# ANALOG JOINT SOURCE-CHANNEL CODING IN MIMO RAYLEIGH FADING CHANNELS

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**ABSTRACT**

In this paper we study the analog transmission of discrete-time continuous-amplitude sources over MIMO Rayleigh fading channels at extremely high data rates. Linear and Decision Feedback MMSE MIMO receivers are used to transform the MIMO channel into several SISO channels. We assume the presence of a limited feedback channel that allows continuous adaptation of the encoder parameters to the time-varying channel SNR. Experimental results show that the decision feedback MIMO receiver performs at 2 dB from the theoretical bound when transmitting Gaussian sources.

## 1. INTRODUCTION

It is well known that the use of separate source and channel coders is optimal for digital communications [1] in many channel models such as the Additive White Gaussian Noise (AWGN) channel. The complexity of those systems, however, can be very high when they are designed to perform close to the Shannon limit. Also, they add significant delays motivated by the long block lengths required to approach the theoretical limits. Moreover, full redesign of the digital system is necessary whenever we want to change the code rate or the distortion target.

It is also well known that analog communications are optimal under some circumstances. For instance, direct transmission of uncoded Gaussian samples over AWGN channels is optimal because Gaussian sources are perfectly matched to Gaussian channels. Thus, it is very interesting to consider analog transmission of discrete-time continuous-amplitude sources when possible. This fact has recently motivated several work [2–5] on finding analog transformations aimed at matching sources with channels. These schemes can perform analog compression at the symbol level and thus no delays are introduced. They also present a very low complexity, making them very attractive for the effective transmission of analog sources such as images and sound.

In the literature, most work on analog joint source channel coding focuses on AWGN channels. An exception is [6], which considers a two-user single-antenna scenario under a flat fading Rayleigh channel. Another exception is [7] where the implementation on a Software Defined Radio testbed of a wireless system based on joint analog source channel coding is presented. Excellent performance over wireless channels is attained when the encoder parameters are continuously adapted to the time-varying Channel Signal to Noise Ratio (CSNR).

In this work, we study the feasibility of analog communications over Multiple Input Multiple Output (MIMO) Rayleigh fading channels. Optimum Maximum Likelihood (ML) decoding of the vector observations is difficult due to the non-linear characteristic of the analog encoding procedure. We circumvent this drawback by considering suboptimal linear and decision feedback receivers that transform the MIMO channel into several Single Input Single Output (SISO) channels. Feeding back the CSNR information of the equivalent SISO channels allows us to adapt the encoder parameters to the channel time-variations and attain a performance close to the theoretical bounds.

The rest of this paper is organized as follows. Section 2 describes the basic principles of analog joint source channel coding and its optimization over SISO channels. Section 3 focuses on analog joint source channel coding over Rayleigh fading MIMO channels. Section 4 presents performance results for different MIMO channels and source distributions while Section 5 is devoted to the conclusions.

## 2. ANALOG JOINT SOURCE-CHANNEL CODING IN SISO CHANNELS

Let us consider the analog transmission of discrete-time continuous-amplitude sources over wireless channels. In this section we focus on SISO Rayleigh fading channels while the MIMO case will be considered in the ensuing section.

Figure 1 shows the block diagram of an $N$:1 analog joint

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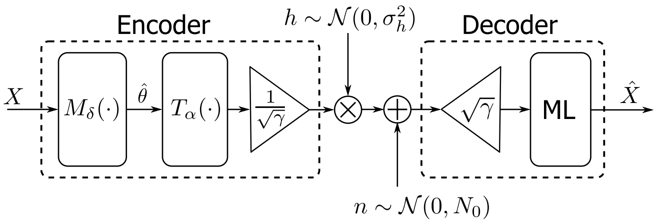


**Fig. 1**. System model for the $N$:1 bandwidth compression scheme under AWGN channels.

source-channel coding system over a Rayleigh fading channel where $N$ independent and identically-distributed (i.i.d.) source symbols are compressed into one channel symbol. Analog source symbols are packed into the source vectors $X = \{x_i\}_{i=1}^N$. The encoding procedure has two steps: the compression function $M_\delta(\cdot)$ and the matching function $T_\alpha(\cdot)$. For the compression function $M_\delta(\cdot)$, a particular type of parameterized space-filling continuous curves, called spiral-like curves, are used to encode the samples. These curves were proposed for the transmission of Gaussian sources over AWGN channels by Chung and Ramstad [2–4]. For the case of 2:1 compression (i.e. $N = 2$), they are formally defined as

$$\begin{cases} x_1 = \text{sign}(\theta)\dfrac{\delta}{\pi}\theta\sin\theta \\ x_2 = \dfrac{\delta}{\pi}\theta\cos\theta \qquad\qquad \text{for } \theta \in \Re, \end{cases} \tag{1}$$

where $\delta$ is the distance between two neighboring spiral arms, and $\theta$ is the angle from the origin to the point $X = (x_1, x_2)$ on the curve. Therefore, each pair of analog samples, $x_1$ and $x_2$, represent a specific point in $\Re^2$ that is matched to the closest point on the spiral. The angle from the origin to that point on the spiral, $\hat{\theta}$, will be the channel symbol for $x_1$ and $x_2$, i.e.

$$\begin{aligned} \hat{\theta} &= M_\delta(X) \\ &= \arg\min_\theta \left\{ \left(x_1 \pm \frac{\delta}{\pi}\theta\sin\theta\right)^2 + \left(x_2 - \frac{\delta}{\pi}\theta\cos\theta\right)^2 \right\}. \end{aligned} \tag{2}$$

It is possible to achieve higher compression rates (i.e. $N > 2$) by extending (1) to generate more complex curves [8, 9].

Next, we use the invertible function $T_\alpha(\cdot)$ to transform the compressed samples. In [2, 4, 10], the invertible function $T_\alpha(\hat{\theta}) = \hat{\theta}^\alpha$, with $\alpha = 2$ was proposed. However, as shown in [5], the system performance can be improved if $\alpha$ is optimized together with $\theta$. We have determined that using $\alpha = 1.3$ provides a good overall performance, whereas using different values of $\alpha$ for each CSNR does not improve the SDR significantly. The channel symbol, $s$, is thus constructed as $s = T_\alpha(\hat{\theta})/\sqrt{\gamma}$ where $\sqrt{\gamma}$ is a normalization factor such that the average transmitted power is equal to one. The received observation, $y$, can be expressed as $y = hs + n$ where $h$ is the Rayleigh fading channel response and $n$ is the noise at reception. Both are modeled as complex-valued zero-mean circularly-symmetric Gaussian random variables. The Channel Signal to Noise Ratio (CSNR) is $|h|^2/N_0$ and changes with each channel realization. Normalizing the fading channel, $\sigma_h^2 = E[|h|^2] = 1$, yields an average CSNR equal to $1/N_0$.

At the receiver, Maximum Likelihood (ML) decoding is used to obtain an estimation of the source symbols. Given an observation, $y$, the ML estimate is obtained as the tuple $\hat{X} = (\hat{x}_1, \hat{x}_2)$ belonging to the curve and satisfying

$$\begin{aligned} \hat{X} &= \arg\max_{X \in \text{curve}} \; p(y|X) \\ &= \{X | X \in \text{curve and } T_\alpha(M_\delta(X)) = y\}. \end{aligned} \tag{3}$$

Thus, ML decoding is equivalent to first applying the inverse function $T_\alpha^{-1}$ to the observation, $y$, i.e.

$$\hat{\theta}' = T_\alpha^{-1}(y) = \text{sign}(y)|y|^{-\alpha} \tag{4}$$

and then mapping $\hat{\theta}'$ to $\hat{X} = (\hat{x}_1, \hat{x}_2)$ according to the employed curve function.

System performance is measured in terms of the Signal to Distortion Ratio (SDR) with respect to the CSNR. The distortion is the Mean Squared Error (MSE) between the decoded and source analog symbols, i.e. MSE $= \mathbb{E}\{\|X - \hat{X}\|^2\}/N$. Thus, assuming that the source signal has unit power, the SDR is calculated as SDR $= 10\log(1/\text{MSE})$.

Since our goal is the minimization of the SDR, the bi-dimensional space has to be filled by the spiral in the best possible way for every CSNR value. By changing the value of the curve parameter $\delta$ according to the CSNR, it is possible to optimize this matching and to improve the system performance. When considering ML decoding, high CSNR and $\alpha = 2$, it is possible to obtain an analytic expression for the optimal value of $\delta$ [10]. When $\alpha \neq 2$, however, analytical optimization of $\delta$ is not feasible. Instead, $\delta$ can be numerically optimized by computing the SDR for each CSNR over a wide range of values for $\delta$. Table 1 shows the best values of $\delta$ that were found via computer simulations for different values of the CSNR for a 2:1 compression ratio.

The proposed analog system can be readily modified to consider more general compression rates $N$:$K$, by simply transmitting some of the samples uncoded and the rest coded with $N$:1 compression as explained in [5].

## 3. ANALOG JOINT SOURCE-CHANNEL CODING IN MIMO CHANNELS

In this section we focus on the analog transmission of discrete-time continuous-amplitude sources over MIMO wireless channels. We assume the source symbols are spatially multiplexed over $n_T$ transmit antennas. At each transmit antenna, a vector $X_i$ of $N$ analog source symbols is encoded into the channel symbol $s_i$, $i = 1, \ldots, n_T$ using an

**Table 1**. Optimal values for $\delta$.

| CSNR (dB) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta$ | 9.8 | 8.0 | 5.6 | 5.0 | 4.2 | 4.0 | 3.9 | 3.7 | 3.6 | 3.4 | 3.2 | 3.1 | 3.0 | 2.9 | 2.7 | 2.5 | 2.3 | 2.2 | 2.1 | 2.0 |
| CSNR (dB) | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| $\delta$ | 1.8 | 1.7 | 1.5 | 1.4 | 1.3 | 1.2 | 1.1 | 1.0 | 0.9 | 0.8 | 0.8 | 0.8 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.5 | 0.4 | 0.4 |

$N$:1 analog encoder such as the one described in the previous section. Channel symbols are transmitted over a frequency flat MIMO channel with $n_R \geq n_T$ antennas. The observed symbols at the MIMO channel output are given by

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n}$$

where $\mathbf{s}$, $\mathbf{y}$ and $\mathbf{n}$ are the vectors that represent the channel symbols, the received symbols and the spatially white additive Gaussian noise, respectively. $\mathbf{H}$ is the $n_R \times n_T$ MIMO channel matrix. We assume a spatially white Rayleigh fading MIMO channel whose coefficients $h_{ij}$ are complex-valued zero-mean circularly-symmetric Gaussian i.i.d. random variables. Channel symbols are normalized so that the radiated power at each antenna is $1/n_T$ (i.e. total radiated power is one). This way, the MIMO CSNR is

$$\text{CSNR}(\mathbf{H}) = \frac{Tr\{\mathbf{HH}^H\}}{n_T n_R N_0}$$

where $Tr\{\cdot\}$ denotes the trace operator and the superindex $^H$ represents conjugate transposition. If MIMO channels are normalized so that $E_{\mathbf{H}}\left[Tr\{\mathbf{HH}^H\}\right] = n_T n_R$, the average CSNR is $1/N_0$

The optimum MIMO receiver calculates the Maximum Likelihood (ML) estimate of the source symbols $\mathcal{X} = \{X_i\}_{i=1}^{n_T}$ from the observations, i.e.

$$\hat{\mathcal{X}} = \arg\max_{\mathcal{X}} \; p(\mathbf{y}|\mathcal{X}) \tag{5}$$

where $p(\mathbf{y}|\mathcal{X})$ is the likelihood of $\mathcal{X}$ with respect to $\mathbf{y}$. However, given the non-linear characteristic of the analog channel encoder, an analytical solution to (5) is extremely difficult. Instead, in this work we focus on two suboptimal MIMO receivers that equalize the MIMO channel: Minimum Mean Squared Error (MMSE) linear receiver and MMSE Decision Feedback (DF) with ordering non-linear receiver.

Figure 2 shows the block diagram of an analog MIMO transmission system with an MMSE linear receiver. The MMSE filter that minimizes the Mean Squared Error between the channel symbol vector $\mathbf{s}$ and the estimated symbol vector $\hat{\mathbf{s}} = \mathbf{Wy}$ is given by

$$\mathbf{W}_{\text{MMSE}} = \left(\mathbf{H}^H\mathbf{H} + n_T N_0 \mathbf{I}_{n_T}\right)^{-1}\mathbf{H}^H \tag{6}$$

Notice that $\mathbf{W}_{\text{MMSE}}$ does not completely cancel the spatial interference, i.e. at $\hat{s}_i$ the desired symbol $s_i$ is corrupted by a residual spatial interference that adds to the Gaussian noise.

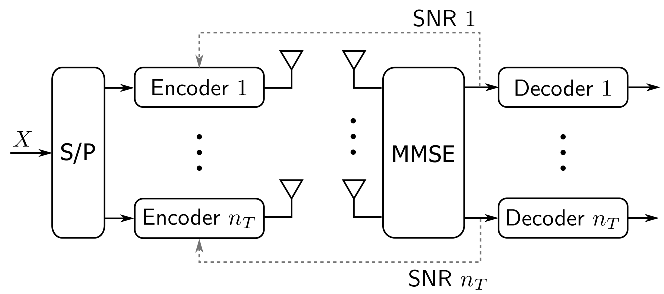


**Fig. 2**. Analog MIMO system with MMSE detection.

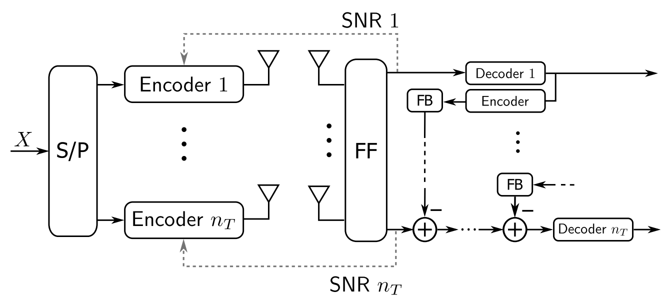


**Fig. 3**. Analog MIMO system with DF detection.

It is shown in [11] that the equivalent CSNR at each output of the MMSE linear receiver can be expressed as

$$\text{CSNR}_i = \frac{\mu_i^2}{\mu_i - \mu_i^2} = \frac{\mu_i}{1-\mu_i}, \;\; i = 1, \ldots, n_T \tag{7}$$

where $\mu_i = (\mathbf{W}_{\text{MMSE}}\mathbf{H})_{ii}$. It is assumed that our system setup provides a feedback channel that enables us to send these $\text{CSNR}_i$ values to the transmitter (see Figure 2). This way we can implement an adaptive coded scheme where the $\delta$ parameter of each analog joint source-channel encoder is continuously updated following Table 1.

Figure 3 plots the block diagram of an analog MIMO transmission system with a DF receiver. Both the Feed Forward (FF) and the Feed Backward (FB) filters are optimized according to the MMSE criterion. The FF filter is obtained from the Cholesky factorization of

$$\mathbf{H}^H\mathbf{H} + n_T N_0 \mathbf{I}_{n_T} = \mathbf{L}^H \boldsymbol{\Delta} \mathbf{L}$$

where $\mathbf{L}$ is an $n_T \times n_T$ lower triangular matrix and $\boldsymbol{\Delta}$ is a $n_T \times n_T$ diagonal matrix. If we define the whitening filter as $\mathbf{B}^H = \boldsymbol{\Delta}^{-1}\mathbf{L}^{-H}$, the FF filter is the product of the matched and whitening filters, i.e. $\mathbf{W}_{\text{MMSE}}^{\text{DF}} = \mathbf{B}^H\mathbf{H}^H$. The overall response of the FF filter and the channel is

$$\mathbf{W}_{\text{MMSE}}^{\text{DF}}\mathbf{H} = \mathbf{L} - n_T N_0 \mathbf{\Delta}^{-1}\mathbf{L}^{-H} \tag{8}$$

In order to simplify the derivation of the DF receiver, we will assume that there are no decoding errors. Under this assumption, the spatially causal component of the interference in (8) can be successively removed with the FB filter $\mathbf{L} - \mathbf{I}_{n_T}$ without altering the noise statistics at the decoder inputs. An advantage of analog coding is that there is no delay in the encoding and re-encoding steps which significantly simplifies the implementation of DF MIMO receivers.

Similarly to the case of linear receivers, we assume the instantaneous CSNR at the decoder inputs is known at the transmitter thanks to the presence of a limited feedback channel (see Figure 3). This allows the continuous update of the $\delta$ parameter following Table 1. Equation (7) is also valid to calculate the CSNR value with

$$\begin{aligned}\mu_i &= \left(\mathbf{B}^H\mathbf{H}^H\mathbf{H} - \mathbf{L} + \mathbf{I}_{n_T}\right)_{ii} \\ &= \left(\mathbf{I}_{n_T} - n_T N_0 \mathbf{\Delta}^{-1}\mathbf{L}^{-H}\right)_{ii}\end{aligned} \tag{9}$$

Finally, notice that decoding ordering is important in the performance of DF MIMO receivers [12]. Ordering can be interpreted as a permutation of the columns of the MIMO channel matrix, i.e. $\bar{\mathbf{H}} = \mathbf{HP}$ where $\mathbf{P}$ is a permutation matrix. Contrarily to [12], the optimum ordering in our case is the one that minimizes the MMSE at the decoder input, i.e.

$$\begin{aligned}\text{MMSE} &= N_0 Tr\left\{\mathbf{\Delta}^{-1}\left(\mathbf{I}_{n_T} - N_0\mathbf{L}^{-H}\mathbf{L}^{-1}\mathbf{\Delta}^{-1}\right)\right\} \\ &\approx N_0 Tr\left\{\mathbf{\Delta}^{-1}\right\}\end{aligned} \tag{10}$$

where the approximation holds when $N_0 << 1$. Thus, the optimum ordering is

$$\mathbf{P}_{opt} = \arg\min_{\mathbf{P}} \; N_0 Tr\left\{\bar{\mathbf{\Delta}}^{-1}\right\} \tag{11}$$

where $\bar{\mathbf{\Delta}}$ results from the Cholesky factorization of $\bar{\mathbf{H}}^H\bar{\mathbf{H}} + n_T N_0 \mathbf{I}_{n_T}$. This optimization problem can be readily solved by searching over the $n_T!$ possible permutation matrices and selecting the one that minimizes the MMSE cost function (10).

## 4. SIMULATION RESULTS

Computer simulations were carried out to illustrate the performance of the proposed analog MIMO transmission methods. We considered two types of source distributions: Gaussian and Laplacian. These distributions are typically encountered in practical applications such as image transmission or Compressive Sensing. The theoretical bound is the minimum attainable SDR for a given CSNR and is known in the literature as the Optimum Performance Theoretically Attainable (OPTA) [13]. The OPTA is calculated by equating the rate

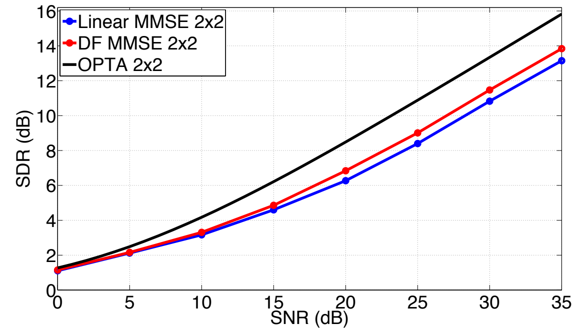


**Fig. 4**. OPTA and SDR in MIMO 2x2 and Gaussian sources.

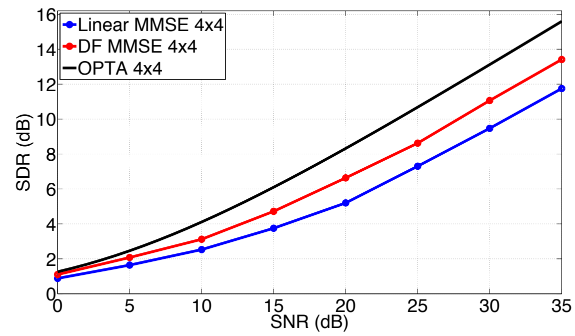


**Fig. 5**. OPTA and SDR in MIMO 4x4 and Gaussian sources.

distortion function, which determines the minimum number of bits that must be employed to encode a source in order to attain a certain distortion, to the channel capacity, that specifies the maximum number of bits that can be transmitted through the channel without errors for a given CSNR. For a generic $N$:$K$ system, this results in

$$\log\left(\frac{C}{\text{MSE}}\right) = \frac{N}{Kn_t}\mathbb{E}_{\mathbf{H}}\left\{\log\det\left(\mathbf{I}_{n_R} + \frac{\text{CSNR}}{n_T}\mathbf{HH}^H\right)\right\} \tag{12}$$

where $\mathbb{E}_{\mathbf{H}}\{\cdot\}$ represents expectation with respect to $\mathbf{H}$ and the constant $C$ in the rate distortion function takes values $C = 1$ for Gaussian sources and $C = e/\pi$ for Laplacian sources.

We consider symmetric MIMO channels with $n_T = n_R = 2$ and $4$ transmit and receive antennas. The source symbols are encoded using the rate 2:1 analog code described in Section 2. Then, the analog coded symbols are normalized and transmitted over a spatially white flat Rayleigh fading MIMO channel. At the receiver, either the linear MMSE or the DF MMSE MIMO receivers described in the previous section are employed. In both cases ML decoding of the analog symbols is used.

Figures 4 and 5 show the achieved SDR for different values of average CSNR for 2 and 4 transmit and receive antennas, respectively. Source symbols are assumed to be Gaussian

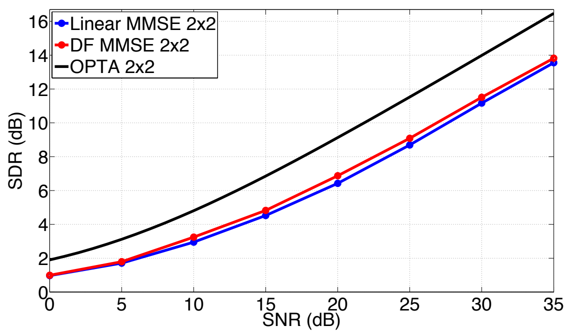


**Fig. 6**. OPTA and SDR in MIMO 2x2 and Laplacian sources.

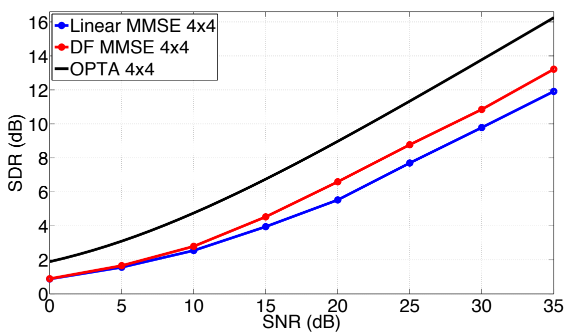


**Fig. 7**. OPTA and SDR in MIMO 4x4 and Laplacian sources.

distributed. The performance of the two considered MIMO receivers, linear MMSE and DF MMSE, are compared to the OPTA. It is apparent from these figures the excellent performance of the proposed analog coding techniques. Indeed, the SDR obtained with the DF MMSE MIMO receiver stays within 2 dB from the OPTA in all cases. The performance of the MIMO linear MMSE receiver is slightly worse than that of DF receivers, although differences increase with the number of antennas. Notice that the superior performance of DF MIMO receivers is due to the non-linear decoding and re-encoding operations carried out during the decision feedback stage.

Figures 6 and 7 show simulation results for the same antenna configurations when the source follows a Laplacian distribution. In this case, the performance of DF MMSE MIMO receivers is not as close to the OPTA as when the source is Gaussian, but stays within 3 dB from the theoretical limits. Again, the performance of linear MMSE receivers is worse than that of DF receivers and the degradation increases with the number of antennas.

## 5. CONCLUSIONS

In this work we have investigated the analog transmission of discrete-time continuous-amplitude sources over MIMO Rayleigh fading channels at extremely high data rates. Source symbols are spatially multiplexed over $n_T$ transmit antennas and two different types of MIMO receivers have been considered: linear MMSE and DF MMSE. Simulation results show the excellent performance of the DF MMSE MIMO receiver which attains a SDR that is only 2 dB under the OPTA bound when the sources are Gaussian and slightly worse for Laplacian sources.

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