AN INVARIANT SIMILARITY REGISTRATION ALGORITHM BASED ON THE ANALYTICAL FOURIER-MELLIN TRANSFORM

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ABSTRACT

Here, we intend to introduce an invariant similarity registration method based on Analytical Fourier-Mellin Transform (AFMT) computed on the image functions. The well-known phase correlation method which estimates the geometrical transformation parameters between tow images from the corresponding Fourier-Mellin spectrums is compared to the proposed one.
In order to illustrate the best performance of the geometric parameters estimation, we apply our method to the construction of Punic Panorama.

Index Terms— Analytical Fourier-Mellin Transform (AFMT), Registration, Phase correlation, Mosaicing.

1. INTRODUCTION

Image registration is the process of overlaying two or more images of the same scene taken at different times, from different viewpoints, and/or by different sensors. It geometrically aligns two images the reference image and the transformed image. The present differences between images are introduced due to special imaging situation. Image registration is a fundamental step in all image analysis tasks in which the final information is gained from the grouping of a variety of data sources. Generally speaking, registration methods can be reformulated by combining the following four components: a space of primitives, a research space, a similarity measure and a search strategy [5]. The selection of the similarity measure is closely linked to the selection of matching primitives and therefore to the space of primitives. Among the typical similarity measures, we can mention those related to the correlation, to the sum of absolute differences and to phase correlation. In order to measure the similarity between two images, it is common to calculate the correlation directly on the images' grayscales. This method consists in looking for, without prior knowledge, a model in a search window. The reference shape is usually a pixel block but may also be the entire image. We assume that all pixels follow the same movement; therefore the optimal matching is obtained by maximizing the correlation function. This method of computing similarity in temporal domain has a significant complexity which could be reduced in the frequency domain. Switching from temporal domain to frequency domain is applicable only if the Fourier transform exist in the geometrical transformation group.
The AFMT same as Fast Fourier Transform (FFT) will reduce the complexity of phase correlation estimation but in the planar similarity transformations group[1] ($\mathbb{R}_+^* \times S^1$) instead of the translation group($\mathbb{R}, +$). We will introduce a direct phase correlation algorithm based on AFMT which is invariant under rotation and scale. The proposed algorithm will be compared to the well known phase correlation algorithm obtained by applying the FMT to the images' classical Fourier spectrums.
In this article, we study how the parameters' estimation will be affected by neglecting the phase and considering only the Fourier magnitude as input to the conventional phase correlation algorithm. In order to illustrate the best performance of the geometric parameters estimation, we apply our method to generate a panoramic view of the Punic archeological site.
The present paper will be organized as follows. In the second section, we will introduce a theoretical presentation of the AFMT and correlation expression in the frequency domain. In the third section, we will describe the algorithm of phase correlation based on the AFMT spectrum. The fourth section is devoted to the simulation of the phase correlation algorithm based on AFMT images. In the fifth section we will compare two phase correlation algorithms. We will conclude by representing the mosaicing algorithm using the Phase correlation based on the AFMT of images.

2. THE ANALYTICAL FOURIER-MELLIN TRANSFORM

In order to introduce the proposed invariant similarities registration algorithm based on AFMT, it is useful to recall some definitions and notations which are formulated in [2]

[1] G is the direct product of the additive group of the real line and the multiplicative group of positive and non-zero real numbers. *G* is formed by all planar and positive similarities centered on the origin of axes and is equivalent to the polar coordinate space.

and [3]. Thus, we denote by $L^1\,(G,\mu)$ the normed vector space of complex functions defined on a given group G.

$$f\ in\ L^1\,(G,\mu)\ \Leftrightarrow\ \int_G |f(x)|d\mu(x) < +\infty \qquad (1)$$

f represents the gray level of an image at the geometrical location x. The abstract harmonic analysis theory extends the classical Fourier Transform to the Fourier Transform on a given group G with the following definition:

Definition: The Fourier Transform on a commutative group G of a given function f belonging to $L^1\,(G,\mu)$ is defined as:

$$\hat{f}(\lambda) = \int_G f(x)\,|T_\lambda(x)|^{-1}d\mu(x) \qquad (2)$$

Where $d\mu$ the normalized invariant is measure of G and $T_\lambda(x)$ represents all irreducible and unitary representation of G. It is known that the direct of planar similarities forms a group which is equivalent to the space of polar coordinates which could be parameterized as follow:

$$\mathrm{G} = \left(\mathbb{R}_{+,}^{*} S^1\right) = \{(r,\theta)\mid r > 0\ and\ 0 < \theta < 2\pi\} \qquad (3)$$

G forms a compact and commutative low under the following operation:

$$(r,\theta).(r',\theta') = (rr',\theta+\theta')$$

Its unique normalized invariant measure (Haar Measure) can be written as:

$$d\mu(r,\theta) = \frac{dr}{r}d\theta$$

Thus, the Fourier transform of f on G can be defined as:

$$\hat{f}(k,v) = M_f(k,v) = \int_0^\infty \int_0^{2\pi} f(r,\theta)\,e^{-ik\theta}r^{iv}\frac{dr}{r}d\theta \quad (4)$$

for $k \in \mathbb{Z}$ and $v \in \mathbb{R}$. It is called in literature the Fourier-Mellin Transform of the irradiance distribution $\mathrm{f(r,\theta)}$ in a two-dimensional image expressed in polar coordinates. The origin of the polar coordinates is usually taken in the image center of gravity in order to achieve invariance under translations. The integral (4) diverges in general, since the convergence is indeed under the assumption that $\mathrm{f(r,\theta)}$ is equivalent to Kr^{α} ($\alpha > 0$ and K is a constant) in a neighborhood of the origin. A rigorous approach has been introduced to solve (undergoes) the singularity at the origin of coordinates. Ghorbel in [3] suggested computing the Analytical prolongation of Fourier Mellin Transform (AFMT) which is defined for $k \in \mathbb{Z}$ and $v \in \mathbb{R}$ as:

$$\mathrm{M}_{\mathrm{f}_\sigma}(\mathrm{k},\nu) = \frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} \mathrm{f(r,\theta)e^{-ik\theta}r^{\sigma-i\nu}}.\frac{\mathrm{dr}}{\mathrm{r}}.\mathrm{d\theta} \qquad (5)$$

Where $\sigma > 0$ is a fixed and strictly positive real number.

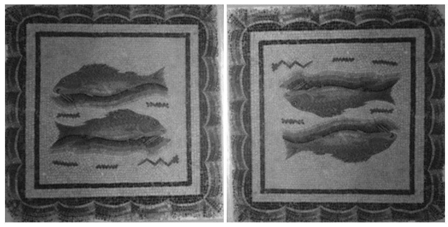
AFMT gives a sole description of images which can be retrieved with the inverse of Analytical Fourier Mellin Transform. Here we recall its expression:

$$\mathrm{f}_\sigma(\mathrm{r},\theta) = \int_0^\infty \sum_{\mathrm{Z}} \mathrm{M}_{\mathrm{f}_\sigma}(\mathrm{k},\upsilon).\mathrm{r}^{-\sigma+i\upsilon}.\mathrm{e}^{ik\theta}\mathrm{d}\nu \qquad (6)$$

This can be used for the fast computation of the G-correlation. By considering the AFMT of two images represented by the two functions $f1_\sigma$ and $f2_\sigma$ for the same analytic prolongation denoted respectively by M_{f1_σ} and M_{f2_σ} and by assuming that the product of the two spectrum $M_{f1_{\sigma.}}\ M_{f2_\sigma}{}^{*}$ [2] is in $L^1(\mathbb{Z}\times\mathbb{R})$, we apply the Inverse of AFMT to this product, we obtain the following correlation:

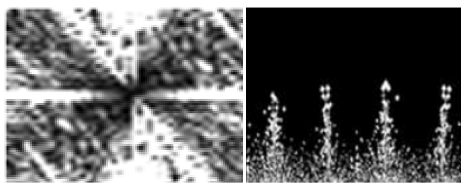
$$\mathrm{C_{TFM}}(\alpha,\beta) = \int_0^\infty \textstyle\sum_{\mathrm{Z}} M_{f1_{\sigma.}}\, M_{f2_\sigma}{}^{*}.\alpha^{-\sigma+i\upsilon}.\mathrm{e}^{ik\beta}\mathrm{d}\nu \qquad (7)$$

3. PHASE CORRELATION BASED ON THE AFMT OF SPECTRUMS



(a) (b)

Figure (1): (a) Image template (b) Image with rotation 180°



(a) (b)

Figure (2): (a) the spectrum of image (a) figure (1). (b) The conversion of the spectrum (a) into log-polar coordinates.

To extract the transformation parameters, a traditional phase correlation algorithm proposes to use the FMT of spectrum. Since the FMT diverges near the origin, we choose to use the AFMT. The use of spectrum instead of images is possible because the transformation between images engenders another one between the spectrums as shown in equation (8). If the image has been rotated with φ and scaled with a factor ρ then the Classical Fourier Transform (F_{2D}) of this image is rotated with the same angle φ and scaled with ρ^{-2}.

$$\|F_{2D}(f\circ(\rho,\varphi))\| = \|(F_{2D}(f))(\rho^{-2},\varphi)\| \qquad (8)$$

So the problem becomes estimating the geometrical parameters assuming $\alpha = \rho^{-2}$ and $\beta = \varphi$. To calculate the AFMTs of spectrums, one must have images in a polar format. In this context, the choice of images' spectrums is suitable since theirs gravity centers are known. Prior starting the calculation of AFMTs, one must perform a pre-processing consisting in the segmentation of the spectrums and their conversion into a polar format. The segmentation of the spectrums is used to easily find the envelope of the

[2] $M_{f2_\sigma}{}^{*}$ is the conjugate of M_{f2_σ}

spectrum which will help us to calculate the transformation parameters. Once the spectrums are truncated and segmented, we can convert them into a polar representation. Figure (4.b) illustrates the conversion of the spectrum in figure (4.a) into log-polar coordinates. After this processing, we calculate the respective AFMTs. We consider the AFMTs of two spectrums represented by the functions $Sf1_\sigma$ and $Sf2_\sigma$ for the same analytic prolongation denoted respectively by M_{Sf1_σ} and M_{Sf2_σ}.

It is noteworthy that two objects with grayscales $Sf1_\sigma$ and $Sf2_\sigma$ have the same shape if and only if there is a similarity as follows:

$$\forall(r,\theta) \in G,\ Sf1_\sigma(r,\theta) = Sf2_\sigma\left(\frac{r}{\alpha}, \theta - \beta\right) \tag{9}$$

The action of vector planar similarities on the Fourier-Mellin space is reflected by:

$$M_{Sf_1^\sigma}(k,v) = \alpha^{\sigma - iv}.e^{-ik\beta}.M_{Sf_2^\sigma}(k,v) \tag{10}$$

This equation shows that the AFMT converts the internal action of a similarity into an external action that has an effect on the amplitude and the phase of spectrum image. This equation can be considered as the shift theorem for the planar similarity group. While calculating the normalized cross-spectrum using equation (11), only information on the difference of phases is preserved [2]:

$$\Phi(k,v) = \frac{M^*_{Sf_1^\sigma}(k,v)M_{Sf_2^\sigma}(k,v)}{\left|M^*_{Sf_1^\sigma}(k,v)\right|\left|M_{Sf_2^\sigma}(k,v)\right|} = \alpha^{-(iv)}.e^{-ik\beta}, \forall(k,v) \in \hat{G} \tag{11}$$

We define the phase correlation of two objects represented respectively by $Sf1_\sigma$ and $Sf2_\sigma$ in the following manner:

$$C_{Tfm}(\alpha,\beta) = \int_0^\infty \sum_Z \Phi(k,v)\alpha^{iv}.e^{ik\beta}dv \tag{12}$$

We can deduce the images transformation parameters (ρ, φ) if we find (α_0, β_0) maximizing the correlation function.

4. PHASE CORRELATION BASED ON THE AFMT OF IMAGES

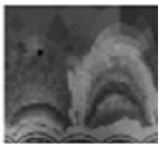


Figure (3): Log-polar sampling of the image in figure 1.b

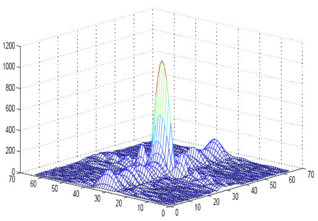


Figure (4): Analytical Fourier Mellin transform of the image in figure (1.b)

From the result presented above, an algorithm is now derived for the rotation and scale estimation. Our idea is to apply the AFMT directly on pictures. So, we apply the log-polar sampling on the two images without going through the steps of calculating the spectrum and its segmentation.

Equation (9) becomes:

$$\forall(r,\theta) \in G,\ f1_\sigma(r,\theta) = f2_\sigma\left(\frac{r}{\rho}, \theta - \varphi\right) \tag{13}$$

Where $f1_\sigma$ and $f2_\sigma$ represent two objects with grayscales which has the same shape. Shall we denote the AFMTs of these two images for the same analytical prolongation respectively by M_{f1_σ} and M_{f2_σ}. Based on the shift theorem in the planar similarities group, we have:

$$M_{f1_\sigma}(k,v) = \rho^{\sigma - iv}.e^{-ik\varphi}.M_{f2_\sigma}(k,v) \tag{14}$$

The phase correlation of two objects $f1_\sigma$ and $f2_\sigma$ can be written as:

$$C_{Tfm}(\rho,\varphi) = \int_0^\infty \sum_Z \Phi(k,v)\rho^{(iv)}e^{ik\varphi}dv \tag{15}$$

With

$$\Phi(k,v) = \frac{M^*_{f2_\sigma}(k,v)M_{f1_\sigma}(k,v)}{\left|M^*_{f2_\sigma}(k,v)\right|\left|M_{f1_\sigma}(k,v)\right|} = \rho^{-(iv)}.e^{-ik\varphi}, \forall(k,v) \tag{16}$$

Then, we search (ρ_0, φ_0) that maximize correlation function. It is preferred to estimate (ρ_0, φ_0) the transformation parameters between images instead of (α_0, β_0) the transformation parameters between spectrums. In phase correlation algorithm based on the AFMT of images, we don't have to calculate the Fourier module $Sf_\sigma(r,\theta)$. We apply log-polar sampling on grayscales images. We estimate the $M_{f_\sigma}(k,v)$, then, we calculate $C_{Tfm}(\rho,\varphi)$.

5. COMPARISON OF TWO PHASE CORRELATION ALGORITHMS

This section compares the results of the transformation parameters estimation with the two algorithms described above. From the figure 1 image (a), we generated 180 images with different rotation angles. Picture (9.a) presents the resulting rotation estimation with Phase correlation algorithm based on the spectrum AFMT; while picture (9.b) presents the algorithm based on the image AFMT. It is obvious that the correlation algorithm based on the AFMT of images is more robust and accurate in the geometric parameters estimation.

Since the conventional algorithm based on the AFMT of spectrum uses only the magnitude, we thought to generate two images (figure 5) that have the same magnitude and two different phases. Figure 5.b is generated using the same magnitude of figure 5.a but the same phase of figure 1.b. Although the divergence between these two images, the conventional algorithm will find them highly correlated since the Fourier phase carries more information than the amplitude [4]. The proposed algorithm based on the image

AFMT will detect the existing difference because it takes in consideration the entire information.

(a) (b)

Figure (5): (a) Mosaic from Bardo museum (Tunisia)
(b) Image generated with the spectrum of 5.a and the phase of image 1.b.

6. EXPERIMENTAL RESULTS AND APPLICATION ON IMAGES MOSAICING

The image registration is the process of aligning two or more images in such a way that objects representing the same structures are superimposed. Numerous scientific domains have benefited from registration applications to refine certain objectives. Mosaicing is one of these applications, and it is mainly used to integrate two or more images of the same scene that are partially overlapping, to reach an integral representation of the scene. Our objective is to present an automatic approach of image registration applied to image mosaicing. This is the phase correlation algorithm based on the image Analytical Fourier Mellin Transform (AFMT). Our idea is to apply the algorithm described in the fourth section but locally in the vicinity of key points. The first step is to detect points of interest in both images using Harris algorithm then we estimates the correlation between all detected points in both images. We chose a match for each point of interest in the reference image; it is the point which gives the highest correlation peak. After this step, we have pairs of points. Now we must define a subset of pairs of points and to test several geometrical transformations and keep the best solution. This approach is based on the RANSAC algorithm (RANdom SAmple Consensu) presented by Martin A. Fischler and Robert C. Bolles in [6]. This algorithm was originally developed to solve problems related to mapping. In our case, the solution we are looking for is an homography expressed from eight independent parameters. Therefore, we need 4 pairs of points to solve the linear system. The principle of the algorithm is as follow. From two sets of points, we define a random subset of four pairs of points. These four pairs of points allow us to calculate a transformation matrix which is applied to the transformed image. We iterate these two previous steps until a maximum number of iterations is reached. The best intermediate solution H' is used to determine precisely the pairs of points set. The more detailed description may be found in [7],[8],[9],[10] and [11]

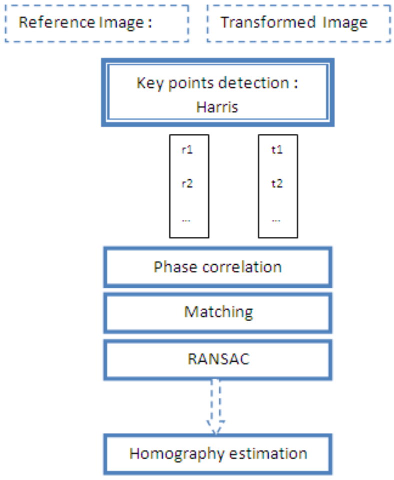


Figure (6): Synopsis of Mosaicing algorithm

(a) (b) (c)

Figure (7): Three pictures of Utica [3]

Figure (8): Mosaic from the three pictures of Utica

7. CONCLUSION

By means of the phase correlation algorithm rotation and scale can be estimated. The conventional inputs for phase correlation algorithm are the Fourier spectrums of reference picture and transformed picture. We demonstrated using practical example that it is more efficient to use images instead of Fourier spectrums since the phase correlation algorithm based on spectrums drops valuable information. The phase correlation algorithm based on the AFMT of images estimates the transformation parameters on the image by passing to the Fourier domain and keeps all the information defining the pictures. Integrating this algorithm within a mosaicing framework proved the robustness of the algorithm and the accuracy of the estimated parameters.

8. REFERENCES

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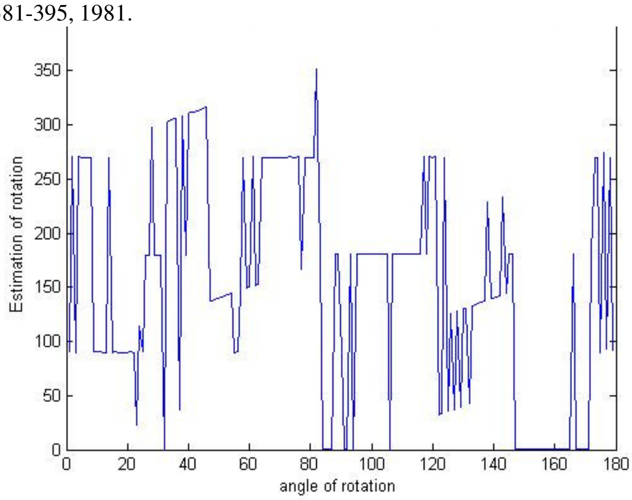
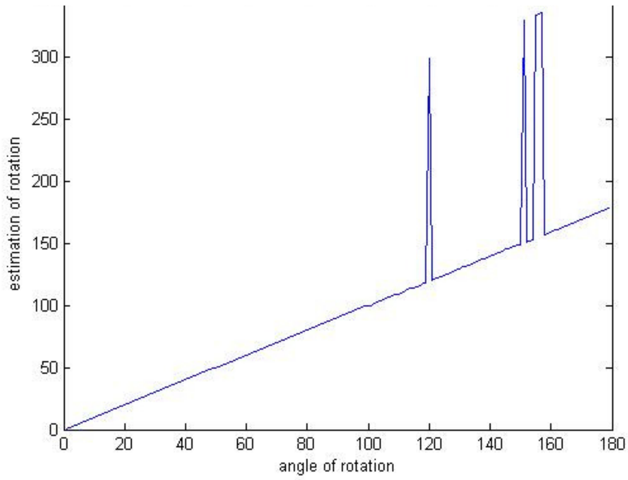


(a)



(b)

Figure (09): Estimation of rotation with phase correlation based on AFMT:
(a) Of spectrum (conventional algorithm), (b) Of images (proposed algorithm)

Figure (10): Mosaic with 9 pictures acquired with different viewpoints,
Panoramic image Of Punic archaeological site