RFI SUBSPACE ESTIMATION TECHNIQUES FOR NEW GENERATION RADIO TELESCOPES

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ABSTRACT

Radio astronomical data are increasingly corrupted by human telecommunication activities. Therefore, Radio Frequency Interferences (RFI) mitigation becomes an important step in the data processing flow, in particular for phased radio telescope array. In this framework, the preliminary step is to retrieve the RFI spatial information. This article presents three new techniques in radio astronomy allowing the estimation of the RFI spatial signature. These techniques are based on subspace decomposition of time-lagged correlation matrices or cyclic correlation matrices, and on a Blind Source Separation approach. Compared to classical methods, these approaches improve the quality of the spatial filtering obtained on the raw uncalibrated sky map.

***Index Terms*—** Radio astronomy, RFI mitigation, Spatial signature vector estimation, Correlation matrix, Cyclostationarity, Alternating Least Squares

1. INTRODUCTION

Radio astronomy studies cosmic objects through the radio waves they emit. New generation radio telescopes, such as the LOw Frequency ARray radio telescope (LOFAR[1]), currently in operation in the Netherlands (see Figure 1), consist in a high number of fixed omnidirectional radiating elements. These antennas are phased together to provide agile multibeam capabilities.

The extreme sensitivity of such systems exposes them to various and growing human telecommunication activities (positionning systems, mobile phone, audio and video broadcasts,...). Even if some frequency bands are protected for radio astronomy, astronomers are interested in exploiting the whole spectrum. In consequence, RFI mitigation is a major issue in recent and future development in radio astronomy. For more details, an extensive overview can be found in [2].

The authors would like to thank the European Commission Framework Program 7 (Project PrepSKA, contract no. 212243), the French funding agency ANR (contract ANR-09-BLAN-0225-04) and the Région Centre for funding part of this work.

Fig. 1. LOFAR superterp. Core of the radio telescope located in Exloo, the Netherlands. LOFAR is composed of 4224 low band antennas (LBA, 30-80MHz) and 2496 high band antenna tiles (HBA, 120-240MHz), distributed over all Europe.

The spatial diversity provided by phased arrays can be used to derive some spatial filtering techniques such as the ones presented in [3, 4]. These approaches are based on a relatively simple model where cosmic sources can be neglected compared to system noise and RFI levels. Moreover, the array is supposed calibrated. In general, typical steps for applying interferences suppression in radio interferometry are [5]:

- Estimating RFI spatial signature vectors
- Nulling the power coming from RFI sources directions

In this paper, we propose and compare three new RFI spatial signature vector estimation approaches for RFI mitigation in radio astronomy. We particularly focus on an extended model which is uncalibrated and contains cosmic sources.

In section 2, we define our data model. Section 3 presents the first technique, based on time-lagged correlation matrices. Section 4 details a technique based on RFI cyclostationary properties. The last technique, based on an Alternating Least Squares approach, is explained in section 5. Section 6 provides comparisons between simulation results. The paper is concluded in section 7.

2. DATA MODEL

We consider a phased antenna array made up of M antennas. $\mathbf{z}(t)$ is the $(M \times 1)$ output data vector of this array. The field of view of this radio telescope contains k_r interferences and k_s cosmic sources. All of these sources are assumed to fulfill the narrow band assumption. Geometrical delays between each antenna and each source can therefore be represented by phase shifts.

The estimated phased antenna array correlation matrix is then expressed by:

$$\begin{aligned}\mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau) &= \left\langle \mathbf{z}(t)\mathbf{z}^H(t-\tau)\right\rangle_T \\ &= \mathbf{G}(\mathbf{A_r}\mathbf{R_r}(\tau)\mathbf{A_r}^H + \mathbf{A_s}\mathbf{R_s}(\tau)\mathbf{A_s}^H)\mathbf{G}^H + \mathbf{N}(\tau)\end{aligned} \quad (1)$$

with $\langle . \rangle_T$ the time averaging operator, T the integration time and $(.)^H$ the conjugate transpose operator. $\mathbf{A_r}$ is the $(M \times k_r)$ matrix containing the k_r normalized spatial signature vectors of the k_r RFI. $\mathbf{R_r}(\tau)$ is the $(k_r \times k_r)$ RFI correlation matrix. $\mathbf{A_s}$ is the $(M \times k_s)$ matrix containing the k_s normalized spatial signature vectors of the k_s cosmic sources. $\mathbf{R_s}(\tau)$ is the $(k_s \times k_s)$ cosmic sources correlation matrix. These cosmic sources are considered Gaussian, centered, white and stationary [6]. $\mathbf{N}(\tau)$ is the $(M \times M)$ system noise correlation matrix, also with Gaussian entries. $\mathbf{G}$ is a diagonal complex gain matrix corresponding to unknown variations in the receiver chain. To reduce notation complexity, this matrix gain will be merged with the spatial signatures in the rest of the paper.

The cosmic sources, interferences and system noise, are supposed to be uncorrelated to each other. Thus, $\mathbf{R_r}(\tau)$, $\mathbf{R_s}(\tau)$ and $\mathbf{N}(\tau)$ are diagonal matrices.

The purpose of this article is the estimation of $\mathbf{A_r}$. Classic approaches, such as the ones mentioned in the introduction, are based on subspace decomposition through a Singular Value Decomposition (SVD) of $\mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau = 0)$. Applied to our model (equation 1), a subspace decomposition would lead to a biased estimation of $\mathbf{A_r}$ due to cosmic sources and uncalibrated system noise contributions (i.e. $\mathbf{R_s}(\tau) \neq \mathbf{0}$ and $\mathbf{N}(\tau) \neq \sigma_n^2\mathbf{I}$, with $\mathbf{0}$ the null matrix, $\mathbf{I}$ the identity matrix and σ_n^2 the system noise power).

The two first proposed approaches are also based on RFI subspace estimation (SE), but with a prior reduction of cosmic sources and system noise influences. This can be done by considering $\mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau)$ at other time-lags than $\tau = 0$ (section 3) or by introducing some RFI cyclostionary properties (section 4). The third method exploits a Blind Source Separation algorithm to get an individual estimation of all the RFI spatial signature vectors.

3. TIME-LAG APPROACH

In radio astronomy, we usually consider that cosmic sources and system noise are almost white by nature [6]. Since cosmic sources are white, there is a short time-lag τ_0 such as their autocorrelation is asymptotically null for $\tau \geq \tau_0$. We therefore have, for $\tau \geq \tau_0$:

$$\begin{aligned}\mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau) &= \left\langle \mathbf{z}(t)\mathbf{z}^H(t-\tau)\right\rangle_\infty \\ &= \mathbf{A_r}\mathbf{R_r}(\tau)\mathbf{A_r}^H + \underbrace{\mathbf{A_s}\mathbf{R_s}(\tau)\mathbf{A_s}^H}_{\to 0} + \underbrace{\mathbf{N}(\tau)}_{\to 0} \\ &\cong \mathbf{A_r}\mathbf{R_r}(\tau)\mathbf{A_r}^H\end{aligned} \quad (2)$$

Applying a SVD to the observations correlation matrix calculated for a time-lag longer than τ_0, we obtain the following decomposition:

$$\mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau) = \mathbf{U}\mathbf{S}\mathbf{V}^H \quad (3)$$

with $\mathbf{U}$ and $\mathbf{V}$ orthogonal matrices, and $\mathbf{S}$ a real diagonal matrix.

By identifying equation 3 with equation 2, the k_r singular vectors of $\mathbf{U}$ corresponding to the k_r dominant singular values (theoretically the k_r non-null singular values) of $\mathbf{S}$ are spanning the same subspace as $\mathbf{A_r}$. The submatrix composed of these vectors is therefore an estimate of the subspace generated by $\mathbf{A_r}$.

This estimation can also be improved by stacking several $\mathbf{R}_{z,z^*}(\tau_i)$, with $i = 1, \ldots, N$. The SVD is then applied on the $M \times MN$ resulting correlation matrix.

Figure 2 shows simulations with synthetic data ($M = 48$ antennas, $k_s = 3$ cosmic sources, $k_r = 3$ RFI). The Interference to Noise Ratio (INR) is -6 dB. In consequence, the RFI are barely visible (see Figure 2.a). The expected RFI subspace is represented at Figure 2.b). The RFI subspace is estimated with the proposed approach by using respectively $N = 1$ and $N = 9$ different time-lags. Figure 2.c) and d) show the RFI SE error relatively to the expected RFI subspace (i.e. difference between estimated $\mathbf{A_r}.\mathbf{A_r}^H$ and true $\mathbf{A_r}.\mathbf{A_r}^H$). In both simulations, the RFI subspace can be retrieved but the stacked approach provides smaller SE errors in the sky map.

4. CYCLOSTATIONARITY APPROACH

Most of the RFI are by nature not stationary. However, most of them present a hidden periodicity due to the periodic characteristics involved in their construction (carrier wave, baud rate, coding scheme...). These parameters are usually scrambled and hidden by the randomness of the message to be transmitted.

An exhaustive overview of cyclostationarity theory and applications can be found in [7, 8].

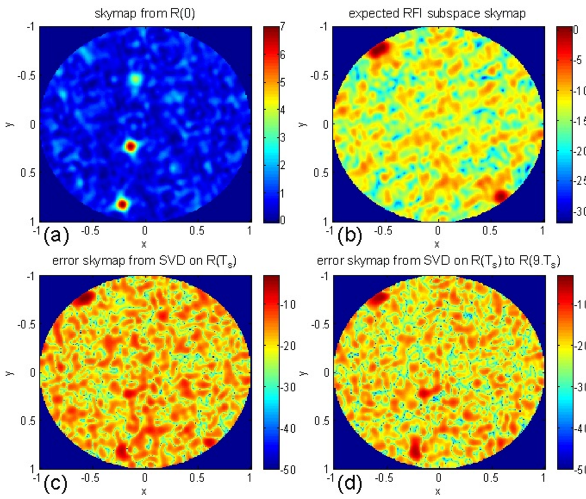

Fig. 2. Time-lagged approach. x and y refer to spatial positions on the celetial sphere. all figures are in dB. (a) Original raw sky map with $k_s = 3$ cosmic sources and $k_r = 3$ RFI. The array is a LOFAR LBA like array with $M = 48$ antennas, and and integration over 8192 samples. The RFI are Binary Phase Shifted Keying (BPSK) modulations. Since the INR = -6dB, only the 3 cosmic sources are visible. (b) Expected RFI subspace sky map to indicate the RFI positions. (c) Sky map of RFI SE error obtained from the SVD of $\mathbf{R}_{z,z^*}(Ts)$ where T_s is the sampling period. (d) Sky map of RFI SE error obtained from the SVD of 9 stacked time-lagged correlation matrices, $\mathbf{R}_{z,z^*}(T_s)$ to $\mathbf{R}_{z,z^*}(9.T_s)$.

Consider a cyclostationary interference impinging a radio telescope. For some specific cyclic frequencies, α, the following expression is non-zero:

$$\mathbf{R}^{\alpha}_{\mathbf{z},\mathbf{z}^{(*)}}(\tau) = \left\langle \mathbf{z}(t)\mathbf{z}^{(*)T}(t-\tau)e^{-j2\pi\alpha t} \right\rangle_{\infty} \quad (4)$$

with $(.)^*$ and $(.)^T$ standing for the conjugate operator and the transpose operator respectively.

$\mathbf{R}^{\alpha}_{\mathbf{z},\mathbf{z}^*}(\tau)$ is called the cyclic correlation matrix, and $\mathbf{R}^{\alpha}_{\mathbf{z},\mathbf{z}}(\tau)$ is called the conjugated cyclic correlation matrix. Cyclic frequencies can be estimated through procedures like the one proposed in [5].

The main advantage of the cyclostationary approach is the asymptotical independence of contributions of non-cyclostationary signals (i.e. cosmic sources and system noise):

$$\mathbf{R}^{\alpha}_{\mathbf{z},\mathbf{z}^{(*)}}(\tau) = \mathbf{A_r}\mathbf{R}^{\alpha,(*)}_{\mathbf{r}}(\tau)\mathbf{A_r}^{(*)T} + \underbrace{\mathbf{A_s}\mathbf{R}^{\alpha,(*)}_{\mathbf{s}}(\tau)\mathbf{A_s}^{(*)T}}_{\to\mathbf{0}} + \quad (5)$$

$$\underbrace{\mathbf{N}^{\alpha,(*)}(\tau)}_{\to\mathbf{0}}$$

$$\cong \mathbf{A_r}\mathbf{R}^{\alpha,(*)}_{\mathbf{r}}(\tau)\mathbf{A_r}^{(*)T} \quad (6)$$

Again, this approach allows us to get an estimate of $\mathbf{A_r}$ using a SVD as explained in section 3, equation 3. But this time, a specific SVD must be applied for each RFI. This constraint provides an access to each individual spatial signature. If available, a set of cyclic and conjugated cyclic correlation matrices can also be stacked to enhance the estimation of each RFI subspace.

Figure 3 shows simulations with synthetic data similar to the previous section. To get sufficient cyclostationary information, the INR has been set to 0dB. The RFI positions have also changed (see Figure 3.a). The capacity of the algorithm to extract one specific RFI subspace is shown on Figure 3.b). A cyclic frequency corresponding to the selected RFI is chosen (in our case, 2 times the carrier frequency). The corresponding conjugated cyclic correlation matrix is calculated. Then, the SVD provides a spatial signature estimation of only the selected RFI. To define the whole RFI subspace, the previous procedure is applied 3 times for 3 different cyclic frequencies. Figure 3.d) shows the RFI SE error relatively to the expected RFI subspace. For comparison, the time-lag approach with $N = 1$ is given at Figure 3.c). With this INR, the cyclic approach provides larger SE error than the time-lag approach but the counterpart is the capability to extract each individual RFI.

The next section describes an approach which provides this capability as well but by only using the time-lagged correlation matrices.

5. ALTERNATING LEAST SQUARES APPROACH

The purpose of this section is to use all statistical information included in the time-lagged correlation matrices. To that aim, we propose to use an iterative Alternating Least Squares (ALS) algorithm that have been proposed, for example, in [9]. The idea can be expressed with defining a set of correlation matrices for N time-lags, using equation 2:

$$\begin{cases} \mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau_1) & \cong \mathbf{A_r}\mathbf{R_r}(\tau_1)\mathbf{A_r}^H \\ & \vdots \\ \mathbf{R}_{\mathbf{z},\mathbf{z}^*}(\tau_N) & \cong \mathbf{A_r}\mathbf{R_r}(\tau_N)\mathbf{A_r}^H \end{cases} \quad (7)$$

Given this set, it is a well-known joint diagonalization problem. The goal is to recursively estimate $\mathbf{A_r}$ and $\mathbf{R_r}(\tau_i)$. Following [10], we now briefly describe the algorithm.

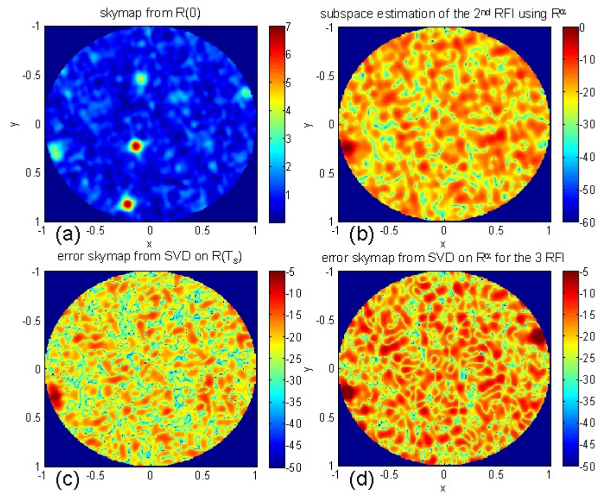

Fig. 3. Cyclostationary approach. (a) Original raw sky map with the same parameters as for Figure 2 except thant INR = 0dB and RFI positions have changed. Both RFI and cosmic sources are now visible. (b) Estimated subspace sky map of 1 RFI obtained by selectionning the right conjugated cyclic frequency corresponding to this RFI (here, 2 times the carrier frequency). (c) For comparison, sky map of RFI SE error obtained from the SVD of $\mathbf{R}_{z,z^*}(Ts)$. (d) Sky map of RFI SE error obtained from 3 consecutive SVD of conjugated cyclic correlation matrix corresponding to each RFI.

5.1. Update of $\mathbf{R_r}(\tau_i)$

The idea is to stack all the columns of the matrices defined in (7). Adapted to our problem, it leads to:

$$\begin{aligned} \mathbf{r_z}(\tau_i) &\triangleq [\mathsf{vec}(\mathbf{R_{z,z^*}}(\tau_1)), \ldots, \mathsf{vec}(\mathbf{R_{z,z^*}}(\tau_N))] \\ \mathbf{dR_r}(\tau_i) &\triangleq [\mathsf{diag}(\mathbf{R_r}(\tau_1)), \ldots, \mathsf{diag}(\mathbf{R_r}(\tau_N))] \\ \mathbf{r_z}(\tau_i) &= (\mathbf{A_r}^* \odot \mathbf{A_r})\, \mathbf{dR_r}(\tau_i) \end{aligned} \quad (8)$$

where $\mathsf{vec}(.)$ denotes the stacking of the columns of a matrix in a vector, $\mathsf{diag}(.)$ is a vector containing all the diagonal terms of the matrix in argument, and $\odot$ is the Khatri-Rao product.

We can deduce the set $\mathbf{R_r}(\tau_i)$ from (8):

$$\mathbf{R_r}(\tau_i) \leftarrow \mathsf{undiag}\left((\mathbf{A_r}^* \odot \mathbf{A_r})^\dagger \mathbf{r_z}(\tau_i)\right) \quad (9)$$

where $\mathsf{undiag}(.)$ reconstructs a set of N diagonal matrices from the matrix in argument, and $\dagger$ is the Moore-Penrose pseudo inverse operator.

5.2. Update of $\mathbf{A_r}$

While all the $\mathbf{R_r}(\tau_i)$ have been estimated, the next step is to estimate $\mathbf{A_r}$. By concatenating horizontally all the matrices of the set, we notice that $\mathbf{A_r}$ is postmultiplied by the concatenation of two matrices. It can be written as:

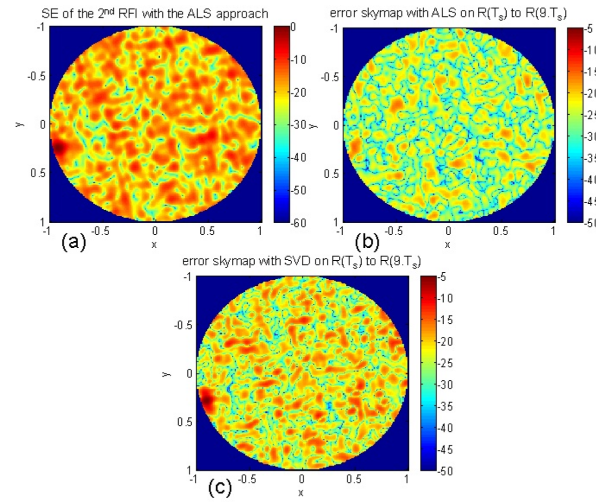

Fig. 4. ALS Approach on a set of $N = 9$ time-lagged correlation matrices. (a) Estimated subspace sky map of 1 RFI obtained by ALS approach. (b) Sky map of RFI SE error obtained with the ALS approach.(c)For comparison, sky map of RFI SE error obtained from the SVD of the same set of matrices.

$$\begin{aligned} \mathbf{R_z} &= \mathbf{A_r}[\mathbf{R_r}(\tau_1)\mathbf{A_r}^H, \ldots, \mathbf{R_r}(\tau_N)\mathbf{A_r}^H] \quad (10) \\ \mathbf{R_z} &\triangleq [\mathbf{R_z}(\tau_1), \ldots, \mathbf{R_z}(\tau_N)] \end{aligned}$$

Finally, we easily find $\mathbf{A_r}$:

$$\mathbf{A_r} \leftarrow \mathbf{R_z}(\tau_i)[\mathbf{R_r}(\tau_1)\mathbf{A_r}^H, \ldots, \mathbf{R_r}(\tau_N)\mathbf{A_r}^H]^\dagger \quad (11)$$

These steps are repeated recursively while reaching convergence.

Figure 4 shows simulations with synthetic data similar to the previous section, but $N = 9$ time-lags are stacked this time. The capacity of the algorithm to extract one specific RFI subspace is shown at Figure 4.a. Figure 4.b shows the RFI SE error relatively to the expected whole RFI subspace. For comparison, the RFI SE error for time-lag approach with the same set of $N = 9$ matrices is given at Figure 4.c. The ALS approach provides both smaller error and indivual RFI extraction capability compared to the time-lag approach. An extensive Monte-Carlo simulation is proposed in the next section.

6. SIMULATIONS

Proposed methods are compared through Monte-Carlo based simulations. The data model used in these simulations is the one described in equation 1, with $k_r = 1$ RFI and $k_s = 3$ cosmic sources. Parameters of these simulations are:

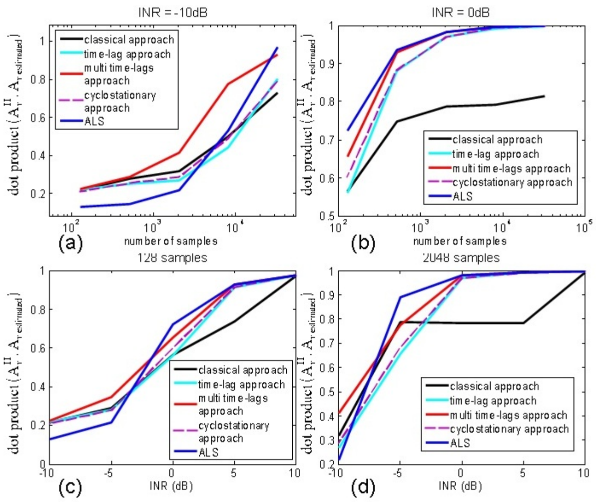

Fig. 5. Simulation results. (a) Fixed INR = -10dB, performances of methods depending on the number of samples over which correlations matrices have been evaluated. (b) Fixed INR = 0dB, performances depending on the number of samples. (c) Fixed number of samples = 128 samples, performances depending on the INR. (d) Fixed number of samples = 2048 samples, performances depending on the INR.

- the number of samples used to estimate the different correlation matrices : from 128 samples to 32768 samples.
- the INR : from -10dB to $+10$dB.
- the calibration, modeled as the noise power fluctuation over antennas : from 0% to 20%.

The spatial signature vector estimation accuracy is measured with the normalized dot product between the estimated vector and the true one. Results have been averaged over 100 iterations for each parameters configuration. As a preliminary result, we noticed an invariance of the methods results with the array calibration.

Figure 5 shows results of these simulations depending on the INR and on the number of samples. All the presented methods show better performances than the classic method. The cyclostationary approach shows similar performances than the time-lag approach. However, the stacking of correlation matrices calculated for different time-lags improves the time-lag approach (referred as multi time-lags approach on Figure 5). The ALS remains the best method in term of estimation of the RFI spatial signature vector.

7. CONCLUSION

Different approaches allowing RFI spatial signature vectors estimation in radio astronomy have been presented. These methods rely on the correlation matrix of a phased antenna array radio telescope. They are based on time-lagged correlation matrices, RFI cyclostationary characteristics, and on an Alternating Least Squares algorithm.

Performances of these methods are evaluated through simulations, compared to a classical approach. The ALS technique, despite its high computational complexity, presents for now the best performances for not too low INR. A next step of this work would be to apply these algorithms onto real data.

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