

# GENERALIZED FREQUENCY DIVISION MULTIPLEXING IN COGNITIVE RADIO

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## ABSTRACT

Generalized Frequency Division Multiplexing (GFDM) is a recent multi carrier modulation technique that can provide low out-of-band radiation, which makes it an attractive choice for the PHY layer of cognitive radios operating in fragmented TV white spaces. Primary, incumbent signals can now be protected with a geo-location database query mechanism. Even then, sensing opportunistic users (OU) operating in the same frequency band is important in cognitive radio operations. In this paper, a simulation based study has been performed to compare GFDM matched filter based sensing characteristics with traditional orthogonal frequency division multiplexing (OFDM) sensing receiver operating characteristic (ROC) curves. Sensing with a GFDM based cognitive radio receiver shows better sensing performance.

**Index Terms**— Cognitive Radio, Flexible PHY, Opportunistic Access, Spectrum Sensing

## 1. INTRODUCTION

Nowadays, radio spectrum is a very scarce resource, and an increasing popularity of wireless devices is making the demand for it even higher. To cope with this huge demand for spectrum, regulatory bodies (like FCC in USA and Ofcom in UK) have recently opened up licensed spectrum for unlicensed access [1, 2]. Unlicensed access in licensed bands should not create interference to incumbent users, and hence new PHY designs and waveforms are being researched which can fill in the TV white spaces (TVWS) in an opportunistic manner. When incumbent users or opportunistic users are inactive over a portion of the spectra, only then other opportunistic users will be allowed to transmit and receive data. A strict specification for such innovative cognitive radio (CR) PHY design is that the opportunistic signal should have extremely low out-of-band radiation into the adjacent incumbent frequency bands.

In the 2010 FCC and Ofcom ruling, requirements of spectrum sensing for protection of incumbent users were eliminated, and a geo-location database mechanism was proposed [2,3]. Information about incumbent users at a particular

position are stored in these geo-location databases. Opportunistic users communicate in the white spaces only and thus provide necessary protection to incumbent users.

The multiband Generalized Frequency Division Multiplexing is a new idea for designing a multicarrier PHY [4]. GFDM is very well suited for a cognitive radio PHY modulation scheme, as pulse shaping filters make the out-of-band leakage into the adjacent incumbent band extremely small. This makes it well suitable for TVWS transmission. Filter Bank Multicarrier (FBMC) [5, 6] is another option for CR-TVWS transmission, which has been well studied; but comparing sensing performance between GFDM and FBMC is out of the scope of this paper. Compared to OFDM, which has rectangular pulse shaping, GFDM with flexible pulse shaping filters, for example, root-raised-cosine (RRC), causes less interference to the adjacent incumbent frequency bands [7]. The pulse shaping filters, however, introduces self intercarrier interference (ICI) to the adjacent subcarriers. This degrades the bit error rate performance. Recent works in [8, 9] have shown that a successive interference canceller can mitigate the ICI and improve the GFDM system performance.

To protect opportunistic users, GFDM signals need to be sensed reliably, so that any other CR signal is not transmitted when a GFDM signal is present in the frequency band of operation. An extensive work has already been done in sensing OFDM signals based on the energy detection principle [10, 11], and in this paper, we evaluate the sensing performance of GFDM signals. Other spectrum sensing techniques like cyclostationary feature based detection [12] are more computationally intensive than simple energy detection and a comparison of these two methods is out of the scope of this paper.

In this paper we use a standard GFDM receiver for sensing an opportunistic signal. Whenever GFDM is used for CR TVWS transmission, it is convenient to use the GFDM receiver as a sensing device for other CR signals. Complementary ROC curves are obtained for sensing with a GFDM sensor and compared with ROC curves of an OFDM sensor.

The rest of the paper is organized as follows. In Section II, the GFDM system model is described. Section III states the basic principles of energy detection, Section IV evaluates the performance of the GFDM sensing as compared to OFDM sensing, and finally the conclusions are given in Section V.

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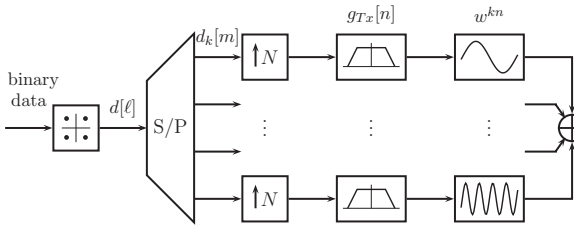


Fig. 1. GFDM transmitter system model

## 2. SYSTEM MODEL

GFDM is a multi-carrier modulation scheme which incorporates a flexible pulse shaping technique. To implement the pulse shaping, the binary data is modulated and divided into sequences of  $K \times M$  complex valued data symbols. Each such sequence  $d[\ell]$ ,  $\ell = 0, 1, \dots, KM - 1$ , is spread across  $K$  subcarriers and  $M$  time slots for transmission. This is represented by means of a block structure,  $\mathbf{D}$ , defined as

$$\begin{aligned} \mathbf{D} &= [\mathbf{d}_0, \mathbf{d}_1 \dots \mathbf{d}_{K-1}]^T \\ &= \begin{bmatrix} d_0[0] & \dots & d_0[M-1] \\ \vdots & & \vdots \\ d_{K-1}[0] & \dots & d_{K-1}[M-1] \end{bmatrix}, \end{aligned} \quad (1)$$

where  $d_k[m] \in \mathbb{C}$  is the data symbol transmitted on the  $k$ th subcarrier and in the  $m$ th time slot.

The GFDM transmitter structure is shown in Fig. 1. In the  $k$ th branch of the transmitter, the complex data symbols  $d_k[m]$ ,  $m = 0, \dots, M - 1$  are upsampled by a factor  $N$ , resulting in

$$d_k^N[n] = \sum_{m=0}^{M-1} d_k[m] \delta[n - mN], \quad n = 0, \dots, NM - 1, \quad (2)$$

where  $\delta[\cdot]$  is the Dirac delta function. We assume  $N = K$ . Also, we have,  $d_k^N[mN] = d_k[m]$  and  $d_k^N[n] = 0$  for  $n \neq mN$ .

The pulse shaping filter  $g[n]$  is applied to the sequence  $d_k^N[n]$ , followed by digital subcarrier upconversion. The resulting subcarrier transmit signal  $x_k[n]$  can be mathematically expressed as

$$x_k[n] = (d_k^N \otimes g)[n] \cdot w^{kn} \quad (3)$$

where  $\otimes$  denotes circular convolution and  $w^{kn} = e^{j \frac{2\pi}{N} kn}$ . Similar to (1), the transmit signals can be expressed in a block structure

$$\begin{aligned} \mathbf{X} &= [\mathbf{x}_0, \mathbf{x}_1 \dots \mathbf{x}_{K-1}]^T \\ &= \begin{bmatrix} x_0[0] & \dots & x_0[MN-1] \\ \vdots & & \vdots \\ x_{K-1}[0] & \dots & x_{K-1}[MN-1] \end{bmatrix}. \end{aligned} \quad (4)$$

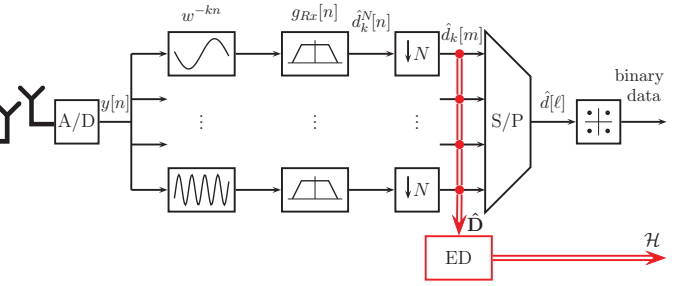


Fig. 2. GFDM receiver system model

The transmit signal for a data block  $\mathbf{D}$  is then obtained by summing up all subcarrier signals according to

$$x[n] = \sum_{k=0}^{K-1} x_k[n]. \quad (5)$$

This is then passed to the digital-to-analog converter and sent over the channel.

The system model for the receiver is shown in Fig. 2. The signal received is  $y[n]$ . The subcarrier receive signal is obtained after digital downconversion and is given as  $\hat{y}_k[n]$ . After convolving with the receiver matched filter  $g[n]$ , the signal is defined as  $\hat{d}_k^N[n]$ , where

$$\hat{y}_k[n] = y[n] \cdot w^{-kn} \quad (6)$$

$$\hat{d}_k^N[n] = (\hat{y}_k \otimes g)[n] \quad (7)$$

The received data symbols,  $\hat{d}_k[m]$  are obtained after down-sampling  $\hat{d}_k^N[n]$  according to  $\hat{d}_k^m = \hat{d}_k^N[n = mN]$ .

The next section describes the principles of energy detection based spectrum sensing and gives the expression for probability of detection and that of false alarm.

## 3. PRINCIPLES OF ENERGY DETECTION

This section details the local spectrum sensing characteristics of GFDM based cognitive radios. Extensive work has already been done on sensing OFDM based primary or incumbent signals [13], [11] and these are applied here to calculate the sensing characteristics of the GFDM signal employed by cognitive devices.

The goal of spectrum sensing now, is to determine if the TV white space is currently occupied by any other cognitive user, i.e., in our case, a GFDM based opportunistic user. Expressed as a binary hypothesis testing problem, we have

$$r[n] = \begin{cases} w_n[n], & \mathcal{H}_0 \\ h[n] * x[n] + w_n[n] & \mathcal{H}_1 \end{cases} \quad (8)$$

where,  $r[n]$  is the signal received by the *new* opportunistic radio's sensing block,  $x[n]$  is the GFDM based CR's transmitted signal,  $h[n]$  is the channel impulse response and  $w_n[n]$  is additive white Gaussian noise (AWGN) with zero mean and

variance  $\sigma_n^2$ . The SNR is defined as  $\gamma = \frac{P}{\sigma_n^2}$  with  $P$  as the opportunistic signal power at the receiver.

Let  $m_s$  denote the number of data symbols that are detected. Under the null hypothesis ( $\mathcal{H}_0$ ), the receiver measures only noise and under the alternate hypothesis ( $\mathcal{H}_1$ ), the receiver measures the opportunistic transmission along with noise.

Using a Neyman-Pearson (NP) test [14], the statistics and the decision rule can be calculated as:

$$L = \sum_{n=1}^{m_s} r^2[n] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{>}} \eta \quad (9)$$

where  $\eta$  is the decision threshold. The test statistic, defined in (9), is the sum of the squared received signal samples. According to the central limit theorem, for large  $m_s$ , the test statistic follows a central chi-squared distribution with  $2m_s$  degrees of freedom under  $\mathcal{H}_0$  and non-central chi-square distribution with a non-central parameter  $\gamma$ , the SNR, under  $\mathcal{H}_1$ .

Here  $L$  is the decision statistic and the number of data samples measured are  $m_s$ . Based on the work by Urkowitz [15] and Sousa [16], the probabilities of detection and false alarm, for spectrum sensing of OFDM based cognitive radio signals, are respectively given as

$$P_d = P \{L > \eta | \mathcal{H}_1\} = Q_{m_s}(\sqrt{2m_s\gamma}, \sqrt{\eta}) \quad (10)$$

$$P_f = P \{L > \eta | \mathcal{H}_0\} = \frac{\Gamma(m_s, \eta/2)}{\Gamma(m_s)} \equiv G_{m_s}(\eta) \quad (11)$$

where  $\Gamma(a, b) = \int_b^\infty t^{a-1} e^{-t} dt$  is the incomplete gamma function and  $Q_m(\cdot, \cdot)$  is the generalized Marcum Q-function, defined as

$$Q_m(a, b) = \int_b^\infty \frac{x^m}{a^{m-1}} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax) dx, \quad (12)$$

where  $I_{m-1}(\cdot)$  is the  $(m-1)$ th order modified Bessel function of the first kind.

Based on (10), (11) the probability of detection for a target probability of false alarm  $\tilde{P}_f$  is given by [16]

$$P_d = Q_{m_s} \left( \sqrt{2m_s\gamma}, \sqrt{G_{m_s}^{-1}(\tilde{P}_f)} \right) \quad (13)$$

The probability of missed detection is hence given as

$$P_m = 1 - P_d = 1 - Q_{m_s} \left( \sqrt{2m_s\gamma}, \sqrt{G_{m_s}^{-1}(\tilde{P}_f)} \right) \quad (14)$$

The theoretical complementary ROC curves are compared with simulated curves for GFDM and OFDM opportunistic signals. In the following section, we describe how the principles of energy detection is applied in case of GFDM receivers acting as sensors. The simulation setup is also described and comparison plots are provided.

## 4. PERFORMANCE EVALUATION

The detailed block diagram of the energy detector is shown in Fig. 3. The GFDM receiver demodulates the received data and the data block  $\hat{\mathbf{D}}$  is passed through a square law device. The summer adds up the energy values of each of the samples along one subcarrier to compute the energy in each of the subcarrier bins. This sensing measurement is compared with a decision threshold to decide whether the subcarrier is empty or occupied. The vector  $\mathcal{H}$  contains the decisions, either  $\mathcal{H}_0$  or  $\mathcal{H}_1$  for all the subcarriers in the CR system.

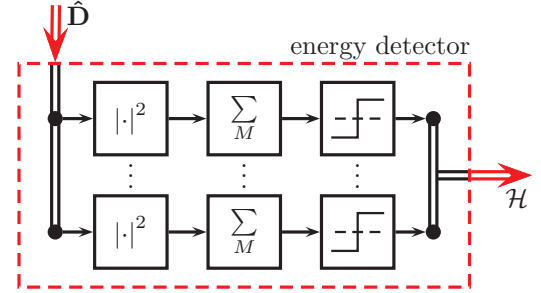


Fig. 3. Energy detector block diagram

The scenario simulated in this paper is where the primary incumbent signals are protected with geo-location database query mechanism, but the opportunistic users in the TV white space need to be sensed before another signal can be transmitted in the same frequency band of operation.

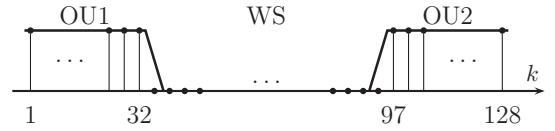
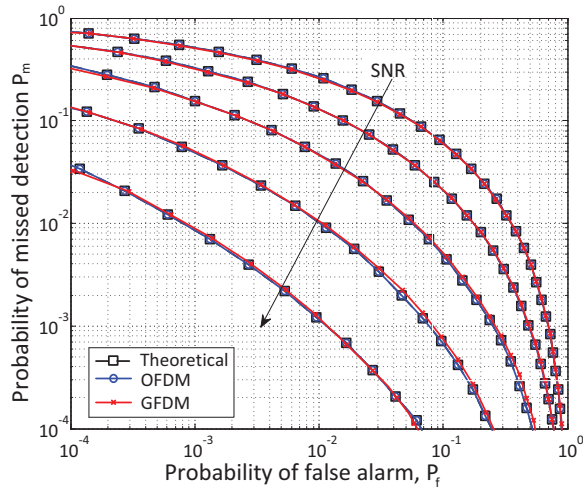


Fig. 4. Simulation setup of white space (WS) for opportunistic usage

The system is simulated with  $K = 128$  subcarriers, where the TVWS is present in subcarriers  $K = 33$  to  $K = 96$ , where the whitespace is in the center of the frequency band and 32 subcarriers on the left as well as on the right side carry opportunistic data. The adjacent subcarriers to the stopband, i.e. the transition band, is not for transmission and hence is not considered in the calculation of the probability of false alarm and that of detection. The simulation parameters are tabulated in Table 1. Multipath channels and cyclic prefix were not considered in the setup.

### 4.1. Synchronous Receiver

Assuming perfect synchronization at the receiver, this system is simulated, once with OFDM and then with GFDM as cognitive opportunistic signals. The OFDM and GFDM systems are sensed with respective OFDM and GFDM sen-



**Fig. 5.** Complementary ROC curves for synchronous GFDM and OFDM with varying SNR.

sors and ROC performance curves are obtained. The complementary ROC curves for GFDM match the complementary ROC curves for OFDM. The self interference generated

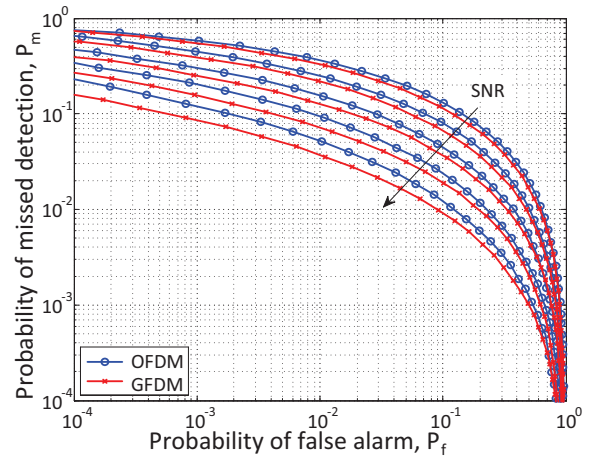
Parameter	Variable	GFDM	OFDM
Samples/symbol	$N$	128	128
Subcarriers	$K$	128	128
Block size	$M$	5	1
Filter type	-	RRC	Rect
Roll-off factor	$\alpha$	0.1	-
Channel	$h$	$h=1$ , AWGN	$h=1$ , AWGN
Cyclic Prefix	CP	No	No
Modulation	-	QPSK	QPSK

**Table 1.** GFDM and OFDM simulation parameters.

in the GFDM system is considered a component of the signal and is not present when the GFDM signal is not present ( $\mathcal{H}_0$ ). These simulated curves follow the theoretical curves from (14) as shown in Fig. 5. This shows that the sensing performance with a GFDM sensor is comparable to ROC curves obtained from traditional OFDM sensors in synchronous systems. The SNR is varied from 0 dB to 4 dB in steps of 1 dB.

#### 4.2. Asynchronous Receiver

A more realistic scenario is where we consider a frequency offset at the receiver. The worst case setup with an offset of half the subcarrier spacing is considered. The OFDM and GFDM signals are sensed by their respective OFDM and GFDM sensors and the complementary ROC is obtained. It is observed in Fig. 6, that the GFDM complementary ROC plots are better compared to OFDM ROC curves. Over the considered range of SNR and  $P_f$ , the probability of missed detection for an OFDM signal is higher than that of a GFDM



**Fig. 6.** Complementary ROC curves for asynchronous GFDM and OFDM with varying SNR.

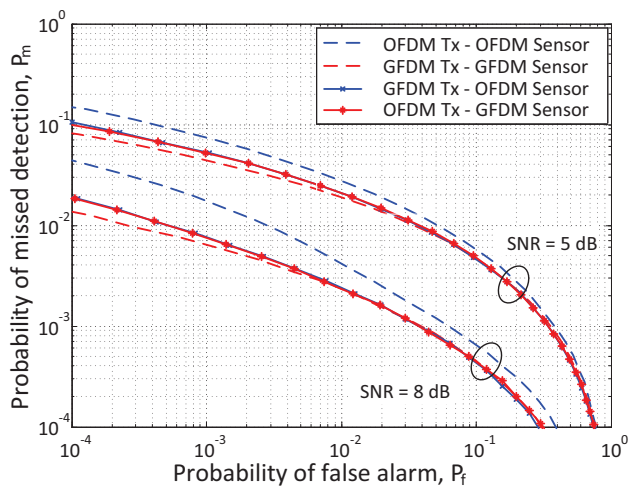
signal. It is also observed that for higher values of SNR, the improvement of the complementary GFDM ROC curves over OFDM ROC curves is larger. This implies that GFDM signals can be better detected as compared to an OFDM signal in asynchronous systems as OFDM is more prone to frequency offset. Hence GFDM sensing is more robust compared to OFDM sensing in more realistic scenarios. The SNR is varied from 0 dB to 4 dB in steps of 1 dB.

#### 4.3. Sensing with GFDM Receiver

In this simulation setup, we have considered an asynchronous CR system. All combinations of transmitting OFDM as well as GFDM and using an OFDM or GFDM receiver for sensing are considered. Based on this we have compared ROC curves for OFDM and GFDM receivers. These are shown in Fig. 7. From the above figure, we see that the sensing ROC performance is best when a GFDM transmission is sensed by a GFDM receiver. The conventional ROC performance curves for OFDM sensing by a traditional OFDM based sensor is also shown here, and its performance is worse than that of the GFDM sensor. The most interesting observation from this study is that when OFDM transmission is sensed by GFDM sensor, then the ROC is better than that of an OFDM based sensor. The steep spectral shape of the GFDM filters improve the sensing performance of a OFDM opportunistic transmission. It is also clear from the above figure, that with higher SNR, sensing with GFDM receiver performance improves.

### 5. CONCLUSION

It is extremely important that the cognitive radio reliably detects not only incumbent active transmissions but also other opportunistic signals. GFDM is an extremely attractive multicarrier modulation scheme suitable for cognitive radio PHY as it has a low out-of-band radiation into the adjacent frequency bands. Traditional OFDM signal detection techniques



**Fig. 7.** Complementary ROC for sensing with OFDM and GFDM receivers at SNR = 5 dB and 8 dB.

and algorithms can be applied to GFDM as well. In this paper, energy detection based spectrum sensing is simulated for the scenarios where OFDM and GFDM are used as opportunistic signals. It is observed that complementary ROC curves for GFDM are better than OFDM and GFDM can be better sensed than OFDM in an asynchronous cognitive radio system. Deriving the theoretical performance in case of asynchronous detection is a work in progress and is kept as an outlook of the simulation study done here. It is also evident that using a GFDM receiver as a sensor also improves the ROC characteristics of a traditional OFDM system. These simulation studies show that compared to conventional OFDM, GFDM is more suitable for cognitive radio PHY, not only because of better spectral shaping, but also because of better sensing characteristics.

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