COGNITIVE RADIO SYSTEM WITH A TWO-USER NON-BINARY NETWORK-CODED COOPERATIVE SECONDARY NETWORK

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ABSTRACT

We investigate the performance of a network coding based secondary network in a cognitive radio system under spectrum sharing constraints. The secondary network is composed of two users that cooperate to transmit their information to a common secondary destination. The outage probability is analyzed under a given maximum interference constraint set by the primary network as well as to the maximum transmit power limit of the secondary users. Theoretical and numerical results show that the adequate use of network coding by the secondary network can provide significant gains in terms of outage probability and diversity order when compared to non cooperative or traditional cooperative techniques.

***Index Terms*—** Cognitive radio, cooperative communications, network coding, spectrum sharing

1. INTRODUCTION

In a cognitive radio context under spectrum sharing constraints, the secondary network may transmit concurrently with the primary network as long as the primary communication is not compromised. A maximum allowable interference level at the primary receiver is defined so that the secondary users (SUs) must take into account this threshold during the transmission in order to adjust their transmit powers. This allows a more efficient use of the frequency spectrum since the interference is limited at the primary receiver [1].

Recently, cooperative communications have emerged as a promising technique to boost the performance of communication systems [2]. The idea behind is to make use of one or more nodes, known as relays, to help the communication between source and destination. The result is a virtual antenna array formed by single-antenna devices. Consequently, the same benefits obtained in multiple antenna systems are achievable. In a cooperative network, the transmission occurs in two phases: first, in the broadcast phase (BP), the source broadcasts its information; then, in the cooperative phase (CP), if the relay correctly decoded the source message it retransmits such message to the destination.

Motivated by the potential great benefits of cognitive radio and cooperative techniques, several works analyzed the performance of cooperative secondary networks under spectrum sharing constraints. For instance, in [3] the authors assume a dual-hop cooperative secondary network under primary interference and show that the primary interference does not affect the secondary diversity gain. In [4], an underlaid cooperative secondary network operating under a multi-user primary network is assumed, in which the secondary system adjusts the transmit power according to the peak interference power at the primary receivers as well as to a maximum transmit power constraint. In [5], the outage probability of a dual-hop cognitive relay network under Nakagami-m fading is investigated, and the authors show that the diversity order is dominated by the fading severity between the two hops of the secondary network. A similar scenario is investigated in [6], where the authors include the effect of the direct link in the secondary cooperative network. In [7] the primary network retransmissions are exploited together with cooperative communications at the secondary network, improving the secondary throughput without harming the primary network.

Recent works have applied the concept of network coding to cooperative networks [8–12]. In a network-coded cooperative network each user broadcast its information in the BP, then transmits a linear combination in the CP composed of its own message and the message(s) from its partner(s). In [10], a network-coded cooperative scheme is proposed where the users send a binary sum (XOR) in the CP. However, the scheme does not increase the diversity gain. An alternative is proposed in [11], called dynamic network coding (DNC), where the linear combinations transmitted during the CP are formed from a non-binary Galois field GF(q). For a scenario with M cooperative users the DNC scheme can achieve a diversity gain of $2M-1$, which is higher than that of binary coding schemes. In [12] a generalization to the DNC scheme is proposed, namely generalized dynamic network coding (GDNC). In the GDNC scheme the users are allowed to transmit several packets in the BP as well as to transmit an arbitrary number of non-binary linear combinations in the CP, resulting in a larger achievable diversity order than DNC.

In this paper we propose the use of network coding by the SUs. The performance in terms of outage probability of

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the so-called cognitive DNC (C-DNC) and cognitive GDNC (C-GDNC) schemes are evaluated, and it is shown that the network-coded cooperation is capable of considerably improving the performance of the secondary network when compared to traditional cooperative techniques as well as to the direct non-cooperative transmission.

The remainder of this paper is organized as follows: Section 2 introduces the system model, while in Section 3 an analytical performance analysis of the proposed scheme is carried out in terms of the outage probability. In Section 4 representative numerical results are provided and insightful discussions are drawn. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

Consider a cognitive network (CN) composed by two SU_s (U_1 and U_2), a common secondary destination D_s, a primary transmitter T_p and a primary destination D_p, as in Fig. 1. The SUs operate in the same band as the primary and it is subjected to transmit power constraints. The quasi-static fading channel between transmitter i and receiver j is denoted by h_{ij}, $i \in \{p, 1, 2\}$ and $j \in \{1, 2, s, p\}$, where p is the primary transmitter or receiver, $\{1, 2\}$ the secondary sources and s the secondary destination. All channels undergo independent Rayleigh fading, thus $|h_{ij}|^2$ follows an exponential distribution with mean power $\lambda_{ij} = d_{i,j}^{\alpha}$, where $d_{i,j}$ represents the normalized distance between the users i and j, with respect to d_{pp} (distance between T_p and D_p). In order to ease the analysis, we assume that all the secondary nodes are approximately at the same distance from the primary nodes. It is also considered a symmetric scenario in the secondary network, where the secondary nodes (U_1, U_2 and D_s) are equidistant.

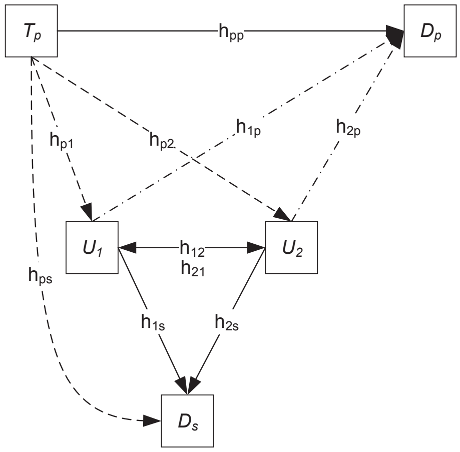

Fig. 1. System Model

The received codeword at user j from the user i is

$$\mathbf{y}_{i,j} = \mathbf{x}_i h_{i,j} + \mathbf{n}_{i,j}, \tag{1}$$

where $\mathbf{x}_i$ is transmitted codeword and $\mathbf{n}_{i,j}$ is the complex zero-mean additive white Gaussian noise with unity variance. Supposing unity bandwidth and that $\mathbf{x}_i$ is complex Gaussian distributed with unity variance, then the mutual information $\mathcal{I}_{i,j}$ between $\mathbf{x}_i$ and $\mathbf{y}_{i,j}$ is [13]

$$\mathcal{I}_{i,j} = \log_2 \left(1 + \frac{|h_{ij}|^2 P_i}{1 + |h_{pi}|^2 P_p}\right), \tag{2}$$

where P_p is the primary transmit power and P_i ,$i \in \{1, 2\}$, is the transmit power of secondary user i and is given by the minimum between the maximal transmit power P_{max} of user i and the transmit power that causes the maximum acceptable interference β at the primary destination, that is

$$P_i = \min \left(P_{max}, \frac{\beta}{|h_{ip}|^2}\right). \tag{3}$$

3. OUTAGE PROBABILITY ANALYSIS

An outage can be defined as the event that the mutual information between nodes i and j is less than an attempted information rate $\mathcal{R}$. The outage probability is thus defined as [13]

$$\mathcal{P}_{out}^{ij} = \Pr \left\{\log_2 \left(1 + \frac{|h_{ij}|^2 P_i}{1 + |h_{pi}|^2 P_p}\right) < \mathcal{R}\right\} \tag{4}$$

where $\Pr\{a\}$ is the probability of event x, $\mathcal{R} = \frac{R_s}{R_{\text{x}}}$, R_s is the attempted information rate in the case of non-cooperative direct transmission, and R_{x} corresponds to the code rate[1] of a given scheme x. For the non-cooperative scheme $R_x = 1$. The outage $\mathcal{P}_{out}^{ij}$ is given in (6) at the top of next page [14], with $\delta = 2^{\mathcal{R}} - 1$ and where Ei is the exponential integral function. As we consider a symmetric scenario in the secondary network, we have that $\left(\mathcal{P}_{out}^{12} = \mathcal{P}_{out}^{1s} = \mathcal{P}_{out}^{21} = \mathcal{P}_{out}^{2s} = \mathcal{P}_e\right)$, where $\mathcal{P}_e$ is the outage probability of an individual link in (6).

Moreover, the diversity order D_{x} of the the transmission scheme x is the asymptotic slope of the outage probability curve as a function of the signal-to-noise ratio (SNR), that is

$$D_{\text{x}} = \lim_{\text{SNR} \longrightarrow \infty} \frac{-\log \mathcal{P}_{out}^{\text{x}}}{\log \text{SNR}}, \tag{5}$$

with $\text{SNR} = \frac{P_i}{\lambda_{ij}}$.

Note that (6) is the outage probability of the SUs in the case of a non-cooperative direct transmission, which has diversity order of 1. Next we discuss three cooperative transmission schemes to be used by the secondary network.

3.1. Cognitive SDF (C-SDF) Scheme

First we consider that the secondary network operates according to the selective decode-and-forward (SDF) protocol [2]. In the BP, each SU sends its own message while in the CP

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[1] The number of information packets divided by the total number of transmitted packets, including the parities.

$$\mathcal{P}_{out}^{ij} = 1 - \frac{\lambda_{pj}}{e^{\frac{\lambda_{ij}\delta}{P_{max}}}} \left[\frac{1 - e^{-\frac{\lambda_{ip}\beta}{P_{max}}}}{\lambda_{pj} + \frac{\lambda_{ij}\delta P_p}{P_{max}}} - \frac{\frac{\beta\lambda_{ip}}{\lambda_{ij}}}{e^{\frac{\lambda_{ip}\beta}{P_{max}}}\delta P_p} e^{\left(\frac{\beta\lambda_{ip}}{\lambda_{ij}}+\delta\right)\left(\frac{\lambda_{ij}}{P_{max}}+\frac{\lambda_{pj}}{\delta P_p}\right)} \mathrm{Ei}\left(-\left(\frac{\beta\lambda_{ip}}{\lambda_{ij}}+\delta\right)\left(\frac{\lambda_{ij}}{P_{max}}+\frac{\lambda_{pj}}{\delta P_p}\right)\right)\right] \tag{6}$$

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each SU sends the message of its partner, if correctly decoded [2]. The outage probability for the C-SDF scheme is

$$\begin{aligned} \mathcal{P}_{out}^{\text{C-SDF}} &= \mathcal{P}_{out}^{12}\mathcal{P}_{out}^{1s} + (1 - \mathcal{P}_{out}^{12})\mathcal{P}_{out}^{2s}\mathcal{P}_{out}^{1s}, \\ &= \mathcal{P}_e{}^2 + (1 - \mathcal{P}_e)\mathcal{P}_e{}^2, \end{aligned} \tag{7}$$

where $P_{out}^{i,j}$ is obtained as in (6), with $R_{\text{C-SDF}} = \frac{1}{2}$. It can be easily shown from (7) that the diversity order of the C-SDF scheme is given by $D_{\text{C-SDF}} = 2$ [2].

3.2. Cognitive DNC Scheme (C-DNC)

In [10, 11] a non-binary network coded cooperative scheme called dynamic network coding (DNC) is proposed where the nodes are able to transmit linear combinations performed over a non-binary field in the CP, as depicted in Fig. 2, for a particular case of $M = 2$ cooperative nodes. Considering for the moment being that the channel between the cooperative nodes is outage-free, the destination receives four messages: I_1, I_2, $I_1 \bigoplus 2I_2$ and $I_1 \bigoplus I_2$. One can see from the received set that the destination is able to recover the messages I_1 and I_2 from any two of the four received messages. Thus, an outage occurs for the message of U_1 when I_1 and at least 2 out of the 3 remainder messages (I_2, $I_1 \bigoplus 2I_2$ and $I_1 \bigoplus I_2$) cannot be decoded. When the inter-user channel fails and U_2 cannot decode the message of its partner, it retransmits its own message. In this situation, upon receiving two copies of the same message, differently from [11], we consider that the destination performs selection combining (SC) between the messages instead of maximal ratio combining (MRC). This is because in such a scenario with the interference imposed by the primary network MRC is no longer optimal [15] and the performance is not considerably improved over the SC scheme, which is a substantially simpler scheme.

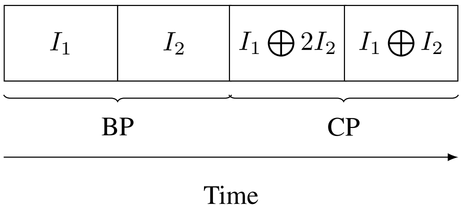

Fig. 2. Packets received by the destination in C-DNC.

The outage probability for the 2-user C-DNC scheme is given in (8) at the top of next page, for $R_{\text{C-DNC}} = \frac{1}{2}$. The diversity order achieved by the 2-user C-DNC scheme is [11]

$$D_{\text{C-DNC}} = 3. \tag{9}$$

3.3. Cognitive GDNC Scheme (C-GDNC)

In [12], the scheme of [11] is extended by considering M cooperative users where each user is able to broadcast k_1 information packets during the BP as well as transmit k_2 linear combinations of its own information and the information of other $M - 1$ users in the CP, if correctly decoded after the BP. For instance, Fig. 3 presents a time schedule example of the GDNC scheme for $M = k_1 = k_2 = 2$. If the inter-user channel fails, the information packet I_1^a will be in outage at the destination, if the direct transmission fails and at least 2 of 3 of the three remaining packets containing I_1^a packets (I_1^b, $\oplus I_1^a$, $\oplus I_1^b$) of U_1 cannot be decoded. If U_2 correctly decoded I_1^a (inter-user channel is not in outage), an outage will occur when the direct transmission fails and at least 4 of the 7 packets received by the destination (I_2^a, I_1^b, I_2^b, $\oplus I_1^a$, $\oplus I_2^a$, $\oplus I_1^b$, $\oplus I_2^b$) cannot be decoded.

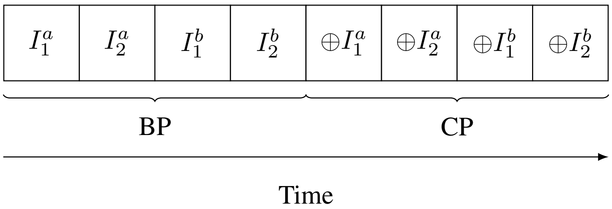

Fig. 3. Packets received by the destination for the C-GDNC scheme, where $M = 2$ users broadcast $k_1 = 2$ information packets (I_i^a,I_i^b) in the broadcast phase as well as transmit $k_2 = 2$ linear combinations ($\oplus I_i^a$,$\oplus I_i^b$), with $i \in \{1, 2\}$

The outage probability of a 2-user C-GDNC scheme with code rate given by $R_{\text{C-GDNC}} = \frac{k_1}{k_1+k_2} = 0.5$ is reproduced in (10), where $C_k^n = \frac{n!}{k!(n-k)!}$ is the binomial coefficient. From (10) it can be found that the diversity order for a of a 2-user C-GDNC scheme with parameters $k_1 = k_2 = 2$ is given by

$$D_{\text{C-GDNC}} = M + k_2 = 4, \tag{11}$$

which is greater than that of C-SDF and C-DNC schemes.

4. RESULTS

Next we present numerical results regarding the performance of the cooperative secondary network. It is assumed that $\lambda_{i,j} = d_{i,j}^{\alpha}$, with $\alpha = 4$, $d_{p,1} = d_{p,2} = d_{p,s} = d_{1,p} = d_{2,p} = 1$ and $d_{1,2} = d_{1,s} = d_{2,s} = \frac{1}{2}$, the attempted secondary transmission rate is $R_s = 1$ bits/s/Hz, and $P_p = 15$ dB.

$$\begin{aligned}\mathcal{P}_{out}^{\text{C-DNC}} &= \mathcal{P}_{out}^{12}(\mathcal{P}_{out}^{1s})^2 + (1-\mathcal{P}_{out}^{12})\left\{\mathcal{P}_{out}^{1s}\left[(\mathcal{P}_{out}^{2s})^2\cdot(1-\mathcal{P}_{out}^{1s}) + 2\cdot\mathcal{P}_{out}^{2s}(1-\mathcal{P}_{out}^{2s})\mathcal{P}_{out}^{1s}\right] + (\mathcal{P}_{out}^{2s})^2\mathcal{P}_{out}^{1s}\right\},\\ &= {\mathcal{P}_e}^3 + (1-\mathcal{P}_e)\left\{\mathcal{P}_e\left[{\mathcal{P}_e}^2(1-\mathcal{P}_e) + 2\mathcal{P}_e(1-\mathcal{P}_e)\mathcal{P}_e\right] + {\mathcal{P}_e}^3\right\}\end{aligned} \tag{8}$$

$$\begin{aligned}\mathcal{P}_{out}^{GDNC} &= \mathcal{P}_{out}^{12}\mathcal{P}_{out}^{1s}\left[C_2^3(\mathcal{P}_{out}^{1s})^2(1-\mathcal{P}_{out}^{1s}) + C_3^3(\mathcal{P}_{out}^{1s})^3\right] + (1-\mathcal{P}_{out}^{12})\mathcal{P}_{out}^{1s}\left[C_4^7(\mathcal{P}_{out}^{1s})^4(1-\mathcal{P}_{out}^{1s})^3\right.\\ &\left.+C_5^7(\mathcal{P}_{out}^{1s})^5(1-\mathcal{P}_{out}^{1s})^2 + C_6^7(\mathcal{P}_{out}^{1s})^6(1-\mathcal{P}_{out}^{1s}) + C_7^7(\mathcal{P}_{out}^{1s})^7\right],\\ &= \mathcal{P}_e^2\left[C_2^3(\mathcal{P}_e)^2(1-\mathcal{P}_e) + C_3^3(\mathcal{P}_e)^3\right] + (1-\mathcal{P}_e)\mathcal{P}_e\left[C_4^7(\mathcal{P}_e)^4(1-\mathcal{P}_e)^3\right.\\ &\left.+C_5^7(\mathcal{P}_e)^5(1-\mathcal{P}_e)^2 + C_6^7(\mathcal{P}_e)^6(1-\mathcal{P}_e) + C_7^7(\mathcal{P}_e)^7\right]\end{aligned} \tag{10}$$

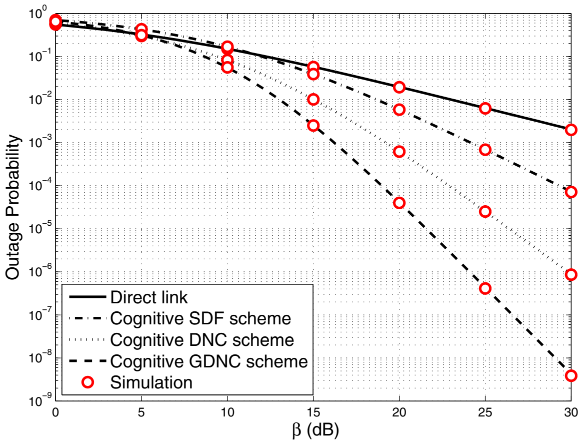

Fig. 4. Outage probability as a function of the primary interference threshold β without maximum transmit power limit.

Fig. 4 presents the outage probability as a function of the primary interference threshold β. In this case we consider that the secondary transmit power is limited only by the interference threshold β, i.e. without any limit to the maximum transmit power ($P_{max} \to \infty$ dB). One can see from Fig. 4 that the C-GDNC scheme presents the best performance among all the other schemes, mainly for $\beta > 10$ dB. Notice that Monte Carlo simulations agree very well with the theoretical results.

Fig. 5 presents the outage probability as a function of the primary interference threshold β, when the SUs have a maximum transmit power contraint $P_{max} = 15$ dB. One can see that the outage probability saturates when $\beta > 20$ dB. In a real scenario, the transmitter has limited transmit power, thus, it is not possible to decrease the outage probability as much as desirable only by increasing the transmit power.

The outage probability as a function of the secondary attempted rate R_s is presented in Fig. 6. We consider $P_p = 15$ dB, $\beta = 30$ dB and $P_{max} \to \infty$ dB. One can see that when $R_s < 4$ bits per channel use (bpcu), the cognitive GDNC outperforms the other schemes. For instance, when $R_s = 1$ bpcu,

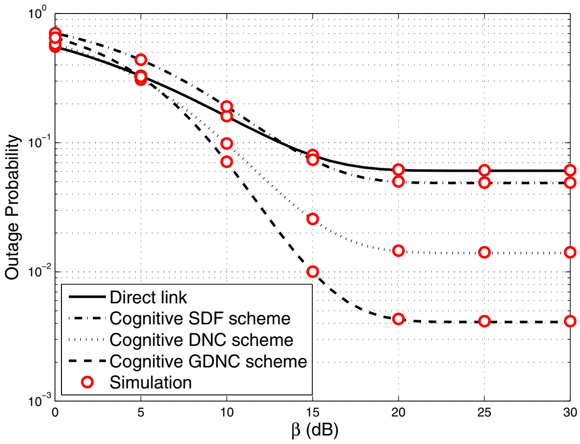

Fig. 5. Outage probability as a function of the primary interference threshold β with a maximum transmit power limit of $P_{max} = 15$ dB for the SUs.

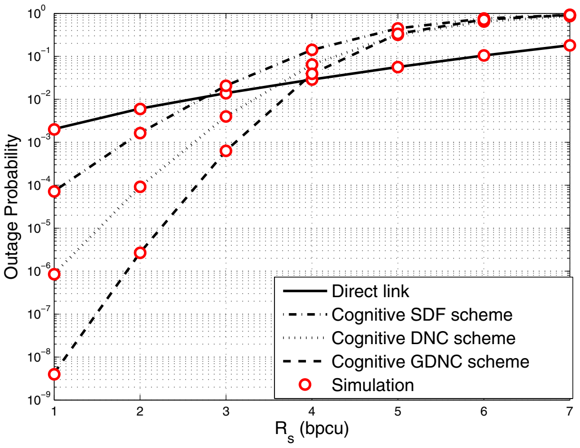

Fig. 6. Outage probability as a function of the secondary attempted rate R_s in bits per channel use (bpcu).

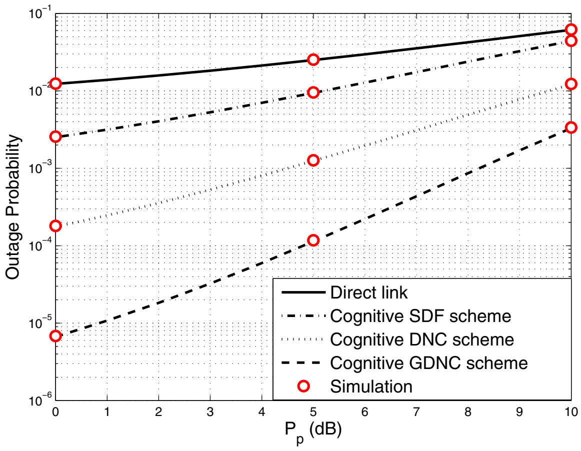

Fig. 7. Outage probability as a function of the primary transmit power P_p.

$\mathcal{P}_{out}^{GDNC} = 4 \cdot 10^{-9}$ while the outage probability for the direct transmission is as large as $\mathcal{P}_{out}^{1s} = 2 \cdot 10^{-3}$. For $R_s > 4$ bpcu, the outage probability of the direct transmission presents the best performance among all the schemes, however, the outage probability at such region is very large.

Fig. 7 presents the outage probability versus the primary transmit power P_p, for $\beta = 10$ dB, $R_s = 1$ bpcu and $P_{max} \to \infty$ dB. It can be seen that the C-GDNC scheme outperforms the others schemes, such advantage decreases with the increment of P_p, even with high interference from the primary.

5. CONCLUSION

We propose the use of network coding in cognitive underlay networks with limits of maximum transmit power and primary interference threshold. We compare the recently proposed DNC and GDNC schemes in cognitive scenarios with the direct transmission and traditional cooperative protocols. It is shown that the cognitive GDNC scheme is capable of outperforming the other schemes in terms of outage probability and achieve the highest diversity order.

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