SAR OPTIMUM WAVEFORM DESIGN FOR TARGET DETECTION BASED ON PRIOR KNOWLEDGE

Bingqi Zhu, Kaizhi Wang, Xingzhao Liu

Department of Electronic Information and Electrical Engineering
Shanghai Jiao Tong University
Shanghai, China
Zhu_bingqi@hotmail.com

ABSTRACT

With the development of SAR technology, it is now possible to transmit proper waveforms according to different missions. In this paper, we apply the optimum waveform design method to meet the application of special known-target detection and aim at enhance the target detection rate. Not only transmitted waveforms, but also both range and azimuth matched filters are designed based on maximizing the signal to interference plus noise ratio (SINR). Three waveform design solutions are given theoretically corresponding to different situations, and numerical simulation results are presented to prove the accuracy and effectiveness of our algorithm. Then conclusions are drawn based on our analysis and simulations.

***Index Terms*—** SAR system, optimum waveform design, SINR, matched filter

1. INTRODUCTION

Synthetic Aperture Radar (SAR) is an active microwave system for detecting and imaging reflecting objects such as nature environment, etc. However, most of the transmitted waveforms such as chirps are applied in the SAR system design without considering the interactions with the environment. In fact, most of time what we want to know is just the existence of a specific target in an area and conventional ways of target detection for SAR images such as template matching and feature-based classification are heavy workload and low efficient [1]. So the idea of designing a cognitive radar system by transmitted waveforms and optimum receivers is suggested in order to maximize the probability of detecting rate. This paper is to design optimum waveforms which enable SAR to detect some certain targets correctly.

The design of optimum waveform for maximizing signal to interference plus noise ratio (SINR) due to a known target in additive Gaussian noise was first investigated in [2] [3]. The problem of matching a known target response in a signal-dependent interference and additive channel noise was first investigated in [4], and it was noted that traditional waveforms such as chirp signals, were inferior in SINR performance for extended targets. Earlier work in signal design for detection and identification includes the works [5] [6]. N. A. Goodman summarized and demonstrated a framework for implementation of closed-loop radar with adaptive waveforms in [7] [8]. S. Kay models the received signal in frequency domain and derives the optimum Neyman-Pearson (NP) detector in [9] [10]. The use of mutual information in designing waveforms was extended for multiple-input multiple-output (MIMO) target recognition and classification applications [11]. The work in [12] used MI-based radar waveform design for multiple extended targets. In [13], a simple and successful design for an adaptive radar phase-coded waveform is presented.

However, the above methods have never been placed in synthetic aperture radar systems, which could acquire broader imaging range and more accurate target-detection rate. In this paper, we transplant SINR-based optimum waveform design method to SAR model. Our goal is to determine both the optimum transmitting waveforms and corresponding matched filter comprised of both range and azimuth filters, so that the output SINR is maximized.

2. PROBLEM FORMULATION

We display all the essential features of our simplified model in Fig. 1. and Fig. 2.

There are range direction and azimuth direction in SAR systems. The azimuth motion of actual antenna is used to 'synthesize' a very long antenna. At each position, a pulse $f(\tau,t_n)$ with finite duration τ_0 is transmitted illuminating a stationary target and surrounding clutter. The impulse response of the target $w(\tau,t)$ is real and deterministic, and $w_c(\tau,t)$ be the ground clutter which is stochastic of known range spectral density $G_c(\omega,t_n)$. Supposing the channel noise $w_n(\tau,t)$ is an additive Gaussian noise with range power spectral density $G_n(\omega,t_n)$. Each return echo passes through the range matched filter $h_r(\tau,t_n)$ and is recorded in an 'echo store'. Then each range signal component of the output $y_s(\tau,t_n)$ is

$$\begin{aligned} y_s(\tau,t_n) &= f(\tau,t_n)\otimes w(\tau,t_n)\otimes h_r(\tau,t_n) \\ &= \int_{-\infty}^{+\infty} S(\omega,t_n)H_r(\omega,t_n)e^{j2\pi\omega\tau}d\omega \quad (0\le t_n\le t_0) \end{aligned} \tag{1}$$

And the component $y_0(\tau,t_n)$ composed of clutter and noise is

$$\left\langle y_0^2(\tau,t_n)\right\rangle = \int_{-\infty}^{+\infty}\left|H_r(\omega,t_n)\right|^2 G_0(\omega,t_n)d\omega \tag{2}$$

Where $\langle\ \rangle$ denotes ensemble average, and

$$G_0(\omega,t_n) = G_n(\omega,t_n)+G_c(\omega,t_n)\left|F(\omega,t_n)\right|^2 \tag{3}$$

Then the output SINR after range matched filter $h_r(\tau,t_n)$ at fast time τ_0 can be written as

$$\begin{aligned} \mathrm{SINR}_{t_n} &= \rho(\tau_0) = \frac{y_s^2(\tau_0,t_n)}{\left\langle y_0^2(\tau_0,t_n)\right\rangle} \\ &= \frac{\left|\int_{-\infty}^{+\infty} S(\omega,t_n)H_r(\omega,t_n)e^{j2\pi\omega\tau_0}d\omega\right|^2}{\int_{-\infty}^{+\infty}\left|H_r(\omega,t_n)\right|^2 G_0(\omega,t_n)d\omega} \end{aligned} \tag{4}$$

According to [4, pp. 578], there exists a function $L(\omega,t_n)$ which meets the equation

$$\left|L(\omega,t_n)\right|^2 = G_0(\omega,t_n) \tag{5}$$

Substitutes (5) into (4), and applies Schwarz's inequality to (5), the SINR can be rewritten as

$$\begin{aligned} \mathrm{SINR}_{t_n} &= \frac{\left|\int_{-\infty}^{+\infty} H_r(\omega,t_n)L(\omega,t_n)L^{-1}(\omega,t_n)S(\omega,t_n)e^{j2\pi\omega\tau_0}d\omega\right|^2}{\int_{-\infty}^{+\infty}\left|H_r(\omega,t_n)L(\omega,t_n)\right|^2 d\omega} \\ &\le \int_{-\infty}^{+\infty}\left|L^{-1}(\omega,t_n)S(\omega,t_n)\right|^2 d\omega \end{aligned} \tag{6}$$

Fig. 1. Basic signal-processing sequence in SAR

The SINR achieves its maximum iff

$$H_r(\omega,t_n) = \frac{[\mu F(\omega,t_n)W(\omega,t_n)e^{j2\pi\omega\tau_0}]^*}{G_n(\omega,t_n)+G_c(\omega,t_n)\left|F(\omega,t_n)\right|^2} \tag{7}$$

where μ is an real nonzero normalization constant. Suppose at each position t_n, the transmit signal is limited to the bandwidth B_r, so the maximum of the SINR is

$$\begin{aligned} \mathrm{SINR}_{t_n} &= \int_{-\infty}^{+\infty}\left|L^{-1}(\omega,t_n)S(\omega,t_n)\right|^2 d\omega \\ &= \int_{B_r}\frac{\left|W(\omega,t_n)\right|^2\left|F(\omega,t_n)\right|^2}{G_n(\omega,t_n)+G_c(\omega,t_n)\left|F(\omega,t_n)\right|^2}d\omega \end{aligned} \tag{8}$$

Now let's consider the azimuth direction. As is shown in Fig. 2, in the azimuth direction $y_s(\tau_m,t)$ should be regarded as the input waveform, where $y_s(\tau,t)$ is determined by (1). After convoluted by transmit signal $f(\tau,t_n)$ and filtered by $h_r(\tau,t_n)$, the ground clutter $w_c(\tau,t)$ and the channel noise $w_n(\tau,t)$ now become into $y_c(\tau,t)$ and $y_n(\tau,t)$, which are determined as

$$\left\langle y_c^2(\tau,t_n)\right\rangle = \int_{-\infty}^{+\infty}\left|H_r(\omega,t_n)\right|^2\left|F(\omega,t_n)\right|^2 G_c(\omega,t_n)d\omega \tag{9}$$

$$\left\langle y_n^2(\tau,t_n)\right\rangle = \int_{-\infty}^{+\infty}\left|H_r(\omega,t_n)\right|^2 G_n(\omega,t_n)d\omega \tag{10}$$

Suppose $Z_c(\tau_m,\upsilon)$ as the power spectral density of $y_c(\tau,t)$ at each azimuth direction; $Z_n(\tau_m,\upsilon)$ the power spectral density of $y_n(\tau,t)$ at each azimuth direction, then the output SINR after azimuth matched filter $h_a(\tau_m,t)$ at slow time t_0 is defined by

$$\mathrm{SINR}_{\tau_m} = \rho(t_0) = \frac{{g_S}^2(\tau_m,t_0)}{\left\langle {g_0}^2(\tau_m,t_0)\right\rangle} \tag{11}$$

where

$$\begin{aligned} g_s(\tau_m,t) &= y_s(\tau_m,t)\otimes h_a(\tau_m,t) \\ &= \int_{-\infty}^{+\infty} Y_s(\tau_m,\upsilon)H_a(\tau_m,\upsilon)e^{j2\pi\upsilon t}d\upsilon \end{aligned} \tag{12}$$

$$\begin{aligned} \left\langle {g_o}^2(\tau_m,t)\right\rangle &= \int_{-\infty}^{+\infty}\left|H_a(\tau_m,\upsilon)\right|^2 Z_0(\tau_m,\upsilon)d\upsilon, \\ Z_0(\tau_m,\upsilon) &= Z_n(\tau_m,\upsilon)+Z_c(\tau_m,\upsilon) \end{aligned} \tag{13}$$

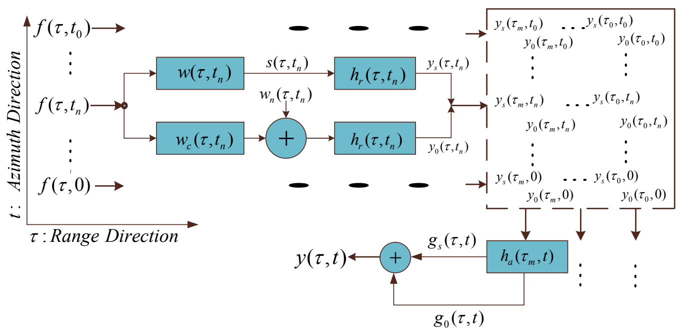


Figure. 2. Simplified model of SAR system

We treat the azimuth direction just the same as the range direction, and thus each azimuth matched filter $h_a(\tau_m,t)$ can be obtained by

$$H_a(\tau_m,\upsilon)=\frac{[\sigma Y_s(\tau_m,\upsilon)e^{j2\pi\upsilon t_0}]^*}{Z_n(\tau_m,\upsilon)+Z_c(\tau_m,\upsilon)} \tag{14}$$

and the maximum SINR of each azimuth direction is

$$\mathrm{SINR}_{\tau_m}=\int_{-\infty}^{+\infty}\frac{|Y_s(\tau_m,\upsilon)|^2}{Z_n(\tau_m,\upsilon)+Z_c(\tau_m,\upsilon)}d\upsilon \tag{15}$$

3. THE OPTIMUM TRANSMIT SIGNALS IN DIFFERENT CONDITIONS

From the above discussion, it is clear that the azimuth matched filter $h_a(\tau_m,t)$ is determined by each transmit signal $f(\tau,t_n)$ and range matched filter $h_r(\tau,t_n)$ designed before. That is to say, once $f(\tau,t_n)$ and $h_r(\tau,t_n)$ are designed, the best azimuth filter $h_a(\tau_m,t)$ is determined as well. As we can see, there are two main factors (receiver noise $w_n(\tau,t)$ and clutter $w_c(\tau,t)$) which are crucial in designing transmit signals. What's more, for each transmit signal, the pulse energy is constrained to a constant

$$\int_{B_r}|F(\omega,t_n)|^2\,d\omega=E_t \tag{16}$$

This section discusses the possible influence by these factors. For completeness, we derive all the optimum signals and matched filters for all cases shown below.

A. Receiver noise is negligible

Once the power spectrum density of receiver noise is small enough compare with the target impulse response and the clutter, i. e. $G_n(\omega,t_n)\equiv 0$. Thus $|L(\omega,t_n)|^2=G_c(\omega,t_n)$. According to (7), (8), (14) and (16), we can conclude:

- Once the $w(\tau,t_n)$ and the $w_c(\tau,t_n)$ is certain, the maximization of SINR at the range direction is a constant, which is

$$(\mathrm{SINR}_{t_n})_{\max}=\int_{B_r}\frac{|W(\omega,t_n)|^2}{G_c(\omega,t_n)}d\omega=const \tag{17}$$

  and it is irrelevant to the form of transmit signals or the matched filters.

- The optimum transmit signal $f(\tau,t_n)$ can be any forms, if only it meets the energy restriction (16).
- The form of the optimum range matched filter $h_r(\tau,t_n)$ is decided by

$$H_r(\omega,t_n)=\frac{\mu e^{-j2\pi\omega\tau_0}W^*(\omega,t_n)}{G_c(\omega,t_n)F(\omega,t_n)} \tag{18}$$

- Since $G_n(\omega,t_n)\equiv 0$, according to (10), it is clear that $Z_n(\tau_m,\upsilon)\equiv 0$. Then the form of the optimum azimuth matched filter $h_a(\tau_m,t)$ is as follow

$$H_a(\tau_m,\upsilon)=\frac{\sigma e^{-j2\pi\upsilon t_0}Y_s(\tau_m,\upsilon)^*}{Z_c(\tau_m,\upsilon)} \tag{19}$$

  where μ and σ are any real non-zero normalization constant.

B. Clutter is negligible

In the absence of clutter, only the target impulse response and receiver noise exist. Thus $G_c(\omega,t_n)\equiv 0$ and it follows

- The SINR at the range direction is

$$\begin{aligned}\mathrm{SINR}_{t_n}&=\int_{B_r}\frac{|W(\omega,t_n)|^2|F(\omega,t_n)|^2}{G_n(\omega,t_n)}d\omega\\&=\int_{B_r}\frac{|W(\omega,t_n)|^2}{G_n(\omega,t_n)}|F(\omega,t_n)|^2\,d\omega\end{aligned} \tag{20}$$

  Introduce the kernel $M(\tau,t_n)$,

$$M(\tau,t_n)=\int_{-\infty}^{+\infty}\frac{|W(\omega,t_n)|^2}{G_n(\omega,t_n)}e^{j2\pi\omega\tau}d\omega \tag{21}$$

  and assume $\lambda_{\max}(t_n)$ and $\varphi(\tau,t_n)$ to be the largest eigenvalue and corresponding eigenfunction of $M(\tau,t_n)$, then the maximization of SINR at the range direction is [2, pp. 1584-85]

$$(\mathrm{SINR}_{t_n})_{\max}=\lambda_{\max}(t_n)E_t \tag{22}$$

- The optimum transmit signal $f(\tau,t_n)$ is attained by choosing

$$f(\tau, t_n) = \sqrt{E_t}\varphi(\tau, t_n) \tag{23}$$

- The optimum range impulse response $h_r(\tau, t_n)$ is computed according to

$$H_r(\omega, t_n) = \frac{\mu e^{-j2\pi\omega\tau_0}[F(\omega, t_n)W(\omega, t_n)]^*}{G_n(\omega, t_n)} \tag{24}$$

- Similar to the case of $G_n(\omega, t_n) \equiv 0$, the $h_a(\tau_m, t)$ is

$$H_a(\tau_m, \upsilon) = \frac{[\sigma Y_s(\tau_m, \upsilon)e^{j2\pi\upsilon t_0}]^*}{Z_n(\tau_m, \upsilon)} \tag{25}$$

when $G_c(\omega, t_n) \equiv 0$.

C. Nothing is negligible

In this situation, both noise and clutter are not negligible and should all be taken into consideration. We use Lagrangian multiplier and construct a new function by combine (8) and (16),

$$P(|F(\omega, t_n)|^2, \lambda) = \int_{B_r} \frac{|W(\omega, t_n)|^2 |F(\omega, t_n)|^2}{G_n(\omega, t_n) + G_c(\omega, t_n)|F(\omega, t_n)|^2} d\omega + \lambda[E_t - \int_{B_r} |F(\omega, t_n)|^2 d\omega] \tag{26}$$

As is proved in [14. pp. 925], we can conclude:

- The optimum transmit signal $f(\tau, t_n)$ in this situation is

$$|F(\omega, t_n)|^2 = \frac{\sqrt{|W(\omega, t_n)|^2 G_n(\omega, t_n)}}{G_c(\omega, t_n)}(A - \sqrt{\frac{G_n(\omega, t_n)}{|W(\omega, t_n)|^2}}) \tag{27}$$

where A is a constant determined by substituting (27) into (16).

- The maximum SINR is

$$(\text{SINR}_{t_n})_{\max} = \int_{B_r} \frac{|W(\omega, t_n)|^2 |F(\omega, t_n)|^2}{G_n(\omega, t_n) + G_c(\omega, t_n)|F(\omega, t_n)|^2} d\omega \tag{28}$$

which the $F(\omega, t_n)$ has just been defined above.

- $h_r(\tau, t_n)$ is expressed as (7).
- And $h_a(\tau_m, t)$ is determined by (14).

Although the optimum result is given above, some other matters need to be paid attention. Unless the bandwidth B_r or B_a is infinite and the resulting spectrum contains no zero-energy bands of finite width, by the Paley-Wiener Theorem, the waveform designed cannot be time limited. In some cases the resulting waveform may be approximately limited to a finite-duration, but most of the times, this approximation is not guaranteed. Fortunately, it is possible to obtain a finite-duration waveform that closely approximates the optimum waveform, i. e. finite impulse response (FIR) filter design techniques [14, pp. 914].

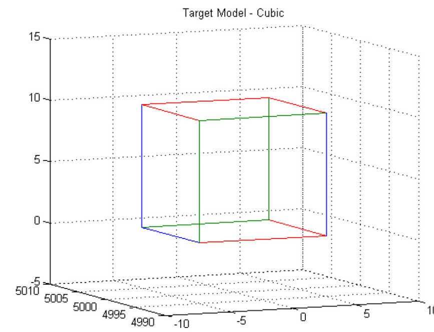


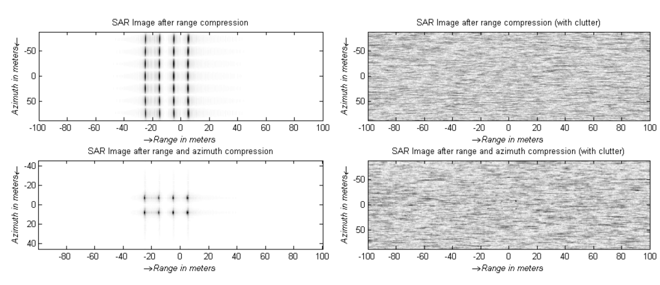
Figure. 3. Target model for simulation

4. SIMULATION RESULTS

In this section, we present numerical example to test the effectiveness of the optimum waveform design algorithm with SAR model.

Suppose there are 8 ideal point-targets which make up of a cubic in space as is shown in Fig. 3. Each point-target's RCS is assumed to be 1 and the cubic is illuminated by a side-look SAR system. If clutter and channel noise can all be ignored, according to the regular SAR signal processing, Fig. 4 (a) is obtained. The Fig. 4 (a) is the range and 2-D compressed image. From Fig.4 (a), we can confirm easily that there exists a cubic in the detection area.

If there is clutter in the detection area large enough to disturb the target, as is show in Fig. 4 (b), we can't figure out easily whether the cubic exists in this large area, and false-alarm rate might be increased dramatically under this circumstance. Now let's use our algorithm. Suppose we know the impulse response $w(\tau, t)$ of cubic, the range spectral density is $G_c(\omega, t_n)$ and the channel noise ignored which corresponds to the situation A in section 3. As is deduced in (18) and (19), the optimum matched filter is determined.



(a) (b)

Figure. 4. The range-compressed image (upper) and the 2-D compressed image of the cubic (lower). (a) The result without clutter. (b) The result with clutter.

Because the form of transmit signal is irrelevant to the SINR, we keep using chirp signals, and the results are in Fig. 6. Fig.6 is the images after filtered by optimum range filter. In Fig. 6 (a), we can see that the optimum range filter can "find" the cubic response although the clutter is huge, and target can be detected within each range signals. We extract single filtered range signal (red) and Fig. 6 (b) is the decibel result of it, which focused very well. Fig. 7 is the image result after convoluted by both range and azimuth optimum filter. We can see that both range direction and azimuth direction have been focused in line and two lines meet in one point, it means that there do exist one estimated target (cubic) in this area. This proves the accuracy of our algorithm.

5. CONCLUSION

It is an important task for SAR system to select transmitting waveforms and matched filters optimally to detect the existence of some certain targets such as tanks or airplanes. Through the preceding discussion, the optimum waveform selection of known-target model with clutter and noise is proposed for SAR based on maximizing the output SINR. We use Lagrange multiplier and the integral equation to maximize the SINR. Each situation is analyzed in detail in section 3 including the noise or the clutter negligible and corresponding optimum waveform resolutions are given concomitantly. What's more, we should notice that some of these resolutions don't guarantee the condition Paley-Wiener Theory required, so some measurement must be taken to closely approximate the theoretically optimum waveform. In section 4, simulations are demonstrated to test the accuracy and effectiveness of our algorithm. The results show that we can detect the target successfully although the clutter is severe. It proves that the algorithm for target detection based on prior knowledge in SAR is quite effective.

6. REFERENCES

[1] C. Oliver and S. Quegan, "Understanding Synthetic Aperture Radar Images," , 1998. :Artech House

[2] Bell, M. R., "Information theory and radar waveform design," IEEE Transactions on Information Theory, 39,5 (Sept. 1993), pp. 1578-1597.

[3] Bell, M. R, "Information theory and radar: Mutual information and the design and analysis of radar waveforms and systems," Ph.D. dissertation, California Institute of Technology.

[4] Pillai, S., Oh, H., Youla, D., and Guerci, J., "Optimum transmit-receiver design in the presence of signal-dependent interference and channel noise," IEEE Transactions on Information Theory, 46, 2 (Mar, 2000), pp. 577-584.

[5] Guerci, J. R. and Pillai, S. U., "Theory and application of optimum transmit-receive radar," In Proceedings of the IEEE 2000 International Radar Conference, Washington, D.C., May 8-12, 2000, pp. 705-710.

[6] Garren, D. A., Osborn, M. K., Odom, A. C., "Enhanced target detection and identification via optimised radar transmission pulse shape," IEE Proceedings – Radar, Sonar and Navigation, 148, 3 (june 2001), pp. 130-138

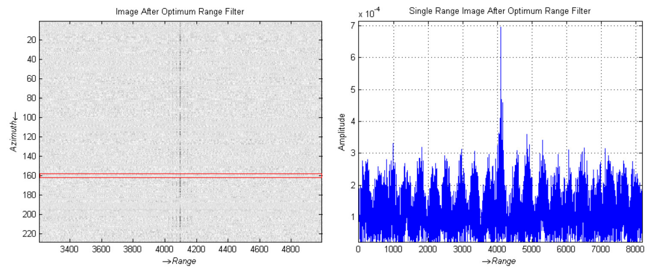


Figure. 6. Image of the cubic after range filter. (a) The image with clutter after optimum range filter. (b) Single range signal after optimum range filter.

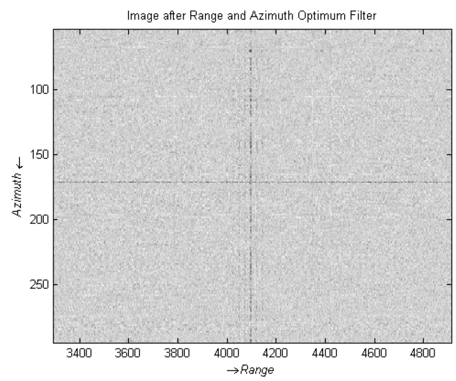


Figure.7. Image of the cubic after filtered by both optimum range and azimuth. The meeting-point of two lines indicates that there exists one estimated target (cubic) in the area.

[7] N. A. Goodman, "Closed-loop radar with adaptively matched waveforms," International Conference on Electromagnetics in Advanced Applications, 2007, Torino, pp. 468-471.

[8] N. A. Goodman, Phaneendra R. Venkata, "Adaptive waveform design and sequential hypothesis testing for target recognition with active sensors," IEEE Journal of Selected Topics in Signal Processing, 2007,1 (1), pp. 105-113.

[9] Kay, S. "Optimal signal design for detection of Gaussian point targets in stationary Gaussian clutter/reverberation," IEEE Transactions on Selected Topics in Signal Processing, (June 2007), pp. 31-41.

[10] Kay, S. "Waveform design for multistatic radar detection," IEEE Transactions on Aerospace and Electronic Systems, (July 2009), pp. 1153-1166.

[11] Yang Yang, Rick S. Blum, "MIMO radar waveform design based on mutual information and minimum mean-square error estimation," IEEE Transactions on Aerospace and Electronic Systems, 43,1 (Jan. 2007), 330-343.

[12] Leshem, A., Naparstek, O., and Nehorai, A. "Information theoretic adaptive radar waveform design for multiple extended targets," IEEE Journal of Selected Topics in Signal Processing, 1, 1 (June. 2007), pp. 42-55.

[13] J. D. Zhang, X. H. Zhu and H. Q. Wang, "Adaptive radar phase-coded waveform design," Electronics Letters, 2009,45 (20), pp. 1052-1053.

[14] R. A. Romero, J. Bae, N. A. Goodman., "Theory and application of SNR and mutual information matched illumination waveforms," IEEE Transactions on Aerospace and Electronic Systems, 47, 2, (Apr. 2011), pp. 912-926.