DOA ESTIMATION AND SIGNAL SEPARATION USING ANTENNA WITH TIME VARYING RESPONSE

Tsvi G. Dvorkind, Eran Greenberg

RAFAEL Advanced Defense Systems LTD
P.O.B. 2250, Haifa, Israel

ABSTRACT

In this paper we suggest a new algorithm for Direction of Arrival (DOA) estimation and signal separation using a novel antenna element with a time variant radiation pattern. With the suggested approach, signals arriving from various spatial directions are acquired in this sensor with different time varying signatures, due to the antenna's continuously changing radiation pattern. We show that if the radiation pattern is varied in a periodical manner and sufficiently fast compared to the bandwidth of the received signals, then multiple sources of radiation can be detected and their direction of arrival estimated. The suggested approach is a novel alternative to array based signal processing, as it allows to perform spatial processing tasks without exploiting multiple sensing elements.

***Index Terms*—** DOA, MUSIC, reconfigurable antenna

1. INTRODUCTION

Detecting and separating signals received from multiple directions usually relies on array based signal processing. The well known phased-array approach allows rapid and accurate beam scanning, simply by electronic adjustment of the phases of the individual elements [1]. An array of sensors is also useful for signal direction of arrival (DOA) estimation. For example, subspace approaches such as MUSIC [2] and ESPRIT algorithms [3] are widely used for estimating DOA of narrow-band signals. Despite the advantages of array-based techniques, they have their limitations in terms of degrees of freedom, as the number of different spatial sources we are able to detect is upper bounded by the number of sensors we utilize within the array.

In this paper we examine the possibility to perform DOA estimation and source separation by continuously altering the spatial response of a **single** sensor. In this way, we obtain spatial diversity over time, rather than by using a set of different elements, as is common with array based signal processing. The goal is to increase functionality in terms of number of sources that we can estimate and differentiate between, and to reduce the front-end size by applying fewer sensing elements.

The advantage of the suggested approach stems from the continuity of the time axis, which allows obtaining a continuum of spatial configurations, as opposed to the finite dimensionality achieved with a limited number of elements in array based approaches. Our main discussion concerns an RF-based sensor (*i.e.,* antenna). Yet, the suggested concept can also be applied in additional contexts. For example, one may continuously alter the spatial response of a microphone in order to create different time-varying responses for acoustic signals arriving from distinctive directions.

Traditionally, spatial variation of the antenna radiation pattern can be achieved by simple mechanical scanning, *i.e.,* by moving the feeder, the reflector, or the entire antenna assembly [4]. Typical scanning rates in a mechanically rotating system is no larger than a few tens of Hertz. The shortcomings of a slowly rotating mechanical system are obvious, in terms of maintenance and scanning rate. In contrast, as we will show later on, we will usually require variations in the rates of tens of kHz at least, which can only be accomplished by an electrical means.

A more general approach to achieve radiation pattern variability is with reconfigurable antennas, where a single element characteristic can be altered through electrical, mechanical, or other means [5]. By designing the current distribution on the antenna structure, one can directly control its spatial radiation pattern. A discussion on the hardware aspects of the electrically reconfigurable element which we propose, is postponed to a future paper. In this work we explore options for altering the antenna radiation pattern, focusing on signal processing approaches for performing DOA and signal separation with such an element.

In a previous contribution [6] we examined the use of a time-variable sensor in order to perform channel sounding. In that work, continuous-wave (CW) signals were transmitted, and the DOA, as well as the temporal impulse response of the channel, were estimated using a variable length dipole antenna. In that contribution, the transmitted signal was known and constrained to be narrow-band, which simplified the estimation process.

In this contribution we exploit the time-varying sensor to resolve a different problem. Assuming that many, possibly wide-band, signals are received at our time-variant sensor, we estimate the DOA's, and decompose the received signal mixture into its individual components using the single sensing element. Unlike to the problem treated in [6], we have no control of the received signals, but we assume that an upper bound on the bandwidth of any signal is known a-priori. We show that if the reception pattern of the antenna is altered periodically and sufficiently fast, we can separate the mixture and estimate the DOA of each signal.

The outline of this paper is as follows. In Section 2 we state our problem, motivating the use of a time varying antenna for spatial signal processing. In Section 3 we suggest to alter the spatial response of our sensor in a periodic manner, and show that by this approach the spatial processing task takes on a simple mathematical form. In Section 4 we present a simulation of a dipole antenna with a periodically time varying response, showing that DOA estimation and mixture decomposition can be accomplished with such an element. Section 5 discusses future work direction and concluding remarks follow in Section 6.

2. MOTIVATION

Let $r(\phi_l, \theta_l)$ be the (possibly complex) reception gain of a sensor due to stimulus arriving from azimuth angle ϕ_l and elevation angle θ_l. The sensor response usually depends on a variety of parameters. For example, for a dipole antenna the response $r(\phi, \theta)$ depends on the ratio between dipole length and wavelength of the RF signal, and on the dielectric properties of the antenna.

Let $s_l(t)$ be the lth signal arriving to the sensor from the lth spatial direction, such that the received waveform at the sensor is of the form $y(t) = \sum_{l=1}^{L} r(\phi_l, \theta_l)s_l(t)$. The L arriving signals can describe the multipath effect in a channel sounding problem [6]. In other applications, $\{s_l(t)\}_{l=1}^{L}$ can represent waveforms due to distinct transmitters.

Our goal is to estimate the signals $\{s_l(t)\}_{l=1}^{L}$ and their spatial arrival angles $\{\phi_l, \theta_l\}$. In our treatment, we assume that the angles are constant during the observation interval.

Naturally, by receiving $y(t) = \sum_{l=1}^{L} r(\phi_l, \theta_l)s_l(t)$ which is the sum of all spatial paths, there is no way to decompose this mixture to its individual components without some additional prior knowledge. In order to resolve this, as in [6], we suggest to add time dependence to the spatial response $r(\phi_l, \theta_l)$, by continuously altering the spatial behavior of the sensor.

Once the spatial response becomes time dependent, the received signal model takes on the form

$$y(t) = \sum_{l=1}^{L} r(t, \phi_l, \theta_l)s_l(t) = \sum_{l=1}^{L} r_l(t)s_l(t), \tag{1}$$

where we define $r_l(t) \triangleq r(t, \phi_l, \theta_l)$ for brevity. If we have prior knowledge of the time variable response of the sensor for each possible direction, *i.e.,* we have a database of all possible 'spatial signatures' $\{r_l(t)\}$, then it may become possible to decompose the mixture (1) into its individual components $\{s_l(t)\}_{l=1}^{L}$, while estimating the arrival angles $\{\phi_l, \theta_l\}$ during the process.

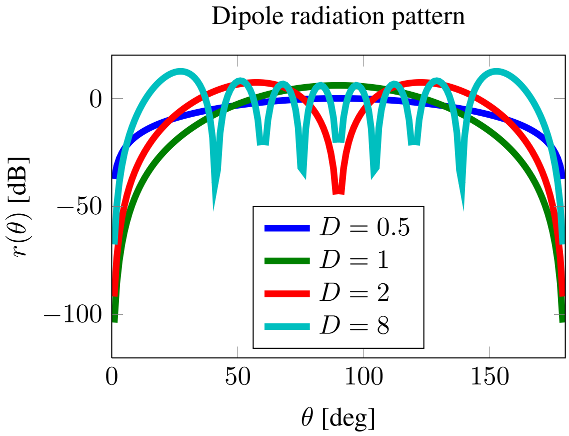

Fig. 1. Dipole antenna response for several choices of element length D.

3. DOA AND SIGNAL SEPARATION USING PERIODICITY

In this section we show that by altering the radiation pattern of the antenna in a periodic manner, then under mild conditions on the signals and simple pre-processing steps, it becomes possible to transform the mixture (1) into a linear measurement model with a time invariant mixing matrix.

Assume that the antenna variable response is periodically altered, with time period T. Hence, for any direction, the spatial signatures are also periodic, *i.e.,* for any lth DOA, and $\forall n \in \mathbb{Z}$,

$$r_l(t) = r_l(t - nT).$$

As an example, consider the radiation intensity of a thin dipole antenna, which is given by [1]

$$\mu \cdot \left(\frac{\cos(\pi D \cos\theta) - \cos(\pi D)}{\sin(\theta)} \right)^2. \tag{2}$$

Here μ, θ and D are, respectively, a current dependent constant, elevation angle, and the length of the antenna normalized by wavelength.

Assuming for simplicity that $\mu = 1$, we plot in Fig. 1 the dipole spatial response for all elevation angles, and for several choices of the element length D.

In order to obtain the spatial signatures $r_l = r(t, \theta_l)$ we alter the length of the sensor. Note that since the dipole antenna pattern is invariant in the azimuth direction, we discuss here spatial estimation in terms of elevation angle only. We assume that a q bit decoder will be utilized, which

$r(t, \theta)$ for varying D

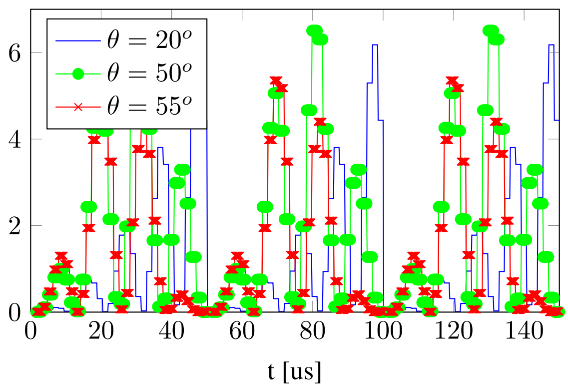

Fig. 2. Spatial signature obtained for signals arriving from three different angles. The length of the dipole element is time varying according to (3).

will periodically switch the length of the dipole between $N = 2^q$ different lengths. Accordingly, the length D of the sensor becomes time dependent, and takes on the form:

$$D(t) = \sum_{m} \sum_{n=1}^{N} nD_0 \cdot \text{rect}\left(t - \frac{n-1}{N}T - mT\right), \quad (3)$$

where D_0 is the length increment and rect(t) is the support function over $[0, T/N]$. Using (2), we have that for a wavefront arriving from angle θ_l, its spatial signature becomes

$$r_l(t) = \left(\frac{\cos(\pi D(t) \cos\theta_l) - \cos(\pi D(t))}{\sin(\theta_l)}\right)^2, \quad (4)$$

with $D(t)$ of (3). In Fig. 2 we show three such spatial signatures which are obtained for three signals arriving from $\theta_1 = 20°, \theta_2 = 50°$ and $\theta_3 = 55°$, where we used $N = 2^5, D_0 = 5/32, T = 1/20_{kHz}$.

3.1. From periodicity to standard form

Exploiting the periodicity of the $\{r_l(t)\}_l$ functions, we can express them using their Fourier series expansion, yielding that the received signal $y(t)$ of (1) is

$$y(t) = \sum_{l=1}^{L} \sum_{k} v_l[k] \exp(j2\pi f_p kt) s_l(t), \quad (5)$$

where $f_p = 1/T$ is the rate in which we alter the radiation pattern and $v_l[k]$ is the kth Fourier coefficient of the periodic function $r_l(t)$. We note that for T-periodic and piece-wise constant $r_l(t)$ of the form $r_l(t) = \sum_m \sum_{n=0}^{N-1} a_n \text{rect}(t - nT/N - mT)$, it is a manner of straightforward calculation to show that the kth Fourier coefficient of $r_l(t)$ is proportional to sinc$(k/N)A^d[k]$, where A^d is the N-length DFT of $\{a_n\}$. Taking a Fourier transform on both sides of (5) we thus have:

$$Y^F(f) = \sum_{l=1}^{L} \sum_{k} v_l[k] S_l^F(f - kf_p),$$

implying that in the kth frequency band we have a weighted sum of all impinging signals. By choosing scanning period T such that the scanning rate f_p is larger than twice the maximal bandwidth of the received signals, the frequency bands will not overlap. We summerize this in the following proposition:

Proposition 1 *Assuming that for all L signals there is an f_{max} such that $S_l^F(f) = 0$ for all $|f| \geq f_{max}$, and assuming that the sensor's spatial pattern is periodically varied with period T satisfying $1/T = f_p > 2f_{max}$, then for any $k_0 \in \mathbb{Z}$ and $f \in \left[\left(k_0 - \frac{1}{2}\right) f_p, \left(k_0 + \frac{1}{2}\right) f_p\right]$ we have*

$$Y^F(f) = \sum_{l=1}^{L} v_l[k_0] S_l^F(f - k_0 f_p).$$

Note that Prop. 1 resembles the Nyquist sampling theorem, with the scanning interval, T, replacing the time-domain sampling period. Under the conditions of Prop. 1, it is possible to transform the measured signal $y(t)$ of (1) into a standard matrix form, which is usually obtained only with an array of sensing elements, but here it is established with a single, time-varying, sensor. Specifically, by demodulating the kth band to baseband and low pass filtering, the resulting signal

$$y_k(t) = h_{LP}(t) * (y(t) \exp(-j2\pi f_p kt))$$

satisfies

$$y_k(t) = \sum_{l=1}^{L} v_l[k] s_l(t). \quad (6)$$

Repeating the process for K different bands, then with $K \geq L$ should produce enough equations to resolve the mixture. A schematic of the suggested detection circuit is presented in Fig. 3. Rewriting (6) in a matrix form for K different bands, we obtain

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) \quad (7)$$

where $\mathbf{y}(t) = [y_{k_0}(t), \ldots, y_{k_K}(t)]^T$ are the signals from the selected K bands, $\mathbf{s}(t) = [s_1(t), \ldots, s_L(t)]^T$ is the vector obtained by the impinging signals, and $\mathbf{A} \in \mathbb{C}^{K \times L}$ is the mixing matrix, with elements $A[k, l] = v_l[k]$.

As the directions of the impinging signals are not known in advance, there are several possible approaches to estimate the DOAs and the signals $\mathbf{s}(t)$.

- We may construct a dictionary matrix $\mathbf{D} \in \mathbb{C}^{N \times \widetilde{K}}$, with $\widetilde{K} \gg K$, that describes many possible directions of arrival on a very fine grid. Thus, the matrix $\mathbf{A}$ of the valid DOAs consists of a subset of columns of $\mathbf{D}$. In this case,

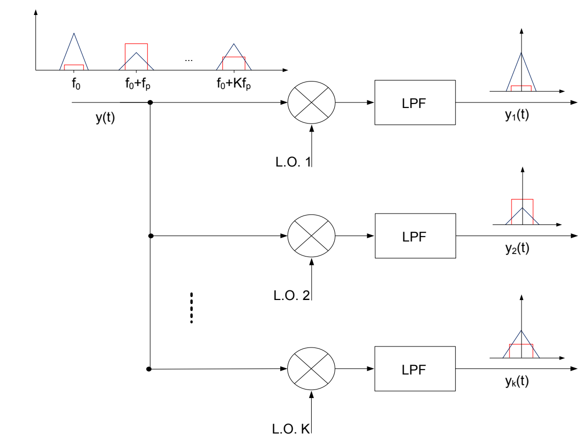

Fig. 3. Schematics of demodulating the different bands to baseband, resulting in $\{y_k(t)\}$ of (6).

the model $\mathbf{y}(t) = \mathbf{D}\mathbf{s}(t)$ has a sparse support solution and it is possible to reconstruct the signals, and their support (which indicates the DOAs) using sparsity based approaches. Furthermore, as a sparse support pattern is valid for all time instances, we can apply a multiple measurement vector setting, as in [7].

- Another route to tackle our problem will be to view (7) as a blind source separation problem (BSS), while treating $\mathbf{A}$ as unknown mixing matrix. If we assume, for example, that the impinging signals $\{s_k(t)\}$ are statistically independent and non-Gaussian, then we can rely on independent component analysis (ICA) techniques to estimate the mixing matrix $\mathbf{A}$ and the signals [8], or one of the many other methods to resolve BSS problems.
- Assuming that the signals $\{s_k(t)\}$ and the additive noise are uncorrelated, we can apply the MUSIC technique [2] for estimating the columns of $\mathbf{A}$. Then, we can reconstruct the signals as well, using a simple least squares (LS) fit.

In this contribution we examine the third option.

4. SIMULATION

We simulate a dipole antenna sensor, with a periodically altered length, as in (3). In practice, this can be accomplished by electrical means, and we will discuss the implementation in a future publication. In this example, by switching between $N = 32$ different states, every $T = 1/20_{kHz}$ seconds, and using antenna length increments of $D_0 = 5/32$ we obtain a variable and periodic spatial response, with maximal element length of 5 wavelengths. As stated in Prop. 1, for this spatial scanning rate signals with bandwidth $f_{max} < \frac{1}{2T} = 10_{kHz}$ will not be aliased. Assuming that the carrier frequency is sufficiently high, we ignore the dipole frequency dependency along the 10_{kHz} bandwidth of the signals. We simulate

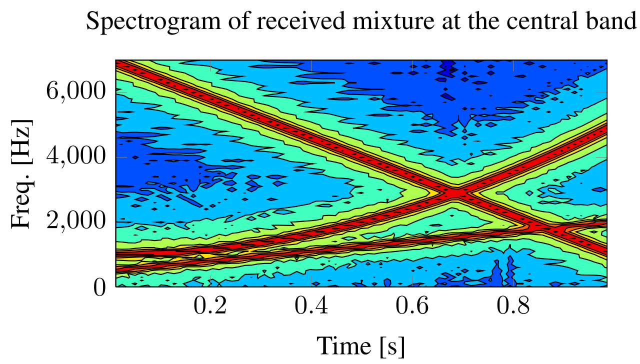

Fig. 4. Received signal mixture is comprised of three chirps.

$L = 3$ chirp signals arriving from three different directions to the dipole antenna. The first, a chirp starting from frequency of 0.5_{kHz} and ending at 2_{kHz} arrives to the sensor from angle of $\theta = 20°$. The second signal is a chirp from 7_{kHz} to 1_{kHz} arriving from $\theta = 50°$, and the third being a quadratic swept-frequency signal, starting from 1_{kHz} to 5_{kHz} and arriving from $\theta = 55°$.

By preprocessing the received signal at the antenna as in Fig. 3, the model (7) results. In Fig. 4 we present a spectrogram of the received signal mixture, as obtained from the central frequency band.

To decompose our mixture and estimate the DOA we use bands $k = \{-10, \dots, 10\}$ around the central frequency. Thus, we end up with $K = 21$ measurements, resulting in an $\mathbf{A} \in \mathbb{C}^{21\times 3}$ size matrix for the model (7). Adding white Gaussian noise to the measurements we then apply the MUSIC algorithm [2] for DOA estimation, at various SNR conditions. For each SNR value we average over 100 Monte-Carlo trials the obtained MUSIC pseudo spectrum. The latter was calculated as $P_{MUSIC}(\theta_i) = 10\log_{10}\|P_n\mathbf{a}_i\|^{-2}$, where $\mathbf{a}_i$ is the expected ith unit length response due to a signal arriving from the ith angle, and P_n is the noise subspace projection matrix. The vectors $\{\mathbf{a}_i\}$ replace the notion of the steering vectors in array-based signal processing, and were constructed on a $0.5°$ grid of possible arrival angles. As a conservative approach, we assume that the maximal number of impinging signals is twice their actual amount, such that the noise subspace was constructed from $21 - 2\times 3 = 15$ eigenvectors. In Fig. 5 we plot the resulting pseudo-spectrum. As can be seen, though at SNR of -10_{dB} it is hard to distinguish between the signals arriving from $\theta = 50°$ and $\theta = 55°$, this becomes possible at larger values of SNR.

After obtaining the true arrival angles, we have an estimate of the mixing matrix $\mathbf{A}$ of (7). Let $\mathbf{x}(t) = \mathbf{y}(t) + \mathbf{n}(t)$ be the noisy measurements. Then, we obtain an estimate of the impinging signals with a simple LS fit of the form $\hat{\mathbf{s}}(t) = \left(\mathbf{A}^H\mathbf{A}\right)^{\dagger}\mathbf{A}^H\mathbf{x}(t)$, where $()^H$ and $()^{\dagger}$ indicate Hermitian transpose and the pseudo-inverse operator, respectively. In Fig. 6 we plot the spectrograms of the signals separated from

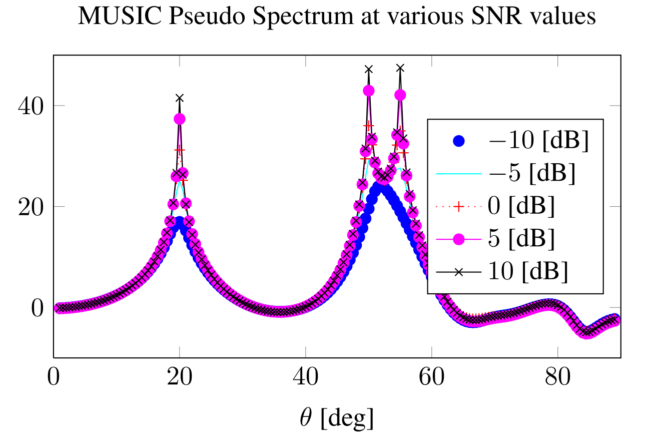

Fig. 5. MUSIC pseudo spectrum at various SNR conditions.

the mixture, as detected for an SNR of 10_{dB}.

5. FUTURE WORK

In the future we intend to investigate (and implement) other types of antennas that will allow us to achieve a time-variant radiation pattern. The first candidate is likely to be the leaky-wave antenna [9], a waveguide structure that radiates energy to free space by leaking. Such a structure includes continuous, discontinuous or periodic aperture that allows radiation. In contrast to array antennas, leaky-wave antennas do not require any complex feeding network; they are fed by a simple transmission line or waveguide connection, while offering directivity and scanning performances sometimes comparable to those of arrays.

6. CONCLUSION

In this work we suggest to periodically alter the spatial response of a single sensor (antenna) along time. In this form, we obtain spatial variability along the time domain, as opposed to classical array based signal processing techniques, for which spatial variability is obtained along the different elements of the array. The proposed approach relies on periodicity of the radiation pattern and was applied in the context of DOA estimation and signal unmixing. This suggests that spatial processing tasks that were traditionally implemented only with an array of elements, can now be accomplished using a single time-variant sensor.

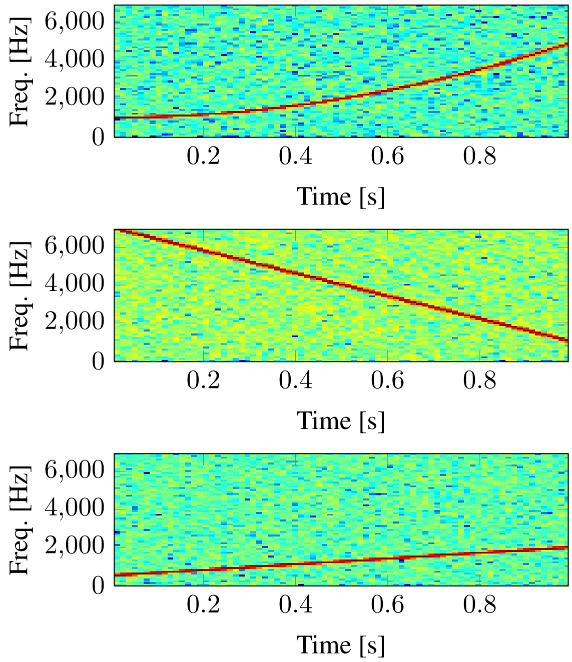

Fig. 6. Spectrograms of the retrieved 3 chirp signals, as obtained after estimating the DOA, and applying LS fit to the measurements.

REFERENCES

[1] C. A. Balanis, *Antenna Theory Analysis and Design*, John Wiley and Sons, Inc., second edition, 1997.

[2] R. Schmidt, *A signal subspace approach to multiple emitter location and spectral estimation*, Ph.D. dissertation, Stanford Univ., 1981.

[3] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” *IEEE. Tran. acous. speech sig. pro.*, vol. 37, no. 7, pp. 984–995, Jul. 1989.

[4] W. M. Cady, M. B. Karelitz, and L. A. Turner, *Radar Scanner and Radoms*, McGraw-Hill Book Company, New York, 1948.

[5] J. T. Bernhard, *Reconfigurable Antennas*, Morgan & Claypool, 2007.

[6] T. G. Dvorkind and E. Greenberg, “Spatial processing with a single antenna,” *IEEE 27-th Convention of Electrical and Electronics Engineers in Israel (IEEEI)*, Nov. 2012.

[7] S. F. Cotter, B. D. Rao, K. Engan, and K. Kreutz-Delgado, “Sparse solutions to linear inverse problems with multiple measurement vectors,” *IEEE Tran. Sig. Proc.*, vol. 53, no. 7, pp. 2477–2488, Jul. 2005.

[8] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso, and E. Moulines, “A blind source separation technique using second-order statistics,” *IEEE Tran. Sig. Proc.*, vol. 45, no. 2, pp. 434–444, 1997.

[9] F. B. Gross, *Frontiers in Antennas: Next Generation Design & Engineering*, McGraw-Hill, 2011.