

# NARROWBAND DETECTION IN OCEAN WITH IMPULSIVE NOISE USING AN ACOUSTIC VECTOR SENSOR ARRAY

V. N. Hari <sup>1</sup>, Anand G. V. <sup>2</sup>, P. V. Nagesha <sup>2</sup> and A. B. Premkumar <sup>1</sup>

<sup>1</sup>School of Computer Engineering, Nanyang Technological University, Singapore 639798

<sup>2</sup>Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India

## ABSTRACT

This paper presents the formulation and analysis of some methods for narrowband detection of underwater acoustic sources in impulsive noise using an array of acoustic vector sensors. Since the array signal vector is unknown due to the unknown location of the source, detection is based on the generalized likelihood ratio test which involves estimation of the signal vector. Different detectors use different signal models which yield different signal estimators. It is shown that the truncated subspace detector (TSD), which uses a truncated normal mode model, yields the best performance.

*Index Terms-* subspace detection, generalized likelihood ratio test, shallow ocean, acoustic vector sensor

## 1. INTRODUCTION

This paper addresses the problem of narrowband detection of an acoustic source in a shallow ocean using an array of acoustic vector sensors (AVS). Detection of sources using a sensor array may be achieved using an optimal Neyman-Pearson (NP) detection strategy if the signal field at the array is known [1]. However, the signal is unknown in a general detection scenario because the source location is unknown, and the environmental parameters may not be known fully. Hence detection of the signal is done by a generalized likelihood ratio test (GLRT), which involves estimation of the signal vector and noise parameters at the array. One form of this test leads to the formulation of a subspace detection algorithm [2]. Two detectors based on the GLRT approach, have been presented recently for AVS arrays [3, 4], viz. the subspace detector (SD) and the approximate signal form detector (ASFD). These were formulated under the assumption that the ambient noise is Gaussian. However, ambient noise in the ocean often has a heavy-tailed distribution [5]. Therefore it is of interest to develop effective methods of detection in such an environment. GLRT detection has earlier been employed in the case of spherically invariant random vector noise [6] and generalized Gaussian noise with interference [7].

It is well-known that AVS arrays have a better direction-finding capability than the conventional APS arrays [8, 9]. This performance advantage arises because an AVS measures not only the acoustic pressure but also all

components of particle velocity at a point in space. Recent work has shown that this advantage of an AVS array can also be exploited to obtain better detection performance than that achieved by an APS array [3, 4].

In this paper we extend the SD and ASFD techniques to underwater signal detection in heavy-tailed noise using an AVS array. The noise distribution is modeled as generalized Gaussian (GG). We also present a new technique called truncated subspace detector (TSD). Each of these detectors is based on a different model of the array signal vector. Different models lead to different estimates of the signal vector. It is shown that the normalized mean square signal estimation error (MSE) of TSD is significantly lower and hence the TSD provides a significantly better performance than both SD and ASFD. The paper is organized as follows. The data model is presented in Section 2. The alternative signal models and corresponding detectors are formulated in Section 3 and expressions for the associated MSEs are derived. A comparative performance analysis of the detectors is presented in Section 4. Conclusions are presented in Section 5.

## 2. DATA MODEL

Consider the detection of a narrowband source with center frequency  $f$  located in a shallow ocean, using an  $N$ -element uniform horizontal linear AVS array. We shall consider three outputs produced by each AVS, viz. the acoustic pressure and two orthogonal horizontal components of particle velocity. The vertical component of particle velocity is not considered since it is found to increase complexity without yielding any significant additional improvement in performance. The sensor array is assumed to be at depth  $z_a$  in the far-field region with respect to the source, which is located at range  $r$ , depth  $z_s$ , and azimuth angle  $\phi$  measured with respect to the axis of the array.

Let  $\mathbf{x}$ ,  $\mathbf{s}$  and  $\mathbf{w}$  denote, respectively, the  $3N \times 1$  data vector, signal vector and noise vector at the AVS array; and let  $\mathbf{x}_n$ ,  $\mathbf{s}_n$  and  $\mathbf{w}_n$  denote, respectively, the  $3 \times 1$  data vector, signal vector and noise vector at the  $n^{\text{th}}$  AVS. Assuming the ocean to be range-independent with water density  $\rho$  and sound speed profile  $c(z)$ , we can express  $\mathbf{s}_n$  as

$$\mathbf{s}_n = [p_n \sqrt{2} \rho c(z_a) v_{xn} \sqrt{2} \rho c(z_a) v_{yn}]^T, \quad (1)$$

where  $p_n$  and  $(v_{xn}, v_{yn})$  denote, respectively, the complex

amplitudes of acoustic pressure and horizontal ( $x, y$ ) components of particle velocity of the signal at the  $n^{\text{th}}$  AVS. The vectors  $\mathbf{w}_n$  and  $\mathbf{x}_n$  are defined similarly. The particle velocity components are scaled by the factor  $\sqrt{2}\rho c(z_a)$  to ensure that all the elements have the same dimension. The signal vector  $\mathbf{s}$  can be written in terms of the normal modes of the oceanic waveguide as [9].

$$\mathbf{s} = \mathbf{A}(\phi)\mathbf{b}, \quad (2)$$

$$\mathbf{A}(\phi) = [\mathbf{a}_1(\phi) \dots \mathbf{a}_M(\phi)], \quad (3)$$

$$\mathbf{a}_m(\phi) = \mathbf{c}_m(\phi) \otimes \mathbf{d}_m(\phi), \quad (4)$$

$$\mathbf{c}_m(\phi) = [1 \exp(ik_m d \cos(\phi)) \dots \exp(i(N-1)k_m d \cos(\phi))]^T \quad (5)$$

$$\mathbf{d}_m(\phi) = [1 \sqrt{2} \zeta_m \cos(\phi) \sqrt{2} \zeta_m \sin(\phi)]^T, \quad (6)$$

$$\zeta_m = k_m/k(z_a) = k_m c(z_a)/2\pi f; m = 1, \dots, M, \quad (7)$$

$$\mathbf{b} = [b_1 \dots b_M]^T, \quad (8)$$

$$b_m = B\psi_m(z_s) \exp(-\delta_m r + jk_m r) / \sqrt{k_m r}; m = 1, \dots, M, \quad (9)$$

where  $k_m$ ,  $\delta_m$  and  $\psi_m(z)$  are respectively the wavenumber, attenuation coefficient and eigenfunction of the  $m^{\text{th}}$  mode,  $M$  is the number of modes,  $d$  is the inter-sensor spacing,  $B$  is a constant, and  $\otimes$  denotes the Kronecker product. The columns of  $\mathbf{A}(\phi)$  are the modal steering vectors, and the elements of  $\mathbf{b}$  are the amplitudes of the modes.

The ambient noise in the ocean is assumed to be spatially white and heavy-tailed, modeled by the following circular complex generalized Gaussian (GG) PDF with variance  $\sigma^2$

$$f_{GG}(\mathbf{w}) = \left( \frac{J(\alpha)}{\sigma^2} \right)^{3N} \exp\left( -\frac{K(\alpha)}{\sigma^\alpha} \sum_{n=1}^{3N} |w(n)|^\alpha \right), \quad 0 < \alpha < 2, \quad (10)$$

$$J(\alpha) = \frac{\alpha \Gamma(4/\alpha)}{2\pi \Gamma(2/\alpha)^2}, \quad K(\alpha) = \left[ \frac{\Gamma(4/\alpha)}{2\Gamma(2/\alpha)} \right]^{\alpha/2}.$$

### 3. GLRT DETECTION IN SHALLOW OCEAN

#### 3.1 Formulation of the detection problem

The detection problem can be cast in the form of the following hypothesis testing problem:

$$\begin{aligned} H_0 : \mathbf{x} &= \mathbf{w} \\ H_1 : \mathbf{x} &= \mathbf{s} + \mathbf{w}. \end{aligned} \quad (11)$$

The joint likelihood functions of the array data vectors under hypotheses  $H_0$  and  $H_1$  are given by

$$f(\mathbf{x}, H_0) = f_{GG}(\mathbf{x}), \quad (12)$$

$$f(\mathbf{x}, H_1) = f_{GG}(\mathbf{x} - \mathbf{s}), \quad (13)$$

where  $f_{GG}(\cdot)$  is defined in (10). The likelihood ratio

$$L(\mathbf{x}) = f(\mathbf{x}; H_1) / f(\mathbf{x}; H_0) \quad (14)$$

yields the following test statistic and decision rule

$$T_{UD}(\mathbf{x}) = \sum_{n=1}^{3N} [ |x(n)|^\alpha - |x(n) - s(n)|^\alpha ], \quad (15)$$

$$\text{Decide } H_1 \text{ if } T_{UD}(\mathbf{x}) > \gamma(P_{FA}) \quad (16)$$

where  $x(n)$  and  $s(n)$  denote the  $n^{\text{th}}$  element of  $\mathbf{x}$  and  $\mathbf{s}$

respectively, and  $\gamma(P_{FA})$  is the threshold corresponding to probability of false alarm  $P_{FA}$  [1]. But (16) is an unrealizable detector (UD) since it requires the knowledge of the signal vector  $\mathbf{s}$  that depends on the unknown source location. However, the performance of UD may be considered as an upper bound on the performance of any realizable detector.

When some parameters of the signal  $\mathbf{s}$  or the noise  $\mathbf{w}$  are unknown, they can be estimated by maximizing the likelihood functions with respect to these unknown parameters. Let  $\theta_j$  denote the unknown parameter vector under hypothesis  $H_j$ ,  $j=0,1$ . The ratio of the maximized likelihoods called the generalized likelihood ratio, is given by

$$L_G(\mathbf{x}) = f(\mathbf{x}; \hat{\theta}_1; H_1) / f(\mathbf{x}; \hat{\theta}_0; H_0), \quad (17)$$

where  $\hat{\theta}_j$  is the maximum likelihood estimate (MLE) of  $\theta_j$ .

Simplification of the generalized likelihood ratio yields the GLRT test statistic. In sections 3.2 and 3.3, two novel detectors based on the GLRT approach are presented.

#### 3.2. Truncated Subspace detector

If  $3N > M$ , where  $N$  is the number of sensors in the array and  $M$  is the number of normal modes, the columns of the modal steering matrix  $\mathbf{A}(\phi)$  are linearly independent and the  $3N$ -dimensional array signal vector  $\mathbf{s}$  belongs to the  $M$ -dimensional modal subspace  $V_M(\phi)$  spanned by the columns of  $\mathbf{A}(\phi)$ . A subspace detector (SD) based on this property of  $\mathbf{s}$  was proposed in [3] for narrowband detection in Gaussian noise. A simpler formulation of the SD is presented here for the general case of non-Gaussian noise.

We can represent  $\mathbf{s}$  in the alternative form

$$\mathbf{s} = \mathbf{A}(\phi)\mathbf{b} = \mathbf{Q}(\phi)\boldsymbol{\beta} = [\mathbf{q}_1(\phi) \dots \mathbf{q}_M(\phi)]\boldsymbol{\beta}, \quad (18)$$

where  $\mathbf{Q}(\phi)$  is a unitary matrix which may be obtained by QR decomposition of  $\mathbf{A}(\phi)$  as

$$\mathbf{A}(\phi) = \mathbf{Q}(\phi)\mathbf{R}. \quad (19)$$

In (18),  $\boldsymbol{\beta} = \mathbf{R}\mathbf{b}$  is a transformed version of the unknown mode amplitude vector  $\mathbf{b}$ . The signal vector  $\mathbf{s}$  belongs to the modal subspace  $V_M(\phi)$  defined as

$$V_M(\phi) = \text{span}\{\mathbf{q}_1(\phi) \dots \mathbf{q}_M(\phi)\}, \quad (20)$$

so that  $\{\mathbf{q}_1(\phi) \dots \mathbf{q}_M(\phi)\}$  constitutes an orthonormal basis of  $V_M(\phi)$ . For a given  $\phi$ , the subspace  $V_M(\phi)$  is known if the modal wavenumbers  $\{k_m; m = 1, \dots, M\}$  are known. Thus the problem of estimating the  $3N$ -dimensional signal vector  $\mathbf{s}$  is reduced to the simpler problem of estimating the  $M$ -dimensional mode amplitude vector  $\boldsymbol{\beta}$ . Maximization of the likelihood function  $f(\mathbf{x}; H_1) = f(\mathbf{x}; \boldsymbol{\beta}, \phi, H_1)$  with respect to the unknown  $M$ -dimensional vector  $\boldsymbol{\beta}$  yields the equation

$$\sum_{n=1}^{3N} |x(n) - \mathbf{r}_n(\phi)\hat{\boldsymbol{\beta}}(\phi)|^{\alpha-2} [\hat{\boldsymbol{\beta}}(\phi) - \mathbf{r}_n^H(\phi)x(n)] = \mathbf{0}, \quad (21)$$

where  $\mathbf{r}_n(\phi)$  is a row vector denoting the  $n^{\text{th}}$  row of  $\mathbf{Q}(\phi)$ , and  $\hat{\boldsymbol{\beta}}(\phi)$  denotes the conditional MLE of  $\boldsymbol{\beta}$  for a given  $\phi$ . The

following closed form solution of (21) is readily obtained if  $\alpha = 2$ :

$$\hat{\boldsymbol{\beta}}(\phi) = \mathbf{Q}^H(\phi)\mathbf{x}. \quad (22)$$

As no closed form solution of (21) is available when  $\alpha \neq 2$ , we use (22) as an approximation to the conditional MLE of  $\boldsymbol{\beta}$ . The approximate MLEs of  $\phi$  and  $\mathbf{s}$  can now be written as

$$\hat{\phi}_{SD} = \arg \max \{ \mathbf{x}^H \mathbf{Q}(\phi) \mathbf{Q}^H(\phi) \mathbf{x} \}, \quad (23)$$

$$\hat{\mathbf{s}} = \mathbf{Q}(\hat{\phi}_{SD}) \mathbf{Q}^H(\hat{\phi}_{SD}) \mathbf{x}, \quad (24)$$

and the test statistic of the SD is thus given by

$$T_{SD}(\mathbf{x}) = \sum_{n=1}^{3N} \left[ |x(n)|^\alpha - |x(n) - \mathbf{r}_n(\hat{\phi}_{SD}) \mathbf{Q}^H(\hat{\phi}_{SD}) \mathbf{x}|^\alpha \right]. \quad (25)$$

The normalized mean square error (MSE) of the signal estimate  $\hat{\mathbf{s}}_{SD}$  can be found from (18) and (24) as

$$\varepsilon_{SD} = \frac{E[(\mathbf{s} - \hat{\mathbf{s}})^H (\mathbf{s} - \hat{\mathbf{s}})]}{E[\mathbf{s}^H \mathbf{s}]} \cong \frac{M}{N\eta}, \quad (26)$$

where  $\eta = \frac{E[\mathbf{s}^H \mathbf{s}]}{N\sigma^2}$  is the signal-to-noise ratio (SNR) and  $E$  is the expectation operator.

The SD can be employed only if the columns of  $\mathbf{A}(\phi)$  are linearly independent, i.e. if  $M \leq 3N$ . Since the number of modes  $M$  increases as the frequency  $f$  is increased [9], the applicability of the SD is limited by an upper cut-off frequency  $f_c$  which depends on the length of the array; a shorter array limits the applicability of the SD to a lower cut-off frequency. Moreover, the SD suffers degradation in performance as  $f$  is increased even if  $f < f_c$ , because an increase in  $M$  leads to an increase in the MSE  $\varepsilon_{SD}$ , as shown by (26).

In order to extend the applicability of the SD to shorter arrays/higher frequencies, as well as arrest the degradation associated with an increase in the number of modes, we propose a detector which uses a truncated model of the signal vector obtained by projecting  $\mathbf{s}$  on a modal subspace  $V_{M'}$  of smaller dimension  $M'$ , where  $M' < M$ :

$$V_{M'}(\phi) = \text{span} \{ \mathbf{q}_1(\phi) \dots \mathbf{q}_{M'}(\phi) \}. \quad (27)$$

The set of spanning vectors of the truncated subspace  $V_{M'}(\phi)$  is a subset of the set of spanning vectors of the full modal subspace  $V_M(\phi)$ . The truncated signal vector is given by

$$\mathbf{s}' = \mathbf{Q}'(\phi) \boldsymbol{\beta}' = [\mathbf{q}_1(\phi) \dots \mathbf{q}_{M'}(\phi)] \boldsymbol{\beta}'. \quad (28)$$

On replacing  $\mathbf{s}$  by  $\mathbf{s}'$  in (13) and then adopting the same procedure as for the subspace detector, we get the following expressions for the conditional MLE of  $\boldsymbol{\beta}'(\phi)$ , the MLEs of  $\phi$  and  $\mathbf{s}'$ , and the test statistic of the truncated subspace detector

$$\hat{\boldsymbol{\beta}}'(\phi) = \mathbf{Q}'^H(\phi) \mathbf{x}, \quad (29)$$

$$\hat{\phi}_{TSD} = \arg \max \{ \mathbf{x}^H \mathbf{Q}'(\phi) \mathbf{Q}'^H(\phi) \mathbf{x} \}, \quad (30)$$

$$\hat{\mathbf{s}}' = \mathbf{Q}'(\hat{\phi}_{TSD}) \mathbf{Q}'^H(\hat{\phi}_{TSD}) \mathbf{x}, \quad (31)$$

$$T_{TSD}(\mathbf{x}) = \sum_{n=1}^{3N} \left[ |x(n)|^\alpha - |x(n) - \mathbf{r}_n'(\hat{\phi}_{TSD}) \mathbf{Q}'^H(\hat{\phi}_{TSD}) \mathbf{x}|^\alpha \right]. \quad (32)$$

where  $\mathbf{r}_n'(\hat{\phi}_{TSD})$  is the  $n^{\text{th}}$  row of  $\mathbf{Q}'(\hat{\phi}_{TSD})$ . It can be shown that the normalized MSE of the estimate  $\hat{\mathbf{s}}'$  is

$$\varepsilon_{TSD} = \frac{E[(\mathbf{s} - \hat{\mathbf{s}}')^H (\mathbf{s} - \hat{\mathbf{s}}')]}{E[\mathbf{s}^H \mathbf{s}]} = \varepsilon_{TSD,1} + \varepsilon_{TSD,2}, \quad (33)$$

$$\varepsilon_{TSD,1} \cong 1 - \frac{E[\mathbf{s}^H \mathbf{s}']}{E[\mathbf{s}^H \mathbf{s}]}, \varepsilon_{TSD,2} \cong \frac{M'}{N\eta}. \quad (34)$$

The normalized MSE  $\varepsilon_{TSD}$  has two components. The first component  $\varepsilon_{TSD,1}$  is due to truncation of the normal mode expansion of the signal vector. The second component  $\varepsilon_{TSD,2}$  is due to noise, and it is analogous to  $\varepsilon_{SD}$  defined in (26). Plots of  $\varepsilon_{TSD}$  versus  $M'$  are shown in Figs. 1 (a), (b) and (c) for SNR = 0, 10 and 20 dB respectively, when a signal of frequency 350 Hz is received by a 10-element AVS HLA. The environmental parameters are as specified at the beginning of Section 4. For this chosen set of parameters,  $M = 15$ . We can draw the following conclusions from (33), (34) and Fig. 1. The noise-induced error component  $\varepsilon_{TSD,2}$  decreases linearly in proportion to  $M'$  as  $M'$  is reduced. The truncation-induced error component increases very slowly as the truncation is increased (i.e. as  $M'$  is reduced) and becomes significant only when  $M'$  is close to 1. This behavior can be attributed to the fact that (i) the modal vectors  $\{\mathbf{a}_1(\phi) \dots \mathbf{a}_M(\phi)\}$  in the expansion of  $\mathbf{s}$  are highly correlated, and (ii) amplitudes  $\{b_{M'+1}, \dots, b_M\}$  of the discarded higher order modes are quite small due to faster attenuation of the higher order modes. The total MSE  $\varepsilon_{TSD}$  is minimum at an optimal value of  $M'$  which is very close to 1. The optimal value of  $M'$  increases very slowly with increasing SNR. It follows that the performance of TSD may be expected to be significantly better than that of SD and that the choice  $M'=1$  is optimal or near-optimal. This prediction is confirmed by the result presented in Section 4. The use of a truncated signal model in TSD also has the additional advantages of (i) reducing the need for channel information to modal wavenumbers of the first few modes only, and (ii) reducing the computational complexity.

### 3.3. Approximate signal form detector

A simple detector called the approximate signal form detector (ASFD) may be formulated [4] by exploiting the fact that the modal wavenumbers  $\{k_m; m = 1, \dots, M\}$  are very close to one another. Using the approximation  $\zeta_m = k_m/k(z_a) \approx 1$  in (7), we get the following approximate expression for the array signal vector

$$\mathbf{s}'' = \mathbf{G}(\phi) \mathbf{p}, \quad \mathbf{G}(\phi) = \mathbf{I}_N \otimes [1 \ \sqrt{2} \cos(\phi) \ \sqrt{2} \sin(\phi)]^T, \quad (35)$$

where  $\mathbf{p} = [p_1 \dots p_N]^T$ , and  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix. On replacing  $\mathbf{s}$  by  $\mathbf{s}''$  in (13) and maximizing the resultant likelihood function  $f(\mathbf{x}; H_1) = f(\mathbf{x}; \mathbf{p}, \phi, H_1)$  with respect to  $\mathbf{p}$  leads to the equation

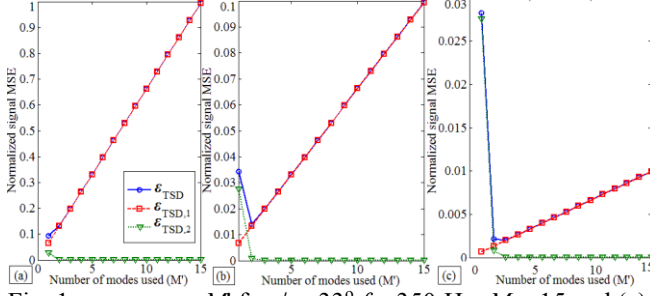


Fig. 1:  $\varepsilon_{TSD}$  versus  $M$  for  $\phi = 33^\circ$ ,  $f = 350$  Hz,  $M = 15$  and (a) SNR = 0 dB, (b) SNR = 10 dB, and (c) SNR = 20 dB.

$$\sum_{n=1}^{3N} |x(n) - \mathbf{g}_n(\phi) \hat{\mathbf{p}}|^{\alpha-2} [\mathbf{g}_n^H(\phi) \mathbf{g}_n(\phi) \hat{\mathbf{p}} - \mathbf{g}_n^H(\phi) x(n)] = \mathbf{0}, \quad (36)$$

where  $\mathbf{g}_n(\phi)$  is a row vector denoting the  $n^{\text{th}}$  row of  $\mathbf{G}(\phi)$ , and  $\hat{\mathbf{p}}$  is the ML estimate of  $\mathbf{p}$ . If  $\alpha = 2$ , the solution of (36) is given by

$$\hat{\mathbf{p}}(\phi) = (\mathbf{G}^H(\phi) \mathbf{G}(\phi))^{-1} \mathbf{G}^H(\phi) \mathbf{x} = \mathbf{G}^H(\phi) \mathbf{x} / 3 \quad (37)$$

No closed form solution of (36) is available when  $\alpha \neq 2$ , but (37) may still be used as an approximation to the conditional MLE. The corresponding MLEs of bearing  $\phi$  and signal vector  $\mathbf{s}$ , and the test statistic of the ASFD are given by

$$\hat{\phi}_{ASFD} = \{\mathbf{x}^H \mathbf{G}(\phi) \mathbf{G}^H(\phi) \mathbf{x}\}, \quad (38)$$

$$\hat{\mathbf{s}}^n = \mathbf{G}(\hat{\phi}_{ASFD}) \mathbf{G}^H(\hat{\phi}_{ASFD}) \mathbf{x} / 3. \quad (39)$$

$$T_{ASFD}(\mathbf{x}) = \sum_{n=1}^{3N} [ |x(n)|^\alpha - |x(n) - \hat{\mathbf{s}}^n(n)|^\alpha ]. \quad (40)$$

The normalized MSE of the signal estimate  $\hat{\mathbf{s}}^n$  is given by

$$\varepsilon_{ASFD} = \frac{E[(\mathbf{s} - \hat{\mathbf{s}}^n)^H (\mathbf{s} - \hat{\mathbf{s}}^n)]}{E[\mathbf{s}^H \mathbf{s}]} = \varepsilon_{ASFD,1} + \varepsilon_{ASFD,2}, \quad (41)$$

$$\varepsilon_{ASFD,1} \cong 1 - \frac{E[\mathbf{s}^n H \mathbf{s}^n]}{E[\mathbf{s}^H \mathbf{s}]}, \quad \varepsilon_{ASFD,2} \cong \frac{1}{\eta}. \quad (42)$$

We note that  $\varepsilon_{ASFD}$  also has two components. The first component  $\varepsilon_{ASFD,1}$ , which is due to incorrect modeling of the signal vector, is very small since the error in the approximation  $\xi_m \approx 1$  is very small. The second component  $\varepsilon_{ASFD,2}$ , which is due to noise, is analogous to  $\varepsilon_{SD}$  and  $\varepsilon_{TSD,2}$  defined in (26) and (34) respectively. The noise-induced error component  $\varepsilon_{ASFD,2}$  is lower than  $\varepsilon_{SD}$  when  $M > N$ . Hence, for a given  $N$ , the ASFD is expected to perform better than the SD at higher frequencies. The TSD is expected to perform better than ASFD at all frequencies since  $\varepsilon_{TSD} < \varepsilon_{ASFD}$ . These predictions are confirmed by the simulation results shown in Section 4. However, the ASFD has the advantage of not requiring any prior information on the modal wavenumbers of the channel.

#### 4. PERFORMANCE ANALYSIS

We have evaluated the performance of the detectors through

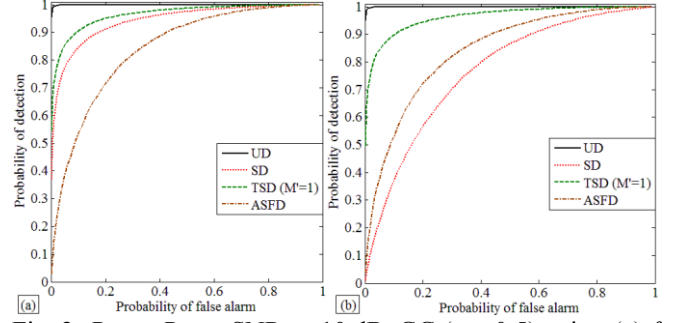


Fig. 2:  $P_D$  vs.  $P_{FA}$  at SNR = -10 dB. GG ( $\alpha = 0.5$ ) noise. (a)  $f = 50$  Hz, (b)  $f = 350$  Hz.

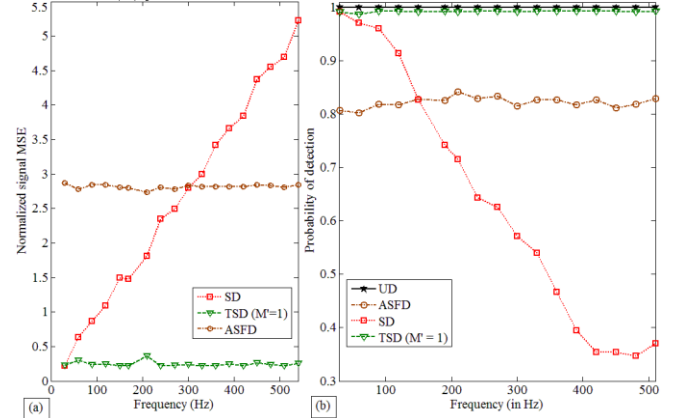


Fig. 3: (a) Normalized signal estimation MSE vs. frequency, and (b)  $P_D$  (at  $P_{FA} = 0.01$ ) vs. frequency for GG ( $\alpha = 0.5$ ) noise with -5 dB SNR.

simulations using a 10-sensor horizontal AVS array with half-wavelength spacing in a Pekeris channel [9] with the following parameters: ocean depth  $h = 70$  m, sound speed in water  $c = 1500$  m/s, bottom sound speed  $c_b = 1700$  m/s, bottom attenuation  $\delta = 0.5$  dB/wavelength, density ratio  $\rho_b/\rho = 1.5$ . The array depth is  $z_a = 40$  m, the source is at a range  $r = 5$  km, depth  $z_s = 40$  m, azimuth  $\phi = 33^\circ$ . The SNR = -10 dB in Fig. 2, and -5 dB in Figs. 3 and 4. In Figs. 2 and 3, the environmental noise is GG ( $\alpha = 0.5$ ) distributed. The probability of false alarm is fixed at  $P_{FA} = 0.01$  in Figs. 3 and 4. TSD results are shown for  $M = 1$ , which is the value that maximizes the probability of detection  $P_D$  of the TSD for the current simulation parameters.

The plots in Fig. 2 show the receiver operating characteristics (variation of  $P_D$  with  $P_{FA}$ ) at an array SNR = -10 dB, for the UD (solid line), SD (dotted line), TSD (dashed line) and ASFD (dot-dashed line) when the signal frequency is (a) 50 Hz (corresponding to  $M = 2$  modes) and (b) 350 Hz ( $M = 15$ ). It can be seen from Fig. 2 that among the realizable detectors considered, the TSD provides the best performance. The ASFD provides a better performance than the SD at the higher frequency, even though the former uses no information about the channel.

In Fig. 3 we study, in greater detail, the variation of performance of the detectors with frequency and the relation between detector performance and signal estimation error.

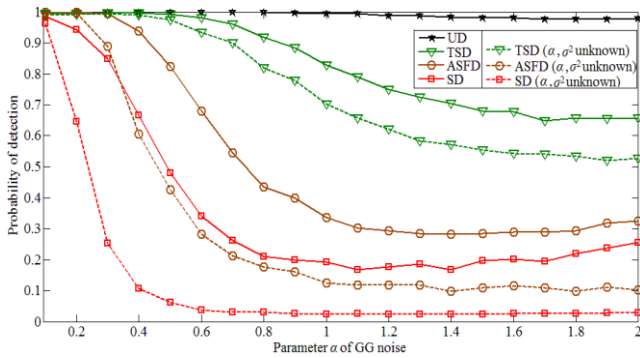


Fig. 4:  $P_D$  vs. parameter  $\alpha$  of GG noise at  $P_{FA} = 0.01$  for  $f = 350$  Hz at SNR = -5 dB.

Fig. 3(a) shows the variation of the signal estimation MSE of the ASFD, SD and TSD methods with frequency, and Fig 3(b) shows the corresponding variation of the  $P_D$  of these detectors. It is seen that there is a very good negative correlation between MSE and  $P_D$  across all detectors and at all frequencies. The errors  $\epsilon_{TSD}$  and  $\epsilon_{ASFD}$  are almost independent of frequency, and  $\epsilon_{TSD} < \epsilon_{ASFD}$ . Accordingly, the detection performance of TSD is uniformly better than that of ASFD. The error  $\epsilon_{SD}$  keeps increasing with frequency due to the increase in the number of modes  $M$ , and consequently the performance of SD keeps degrading as frequency is increased. At very low frequencies the performance of SD is only marginally inferior to that of TSD and better than that of ASFD. At higher frequencies, ASFD performs better than SD.

In Fig. 4 we study the variation of  $P_D$  of the detectors with the parameter  $\alpha$  of the environmental noise. We consider two cases – when the parameters  $\sigma^2$  and  $\alpha$  are known (solid lines) and unknown (dashed lines). When the noise parameters are unknown, detection is done by obtaining ML estimates of the unknown noise parameters. Figure 4 indicates that the performance of all detectors keeps improving as  $\alpha$  is reduced (i.e. as impulsiveness of environmental noise increases). This is so because, for a given SNR, most of the energy of an impulsive noise resides in the outlier values of noise. The detectors are able to effectively reject these impulsive components and thus achieve better performance in more impulsive noise. The difference between the dashed and solid lines represents the degradation in detector performance due to lack of knowledge of the noise parameters. This degradation is lower in the case of the TSD as compared to the ASFD and the SD, showing the greater robustness of TSD. If the noise PDF is estimated in advance using secondary data [4], this degradation can be reduced. Overall, the TSD is shown to be the most effective detection scheme over the whole range of heavy-tailed GG noise PDFs.

## 5. CONCLUSION

This paper proposes two new AVS array-based detection schemes for sources in shallow ocean environments contaminated by impulsive noise, viz. the truncated

subspace detector (TSD) and the approximate signal form detector (ASFD). The detectors involve estimation of the array signal vector which is unknown due to the unknown location of the source. The detection performance is shown to be related to the normalized mean square signal estimation error (MSE) of the associated GLRT. Thus the MSE provides a simple indicator to gauge the effectiveness of a detector without using Monte Carlo simulations. The subspace detector (SD) degrades in performance with increasing frequency, due to an associated increase in the MSE. The TSD and ASFD methods are formulated to obtain improved detection performance by reducing the signal estimation MSE. The TSD is shown to be the most effective and robust detection scheme for a wide range of noise PDFs, if the modal wavenumbers of the ocean channel are known. If this information is unknown, however, detection may be done using the ASFD. As the noise becomes more impulsive, the performance of the detectors improves because they are able to discard more effectively the outlier values arising from noise.

## 6. ACKNOWLEDGMENT:

This work was partly supported by a grant from National Institute of Ocean Technology, Chennai, India, under the Ocean Acoustics Programme.

## REFERENCES

- [1] S. M. Kay, *Fundamentals of Statistical Signal Processing, Vol.II: Detection Theory*, Prentice-Hall, Upper Saddle River, New Jersey, 1998.
- [2] S. Kraut, L. L. Scharf, L. T. McWhorter, "Adaptive subspace detectors," *IEEE Transactions on Signal Processing*, vol. 49, no. 1, pp. 1-16, 2001.
- [3] K. M. Krishna, G. V. Anand, "Narrowband detection of acoustic source in shallow ocean using vector sensor array," *Proc. OCEANS 2009, MTS/IEEE Biloxi*, pp.1-8, Oct. 2009.
- [4] V. N. Hari, G. V. Anand, A. B. Premkumar, A. S. Madhukumar, "Underwater Signal Detection in Partially known Ocean using Short Acoustic Vector Sensor Array," *Proc. OCEANS 2011, Santander*, pp. 1-9, June 2011.
- [5] F.W.Machell, C.S.Penrod, G.E.Ellis, "Statistical characteristics of ocean acoustic noise process", in: E. J. Wegman, S. C. Schwartz, J. B. Thomas (Eds.), *Topics in Non-Gaussian Signal Processing*, Springer, Berlin pp. 29-57, 1989.
- [6] N. B. Pulsone, R. S. Raghavan, "Analysis of an Adaptive CFAR Detector in Non-Gaussian Interference," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no. 3, pp. 903-916, 1999.
- [7] M. N. Desai and R. S. Mangoubi, "Robust Gaussian and non-Gaussian matched subspace detection," *IEEE Transactions on Signal Processing*, vol. 51, no. 12, pp. 3115-3127, 2003
- [8] M. Hawkes and A. Nehorai, "Acoustic vector-sensor beamforming and Capon direction estimation," *IEEE Transactions on Signal Processing*, vol. 46, no. 9, pp. 2291-2304, 1998.
- [9] K. G. Nagananda, G. V. Anand, "Subspace intersection method of high resolution bearing estimation in shallow ocean using acoustic vector sensors," *Signal Processing*, vol. 90, no. 1, pp.105-118, 2010.