RECURSIVE BLIND EQUALIZATION WITH AN OPTIMAL BOUNDING ELLIPSOID ALGORITHM

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ABSTRACT

In this paper, we present an algorithm for blind equalization i.e. equalization without training sequence. The proposed algorithm is based on the reformulation of the equalization problem in a set membership identification problem. Among the Set Membership Identification methods, the chosen algorithm is an optimal bounding ellipsoid type algorithm. This algorithm has a low computational burden which allows to use it easily in real time. Note that in this paper the equalizer is a finite impulse response filter. An analysis of the algorithm is provided. In order to show the good performance of the proposed approach some simulations are performed.

***Index Terms*—** Blind Equalization, FIR equalizer.

1. INTRODUCTION

The equalization problem is depicted on Fig. 1. It consists in the estimation of a source sequence from the knowledge of a received output sequence. Two main groups of methods can be distinguished: equalization with training and equalization without training. The first group needs the use of training sequences known both to receiver and transmitter. When it is not practical to use training sequences, the equalization can only be made using the received signal. This corresponds to the second group of methods: blind equalization methods. Blind equalization has received much interest for the last decades as pointed out by the available contributions (see [1], [2], [3], [4], [5], [6], [7], [8] and references therein). Two specific groups of methods can be roughly distinguished: (1-) Second-Order Statistics (SOS) methods ([4], [5] [7]) and (2-) Higher-Order Statistics (HOS) methods ([2], [3], [8]). In this paper we proposed a novel approach to blind equalization for a SISO channel.

The suggested approach is based on one key idea: rewrite the equalization problem as an identification problem in presence of bounded disturbances. Such an identification problem can be solved using Set Membership Identification methods. Among the Set Membership Identification methods, Optimal Bounding Ellipsoid (OBE) algorithms represent a very popular class of recursive algorithms ([9], [10], [11], [12], [13], [14]). The proposed approach is based on such algorithms. It is a recursive method with a low computational burden and it is then suitable for real-time application.

The paper is organized as follows: in section 2 the channel model is described and the problem of blind channel equalization is formulated. Section 3 is devoted to the presentation of the equalization algorithm: the principle and the algorithm are stated in subsections 3.1 and 3.2, an analysis is provided in 3.3. Section 4 shows simulation results. Section 5 concludes the paper.

2. PROBLEM FORMULATION

On Fig. 1 $\{s_k\}$ is the input sequence, $\{x_k\}$ is the received sequence given by

$$x_k = \sum_i h_i s_{k-i} + n_k$$

where $\{h_i\}$ are the impulse response coefficients which encompasses the effects of the transmitter filter, unknown channel and receiver filter. $\{n_k\}$ is an unknown noise sequence, $\{\widehat{s}_k\}$ denotes the output of the equalizer.

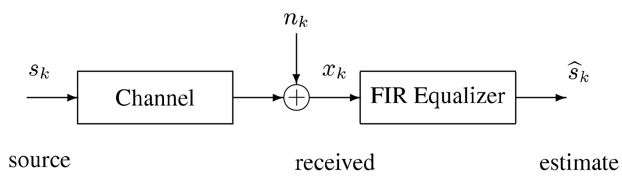

Fig. 1. Equalization problem

The input sequence $\{s_k\}$ is supposed to be unobservable and the objective in this paper is to propose a blind equalization algorithm i.e. an algorithm which allows the estimation of a deconvolution filter such that the transmitted sequence $\{s_k\}$ can be reconstructed reliably.

The proposed algorithm is based on the following usual assumptions:

A.1 The input sequence $\{s_k\}$ is supposed to be independent, identically distributed;

A.2 The input sequence $\{s_k\}$ is drawn from a QAM constellation $\mathbf{C}_{QAM}$;

A.3 The noise sequence $\{n_k\}$ is supposed to be bounded via $|n_k| \leq \delta_n$;

A.4 Sequences $\{s_k\}$ and $\{n_k\}$ are independent;

A.5 There exists a L order Finite Impulse Response (FIR) deconvolution filter F_o, characterized by a coefficient vector θ_o, such that if $n_k = 0$ then

$$s_k = \sum_{i=0}^{L} f_i x_{k-i} = \phi_k^T \theta_o$$

where

$$\theta_o = \begin{pmatrix} f_0 \\ \vdots \\ f_L \end{pmatrix} \quad \text{and} \quad \phi_k = \begin{pmatrix} x_k \\ \vdots \\ x_{k-L} \end{pmatrix}$$

Assumption $A.5$ seems a bit hard nevertheless, due to the fact that inaccuracy in the modeling can be included in the noise n_k, we have observed the algorithm to work well numerous cases where this assumption was not satisfied (this is the case in the example).

In the following we consider the function $dec(.)$ defined by

$$dec(y) = \begin{matrix} \arg\min \\ z \in \mathbf{C}_{QAM} \end{matrix} \left\{ \left| z - y \right|_2 \right\} \tag{1}$$

$dec(y)$ corresponds to the nearest symbol in $\mathbf{C}_{QAM}$ from y.

3. EQUALIZATION ALGORITHM

3.1. Principle

Under assumptions listed in section 2 and in the case where the noise sequence can be neglected ($n_k = 0$), it is stated in [15] that if we estimate a deconvolution filter F, defined by a coefficient vector θ with the same dimension as θ_o and such that the output $\widehat{s}_k$ belongs to $\mathbf{C}_{QAM}$, then there exists $n \in \mathbf{N}$ such that

$$\theta = e^{jn\frac{\pi}{2}} \theta_o$$

where θ_o is the equalizer coefficient vector of F_o defined in assumption [A.5].

In the case where the noise can't be neglected, the output $\sum_{i=0}^{L} f_i x_{k-i}$ of the filter F_o doesn't belong to $\mathbf{C}_{QAM}$. A useful elementary property on the output of the deconvolution filter F_o in a noisy context is provided below.

Property 1

Consider assumptions stated in section 2. If the noise sequence $\{n_k\}$ satisfies $|\sum_{i=0}^{L} f_i n_{k-i}| < 1$ (with F_o defined in assumption [A.5]) then we have

$$dec\left(\sum_{i=0}^{L} f_i x_{k-i}\right) = s_k \tag{2}$$

and

$$dec\left(\sum_{i=0}^{L} f_i x_{k-i}\right) = \sum_{i=0}^{L} f_i x_{k-i} + v_k \tag{3}$$

where $|v_k| \leq \delta_v$ with

$$\delta_v = \sum_{i=0}^{L} |f_i| \delta_n < 1 \tag{4}$$

Proof 1

We have $x_k = \sum_i h_i s_{k-i} + n_k$ and F_o such that $\sum_{i=0}^{L} f_i (x_{k-i} - n_{k-i}) = s_k$, it follows $s_k = \sum_{i=0}^{L} f_i x_{k-i} - \sum_{i=0}^{L} f_i n_{k-i}$. (2) is a consequence of the fact that s_k is drawn from a QAM constellation and $|\sum_{i=0}^{L} f_i n_{k-i}| < 1$. (3) follows with $v_k = -\sum_{i=0}^{L} f_i n_{k-i}$ such that $|v_k| < 1$.

This trivial property is important because this means that at each time the output $\sum_{i=0}^{L} f_i x_{k-i}$ of F_o is close to a point which belongs to the constellation and this point corresponds to s_k. This is illustrated on Fig. 2 for a 4-QAM modulation.

The proposed algorithm draws on (3) and (4): its aim is the estimation of a deconvolution filter F such that the output $\widehat{s}_k$ belongs to a neighborhood of the QAM constellation. This corresponds to the estimation of a coefficient vector θ in such a way that at each time $\widehat{s}_k = \phi_k^T \theta$ satisfies

$$dec(\widehat{s}_k) = \widehat{s}_k + \epsilon_k$$

that's to say

$$dec\left(\phi_k^T\theta\right) = \phi_k^T\theta + \epsilon_k \tag{5}$$

where $|\epsilon_k| \leq \delta$ with $\delta_v \leq \delta < 1$. In the case where the noise can be neglected ($\delta_v = \delta = 0$) this corresponds to the principle stated in [15].

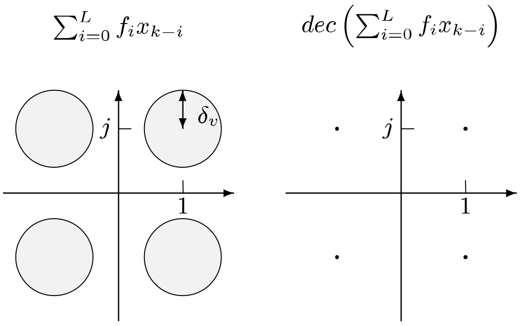

Fig. 2. $\widehat{s}_k = \sum_{i=0}^{L} f_i x_{k-i}$ in a neighborhood of $dec(\widehat{s}_k)$ for a 4-QAM modulation

Problem (5) is similar to an identification problem in presence of bounded disturbances. In this paper we shall investigate an OBE type algorithm ([11], [12], [14]), the reason is that its computational complexity is low and it is adapted to an identification problem in presence of bounded disturbances. The algorithm is given in the next subsection.

3.2. Algorithm

$\hat{\theta}_k$ represents an estimation of the coefficient vector θ. Let us define the a priori and a posteriori predictors as

$$\begin{cases} \hat{s}_{k/k-1} = \phi_k^T\hat{\theta}_{k-1} \\ \\ \hat{s}_{k/k} = \phi_k^T\hat{\theta}_k \end{cases} \tag{6}$$

and their corresponding errors

$$\begin{cases} \epsilon_{k/k-1} = dec(\hat{s}_{k/k-1}) - \hat{s}_{k/k-1} \\ \\ \epsilon_{k/k} = dec(\hat{s}_{k/k}) - \hat{s}_{k/k} \end{cases} \tag{7}$$

The equalization algorithm corresponds to an improved weighted recursive least square algorithm. Its update equation for $\hat{\theta}_k$ is as follows

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \Gamma_k\epsilon_{k/k-1} \tag{8}$$

with

$$\begin{cases} \Gamma_k = \frac{P_{k-1}\phi_k\sigma_k}{\lambda+\phi_k^T P_{k-1}\phi_k\sigma_k} \\ \\ P_k^{-1} = \lambda P_{k-1}^{-1} + \phi_k\sigma_k\phi_k^T \end{cases} \tag{9}$$

and $\epsilon_{k/k-1}$ defined from (6) and (7).

The two weighting terms are λ and σ_k. $0 < \lambda \leq 1$ is the forgetting factor fixed by the user to weight the past information.

σ_k is a switching flag designed in the following manner:

$$\sigma_k = \begin{cases} \frac{\lambda}{\phi_k^T P_{k-1}\phi_k}\left(\left|\frac{\epsilon_{k/k-1}}{\delta}\right| - 1\right) \\ \qquad \text{if } \left(\left|\epsilon_{k/k-1}\right| > \delta\right) \text{ and } \left(\phi_k^T P_{k-1}\phi_k > 0\right) \\ 0 \\ \qquad \text{if } \left(\left|\epsilon_{k/k-1}\right| \leq \delta\right) \text{ or } \left(\phi_k^T P_{k-1}\phi_k = 0\right) \end{cases} \tag{10}$$

The difference with algorithms proposed in [11], [12] and [14] lies in the fact that, here, there is no reference signal. The proposed equalization algorithm is a decision directed equalization algorithm: the value of $dec(\hat{s}_{k/k-1})$ monitors the algorithm behavior.

3.3. Analysis

The algorithm derived in subsection 3.2 is discussed in the present subsection.

δ is a user defined scalar whose role is described below. From (10), σ_k stops the updating of $\hat{\theta}_k$ if the arrival data are meaningless (i.e. $\phi_k^T P_{k-1}\phi_k = 0$). In the following consider the case where $\phi_k^T P_{k-1}\phi_k > 0$.

From the above expressions the prediction $\hat{s}_{k/k}$ can be written as

$$\hat{s}_{k/k} = \hat{s}_{k/k-1} + \phi_k^T\Gamma_k\epsilon_{k/k-1}$$

This gives the following error

$$dec\left(\hat{s}_{k/k-1}\right) - \hat{s}_{k/k} = (1 - \phi_k^T\Gamma_k)\epsilon_{k/k-1}$$

Making use of the expression for Γ_k, it follows

$$dec\left(\hat{s}_{k/k-1}\right) - \hat{s}_{k/k} = \frac{\lambda}{\lambda + \phi_k^T P_{k-1}\phi_k\sigma_k}\epsilon_{k/k-1}$$

Two possible cases arise.

- If $\left|\epsilon_{k/k-1}\right| > \delta$, then using the value of σ_k yields

  $$dec\left(\hat{s}_{k/k-1}\right) - \hat{s}_{k/k} = \frac{\epsilon_{k/k-1}}{\left|\frac{\epsilon_{k/k-1}}{\delta}\right|}$$

  This gives

  $$\left|dec\left(\hat{s}_{k/k-1}\right) - \hat{s}_{k/k}\right| = \delta$$

  We have $\delta < 1$ consequently $dec\left(\hat{s}_{k/k}\right) = dec\left(\hat{s}_{k/k-1}\right)$ and then we get

  $$\left|dec\left(\hat{s}_{k/k}\right) - \hat{s}_{k/k}\right| = \delta$$

- If $|\epsilon_{k/k-1}| \leq \delta$ then $\sigma_k = 0$ thus the adaptation is frozen:

$$\hat{\theta}_k = \hat{\theta}_{k-1}$$

It follows that $\hat{s}_{k/k} = \hat{s}_{k/k-1}$ and $dec\left(\hat{s}_{k/k}\right) = dec\left(\hat{s}_{k/k-1}\right)$ then we get

$$\left|dec\left(\hat{s}_{k/k}\right) - \hat{s}_{k/k}\right| \leq \delta$$

This clearly shows that, via the weighting term σ_k, the proposed algorithm ensures the following property:

$$\forall k \text{ such that } \phi_k^T P_{k-1}\phi_k > 0 \;\; ; \;\; |\epsilon_{k/k}| \leq \delta \tag{11}$$

Consequently δ is a chosen bound on the a posteriori error $\epsilon_{k/k}$. This means that, for $\delta < 1$, the output of the deconvolution filter belongs to a neighborhood of the QAM constellation. This neighborhood is defined by the bound δ.

The choice of the threshold δ determines the ability to reach a deconvolution filter F such that the transmitted sequence $\{s_k\}$ can be reconstructed reliably. From (4) this threshold depends on F_o and on δ_n:

$$\delta_v \leq \delta < 1 \tag{12}$$

with $\delta_v = \sum_{i=0}^{L} |f_i|\delta_n$.

An underestimation of the bound δ is in contradiction with condition (12) while with an overestimation the convergence may be slower. In section 4, simulation experiments are performed with $\delta = 0.99$.

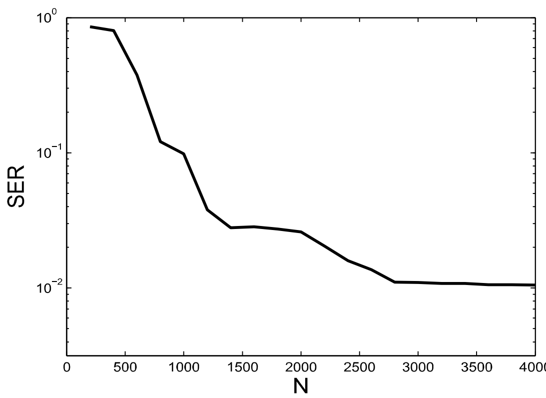

Fig. 3. First simulation: SER for various data set size from 100 Monte Carlo simulations

4. SIMULATION RESULTS

In this section, different simulations are reported to illustrate performance of the proposed method. We consider the 4-QAM modulation case and a channel described by the following filter:

$$H(z) = -1.666 + 0.175 * j + (0.288 + 0.726 * j)z^{-1} + (1.191 + 2.183 * j)z^{-2} + (-0.038 + 0.114 * j)z^{-3}$$

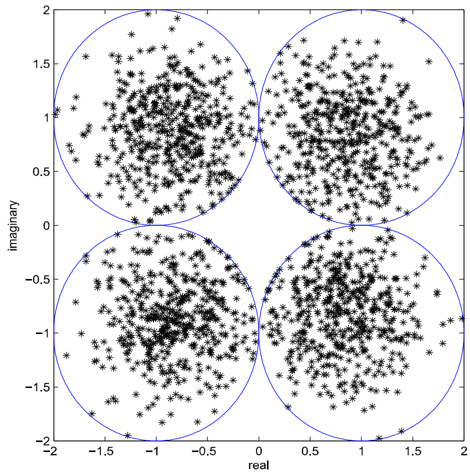

Fig. 4. First simulation: constellation of $\hat{s}_{k/k}$

In each experiment the order of the equalizer filter is $L = 15$. The forgetting factor λ has been chosen equal to 0.99 in order to reduce the impact of bad initial conditions and the threshold δ has been chosen equal to 0.99.

$\{n_k\}$ is a white noise uniformly distributed with $|n_k| \leq \delta_n$ where δ_n is adjusted to have a desired Signal to Noise Ratio (SNR). The proposed blind equalization algorithm has been tested on the basis of Monte Carlo simulations of 100 experiments.

In a first simulation, we have investigated the influence of the data-set size N in the equalization accuracy. Here SNR$= 20dB$ and N varied from 200 to 4000. The Symbol Error Ratio (SER) is depicted in Fig. 3 as a function of N. It appears an important improvement of performance as N grows until $N = 3000$.

Fig. 4 shows the constellation of $\hat{s}_{k/k}$ for $N = 2000$. Each point is in a neighborhood of the 4-QAM constellation. In the beginning, points are on boundaries, then after several iterations they converge inside constellation circles.

In a second simulation, the number of available data was $N = 2000$ and SNR varied from $5dB$ to $30dB$. We compared the proposed method against two methods:

- the Constant Modulus Algorithm (CMA) proposed in [2];
- the Stochastic gradient algorithm (SQD) proposed in [6].

Fig. 5 shows SER curves as a function of SNR. We observe that the proposed algorithm enhances performance of CMA and SQD for high and low SNR. Let us remark that the computational time of the proposed method is less than $1s$ which

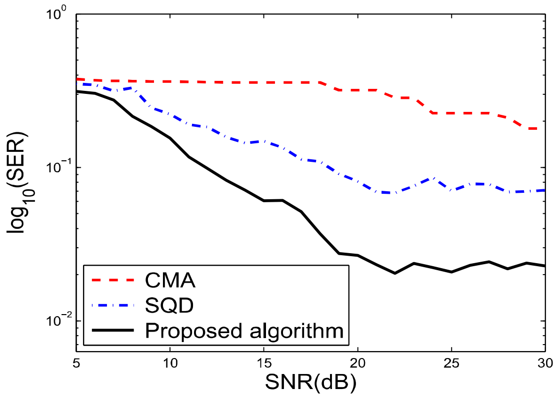

Fig. 5. Second simulation: SER obtained with 3 approaches for various levels of noise and from 100 Monte Carlo simulations

demonstrates the low computational burden of the proposed method.

5. CONCLUSION

In this paper we have investigated the blind channel equalization problem. It is shown that this problem can be written as an identification problem in presence of bounded disturbances. The proposed algorithm is based on a particular type of ellipsoidal algorithms: OBE type algorithms. One important contribution of this paper is the fact that few parameters have to be chosen (the forgetting factor λ and the bound δ) and systematic rules for their selection are provided. Simulation results have demonstrated that the algorithm performs better than some algorithms proposed in literature. In terms of future research, we believe that the proposed algorithm can be extended to other equalizer structures (such as infinite impulse response representation) or to MIMO channel.

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