ \exp(j\mathbf{k}^T \mathbf{r}_{T_M}) \end{bmatrix}. \tag{3}$$

is the transmit spatial steering vector and

$$\mathbf{a}_D(f, \theta, \phi) = \begin{bmatrix} 1 \\ \exp\left(j2\pi\left(f + \frac{2v}{\lambda}\sin\theta\cos\phi\right)T\right) \\ \vdots \\ \exp\left(j2\pi\left(f + \frac{2v}{\lambda}\sin\theta\cos\phi\right)(L-1)T\right) \end{bmatrix}. \tag{4}$$

is the temporal steering vector. The wave vector can be expressed as

$$\mathbf{k} = \frac{2\pi}{\lambda} \begin{bmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \sin\phi \end{bmatrix}. \tag{5}$$

where λ is the carrier wavelength and T is the pulse repetition interval.

The noise $\mathbf{n}$ is assumed to be temporally and spatially white with zero mean and covariance matrix $\sigma^2 \mathrm{I}$, where σ^2 is the variance of the noise and I denotes the identity matrix. The clutter signal contains reflections from stationary objects within the under test range cell. Note that for a fixed iso-range ring, the elevation angle ϕ_c is a definite quantity. The clutter signal can be illustrated as

$$\mathbf{c} = \int_0^{2\pi} \beta(\theta, \phi_c) \mathbf{v}(0, \theta, \phi_c)\, d\theta. \tag{6}$$

where $\beta(\theta, \phi_c)$ is the complex amplitude of the clutter patch echo. Therefore the interference covariance matrix can be expressed as $\Sigma = \Sigma_c + \sigma^2 \mathrm{I}$, where Σ_c is the clutter covariance matrix.

3. LIKELIHOOD RATIO TEST

The following binary hypothesis test is considered to have reliable decision rule, based on received signal $\mathbf{x}$, decide whether it contains only noise and clutter returns (null hypothesis) or signal plus interference signal (alternative hypothesis).

$$H_0 : \mathbf{x} = \mathbf{c} + \mathbf{n}, \quad H_1 : \mathbf{x} = \gamma b \mathbf{v} + \mathbf{c} + \mathbf{n} \tag{7}$$

3.1. Likelihood Ratio Test for Nonfluctuating Target

The likelihood ratio for nonfluctuating target is defined by

$$\Lambda_{\mathrm{NF}}(\mathbf{x}) = \frac{p_{NF}(\mathbf{x}|H_1)}{p(\mathbf{x}|H_0)}. \tag{8}$$

where$p(\mathbf{x}|H_0)$ is the probability density function of received signal under H_0

$$p(\mathbf{x}|H_0) = \frac{1}{\pi^N \det(\Sigma)} \exp(-\mathbf{x}^H \Sigma^{-1} \mathbf{x}). \quad (9)$$

wheredet(.) denoting the determinant of the matrix argument. For nonfluctuating target, $\gamma = 1$, and under alternative hypothesis, the received signal is a complex Gaussian random vector with mean $b\mathbf{v}$, therefore

$$p_{NF}(\mathbf{x}|H_1) = p(\mathbf{x} - b\mathbf{v}|H_0). \quad (10)$$

Substituting (9) and (10) into (8), taking the logarithm, and simplifying the equation yields

$$\ln \Lambda_{\mathrm{NF}}(\mathbf{x}) = 2real(b^* \mathbf{v}^H \Sigma^{-1} \mathbf{x}) - |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}. \quad (11)$$

where$real(.)$ denotes the real part of the argument.Therefore the likelihood ratio test (LRT) for nunfluctuating target is derived as

$$\mathrm{T}_{\mathrm{LRT-NF}}(\mathbf{x}) = 2real(b^* \mathbf{v}^H \Sigma^{-1} \mathbf{x}) - |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v} \underset{H_0}{\overset{H_1}{\gtrless}} \tau_{NF}. \quad (12)$$

whereτ_{NF} is a threshold selected to have a given probability of false alarm**.** The probability of false alarm of nonfluctuating target detector can be easily evaluated as follows

$$P_{FA-NF} = \mathrm{P}[\mathrm{T}_{\mathrm{LRT-NF}}(\mathbf{x}) > \tau_{NF}|H_0] = Q\left(\frac{\tau_{NF} + |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}}{\sqrt{2|b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}}}\right). \quad (13)$$

3.2. Likelihood Ratio Test for Fluctuating Target

In the case of fluctuating target, γ is a zero mean complex Gaussian variable with unit variance. Therefore the received signal under alternative hypothesis is NML dimensional complex zero mean Gaussian random vector with covariance matrix $\Sigma + b^2 \mathbf{v}^H \mathbf{v}$**.**

$$p_F(\mathbf{x}|H_1) = \frac{\exp\left(-\mathbf{x}^H \left(\Sigma + |b|^2 \mathbf{v}^H \mathbf{v}\right)^{-1} \mathbf{x}\right)}{\pi^N \det(\Sigma + |b|^2 \mathbf{v}^H \mathbf{v})}. \quad (14)$$

The likelihood ratio for fluctuating target can be expressed as

$$\Lambda_{\mathrm{F}}(\mathbf{x}) = \frac{p_F(\mathbf{x}|H_1)}{p(\mathbf{x}|H_0)}. \quad (15)$$

Using matrix inversion lemma,

$$(\Sigma + |b|^2 \mathbf{v}^H \mathbf{v})^{-1} = \Sigma^{-1} - \frac{\Sigma^{-1} \mathbf{v} \mathbf{v}^H \Sigma^{-1}}{\frac{1}{|b|^2} + \mathbf{v}^H \Sigma^{-1} \mathbf{v}}. \quad (16)$$

and the following lemma,

$$\det(\Sigma + |b|^2 \mathbf{v}^H \mathbf{v}) = \det(\Sigma)\,(1 + |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}). \quad (17)$$

the log-likelihood ratio for fluctuating target can be derived as the following

$$\ln \Lambda_{\mathrm{F}}(\mathbf{x}) = \frac{|\mathbf{v}^H \Sigma^{-1} \mathbf{x}|^2}{\frac{1}{|b|^2} + \mathbf{v}^H \Sigma^{-1} \mathbf{v}} - \ln(1 + |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}). \quad (18)$$

The likelihood ratio test for fluctuating target can be expressed as

$$\mathrm{T}_{\mathrm{LRT-F}}(\mathbf{x}) = \frac{|\mathbf{v}^H \Sigma^{-1} \mathbf{x}|^2}{\frac{1}{|b|^2} + \mathbf{v}^H \Sigma^{-1} \mathbf{v}} - \ln(1 + |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}) \underset{H_0}{\overset{H_1}{\gtrless}} \tau_F. \quad (19)$$

The probability of false alarm of fluctuating target detector can be easily written as

$$P_{FA-F} = \mathrm{P}[\mathrm{T}_{\mathrm{LRT-F}}(\mathbf{x}) > \tau_F|H_0] = e^{-\left(\tau_F + \ln\left(1 + |b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}\right)\right)\left(1 + \frac{1}{|b|^2 \mathbf{v}^H \Sigma^{-1} \mathbf{v}}\right)}. \quad (20)$$

In Fig. 2, we have plotted the "$P_{FA} - \tau$" curves for the nonfluctuating and fluctuating target detectors. From Fig. 2, there is a good agreement between theory and simulation.
The detection rules require the knowledge of interference covariance matrix Σ. In practice, Σ can be estimated from secondary data $\mathbf{x}_k; k = 1,2,\dots,K$. Under Gaussian interference, the classical sample covariance matrix (SCM) is the ML estimate, given by

$$\hat{\Sigma}_{\mathrm{K}} = \frac{1}{\mathrm{K}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathbf{x}_k \mathbf{x}_k^H. \quad (21)$$

where$K$ is the number of secondary data.
The adaptivedetectors can be obtained by replacing Σ with its SCM estimation $\hat{\Sigma}$.

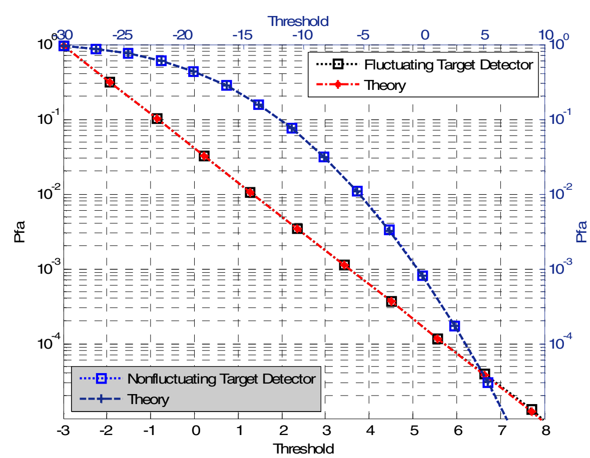

Fig. 2. Probability of false alarm versus threshold for the theoretical and numerical simulation of LRT-F and LRT-NF detectors.

The Kelly detector is a well-known adaptive detection rule which uses the K adjacent range cells of cell under test (CUT) as training data and is expressed as

$$\mathrm{T_{Kelly}}\left(\mathbf{x},\hat{\Sigma}_{\mathrm{K}}\right)=\frac{\left|\mathbf{v}^{H}\hat{\Sigma}_{\mathrm{K}}^{-1}\mathbf{x}\right|^{2}}{(\mathbf{v}^{H}\hat{\Sigma}_{\mathrm{K}}^{-1}\mathbf{v})(\mathrm{K}+\mathbf{x}^{H}\hat{\Sigma}_{\mathrm{K}}^{-1}\mathbf{x})}\underset{H_0}{\overset{H_1}{\gtrless}}\tau_{Kelly}. \quad (22)$$

The performance of the Kelly detector depends on the number of training data.According to well known Reed-Mallett-Brennan (RMB) rule, the amount of training data samples to achieve 3dB optimal signal to interference ratio performance is 2NML. It is obvious that the necessary condition for nonsingularity of the SCM estimation of interference covariance matrix is K ≥ NML.

4. SIMULATION RESULTS

In this section, we verify the above analysis and compare the detectors utilizing simulated data. It's assumed the airborne MIMO radar platform flies at speed of $v = 150\ m/s$. The carrier wavelength is 0.25 meter and the radar transmits pulses with pulse repetition frequency 8000 Hz. The radar uses $M = 4$ uniform linear array transmit antennas with distance 0.125 m. The receive antennas are $N = 4$ omnidirectional antennas with half wavelength distance which collect $L = 8$ pulses in a coherent processing interval. The reflected signal from target is imposed to the receive antennas from azimuth angle $\theta_T = 0$ with Doppler frequency $f_d = 500Hz$. The additive white complex noise is supposed to be zero mean and variance such that $SNR = -8dB$. The simulations have been done for clutter to noise ratio $50dB$. The effective signal to interference ratio for detectors is post processing SINR which defined as [25]

$$SINR_{post} = |b|^2\mathbf{v}^H\Sigma^{-1}\mathbf{v}. \quad (23)$$

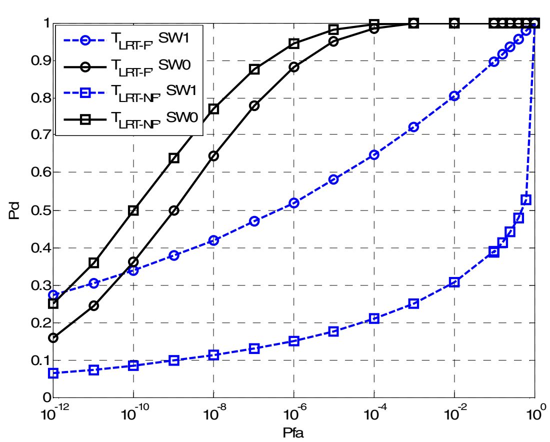

Fig. 3. : Receiver-operating-characteristic (ROC) curves for the LRT-F and LRT-NF detectors, post processing SINR=13dB.

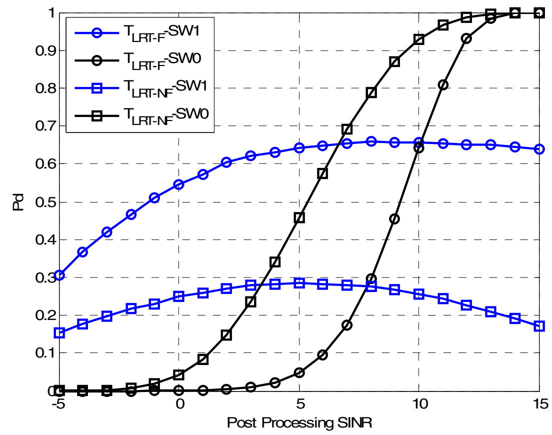

Fig. 4. $\boldsymbol{P_d}$versus$\boldsymbol{SINR_{Post}}$ for the LRT-F and LRT-NF detectors, $\boldsymbol{P_{FA} = 10^{-4}}$.

Assuming $SINR_{post} = 13dB$, the receiver operating characteristics (ROC) of the LRT detectors are plotted in Fig. 3. It is observed that the nonfluctuating target (LRT-NF) detector has the best performance when the Swerling 0 model (constant RCS) is used. Using Swerling I model for target RCS causes that the performance of LRT-NF detector degrades dramatically. As shown in Fig. 3, fluctuating target detector (LRT-F) outperforms LRT-NF detector in Swerling I target RCS model.

Fig. 4 shows the detection performance of proposed detectors when the threshold is set to reach a predetermined false alarm probability, $P_{FA} = 10^{-4}$. The target RCS fluctuations cause to have no detection performance gain when the target signal power exceeds a threshold.

In Fig. 5, the ROC curves for the LRT-F and LRT-NF detectors are plotted to evaluate the performance of Kelly detector while $SINR_{post} = 10dB$. It can be seen that the proposed detectors have better performance than Kelly detector with K=256, 512 training data. For this reason LRT-F

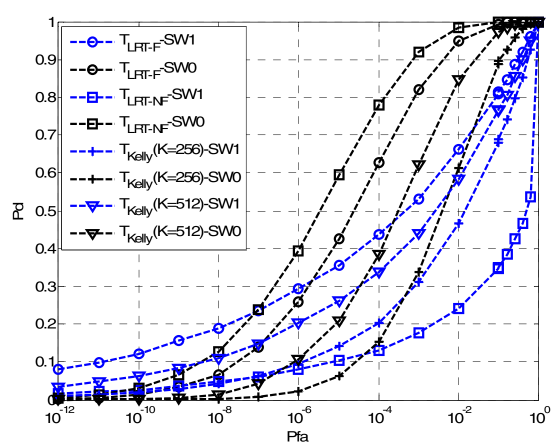

Fig. 5. ROC curves for the LRT-F, LRT-NF, Kelly K=256 and Kelly K=512 detectors, post processing SINR=10dB.

and LRT-NF detectors can be used as benchmark for performance comparisons.

5. CONCLUSION

For colocated MIMO radar on an airborne platform, the LRT detectors with full knowledge of target and interference statistic characteristics are studied under the Gaussian interference assumption. A general spatiotemporal signal model for airborne colocated MIMO radar with arbitrary transmit and receive antennas placement is proposed. Two detection rules for nonflactuating and fluctuating target are proposed which can be used as benchmark for performance comparisons in Swerling 0 and Swerling I RCS model respectively. Simulation results show that the proposed detectors outperform well-known Kelly detector. It is because of Kelly detector estimates the interference covariance matrix using training data and its performance converges to the proposed detectors performances.

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