CARDIOVASCULAR SIGNAL RECONSTRUCTION BASED ON SHAPE MODELLING AND NON-STATIONARY TEMPORAL MODELLING

D. Martín-Martínez, P. Casaseca-de-la-Higuera, M. Martín-Fernández and C. Alberola-López*

Laboratory of Image Processing, E.T.S.I. Telecomunicación, University of Valladolid, Spain

ABSTRACT

Physiological signals, specially those related to cardiovascular function, are usually corrupted due to the number of degradation sources appearing in the acquisition process (noise, movements, etc.). If the power of these artifacts is close to the power of the signal, they cannot be removed and the affected epoch must be set aside. In this paper, we propose a novel methodology for reconstructing corrupted pieces based on signal modelling. The method consists of two stages: 1) estimation of the model parameters from the largest uncorrupted signal and 2) simulation of the model to achieve a new piece able to replace the corrupted one. Results on real data show that reconstructed pieces are valid in terms of statistical similarity, yielding anomaly-free realizations of the stochastic process modelling the acquired signal.

Index Terms— Cardiovascular Signal, Reconstruction, Shape modelling, PCA, Temporal modelling, ARMA Models.

1. INTRODUCTION

The analysis of cardiovascular signals, such as ECG (electrocardiography) and PPG (photoplethysmography), was one of the first areas in medicine where signal processing was applied for diagnostic aid and follow-up [1]. Nowadays, the contribution of the signal processing field to medicine is highly appreciated and interpretation of cardiovascular signals is rarely based on the own signal, but on either magnitudes or new signals resulting from processing (filtering, delineation, spectral analysis, etc.) the original. Although literature holds a wide variety of methods for processing these signals, there still exist some unsolved problems such as the reconstruction of pieces corrupted with high power noise (due to patient's movements, for instance). These degenerated pieces are ignored so far, which implies working with (short) epochs of the signal, instead of the whole acquisition.

The use of theoretical models allows for the achievement of synthetic signals whose features are similar to those of the one used to estimate the model. This methodology is frequently applied to predict the behavior of stocks from economic and financial series [2]. In this context, quite a few theoretical models —AR (autoregressive), ARMA (autoregressive moving average), RSM (regime switch models), etc. [3]— are proposed to this end. Nevertheless, these models should not be directly applied to physiological signals since the premises assumed for financial series (stationarity, etc) can be hardly complied by physiological signals.

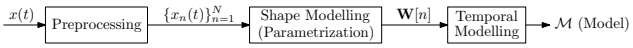

Fig. 1. Block diagram for the methodology proposed to bring theoretical models of any cardiovascular signal $x(t)$.

In this paper, we propose a novel model-based methodology for cardiovascular signal (ECG and PPG) reconstruction. Signals are first divided in heart beat epochs. The shape of each piece is modeled by a parametric curve whose parameters (with non stationary evolution) are considered as univariate ARMA processes after a proper application of PCA (principal component analysis). The evolution model is estimated from the uncorrupted largest piece of the original signal, and used, together with the initial and the boundary conditions provided by the uncorrupted segments, to synthesize a new piece to replace the corrupted one.

The paper is organized as follows: section 2 presents the theoretical contents, with specific sections describing the signal analysis (where the modelling methodology is described) and synthesis. Section 3 summarizes the experimental evaluation of the method on a real case along with a discussion on the obtained results. Finally, section 4 closes the paper by gathering the main conclusions obtained from the developed study.

2. METHODS

Methodology proposed for dealing with the main objective of the study (recreate a piece of a signal that has been corrupted) consists of two stages: the estimation of the theoretical model from an uncorrupted piece of signal, and the simulation of it to take the place of the corrupted piece. From now on, the first stage will be referred to as "analysis", while "reconstruction" will be used to make reference to the second one.

2.1. Analysis

Aiming at a better comprehension of the methods presented, this section is split into two parts: section 2.1.1 gives a deep description of the model, while section 2.1.2 is devoted to the estimation of the model from the available uncorrupted data.

2.1.1. Description of the Theoretical Model

Overall, three stages can be identified (see Fig. 1): preprocessing, shape modelling and temporal modelling. The first stage divides the original signal into a collection of pieces (one per beat), the second one represents every beat by means of a parametric curve,

This work was partially supported by the Ministerio de Ciencia e Innovación and the Fondo Europeo de Desarrollo Regional (FEDER) under Research Grant TEC2010–17982, by the Instituto de Salud Carlos III under the Grant PI11–01492 and by the Centro para el Desarrollo Tecnológico Industrial (CDTI) under the cvREMOD (CEN–20091044) project. The work was also funded by the Junta de Castilla y León under Grants SAN126/VA032/09, SAN126/VA033/09, VA039A10–2, GRS474/A/10 and VA376A11–2.
Asterisk indicates the corresponding author: dmarmar@lpi.tel.uva.es

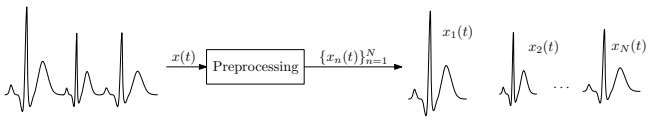

Fig. 2. Preprocessing task over an ECG signal.

and the last one models the temporal evolution of the parameters resulting from the second stage. All these stages are described in what follows.

Preprocessing. One common feature of all cardiovascular signals is their quasi-periodic behavior due to the action of the heart. On the basis of this property, these signals can be divided in pieces, each one corresponding to a beat. This is what the preprocessing stage does: given a signal $x(t)$, a collection of N pieces $\{x_n(t)\}_{n=1}^N$, where n is the ordinal of the corresponding beat, is returned. Fig. 2 shows how pieces corresponding to each beat are separated. To this aim, we do not specify any particular method; whichever delineation procedure could be used for each signal (ECG and PPG) indeed. Additionally, tasks such as filtering and detrending could be also carried out at this stage if necessary.

Shape modelling. This stage can be also termed as "parametrization", since the elements of the collection resulting from the preprocessing are modeled by means of a parametric curve. That is, for $1 \leq n \leq N$,

$$x_n(t) = \Gamma(\mathbf{W}[n], t) + r(t),\ t \in \mathcal{T}, \tag{1}$$

where $\Gamma(\cdot\,, t)$ is a parametric curve, $\mathbf{W}[n] \in \mathbb{R}^M$ is the vector of parameters for the $n-$th beat, $r(t)$ is the residue (which will be obviated from now on), and $\mathcal{T}$ is the domain of $x_n(t)$. Both $\Gamma(\cdot\,, t)$ and $\mathbf{W}[n]$ do depend on the signal nature; here, two examples (ECG and PPG) are presented and summarized in Table 1. The goodness of fit of the proposed shape models can be qualitatively evaluated in Fig. 3-(c), (d), where real examples are drawn. The model proposed for PPG signals is a simplification of the one proposed in [4, 5] for modelling ECG signals, in which five Gaussian curves are considered (one per wave). Despite this model was conceived to model ECG signals, we have not used it, but the Hermite's interpolation to deal with this sort of signals. The rationale of that decision lies on the fact that Gaussian curves cannot cope with the asymmetries of the complexes forming each beat.

After a careful reading of Table 1, one aspect should be clarified: why does not the ECG shape model make use of time-warping? The rationale of this decision lies on the accuracy of the warping landmarks, which is higher for PPG than for ECG. Little errors in this step become more critical if warping transformation is carried out.

Temporal modelling. To find out a model of the temporal evolution for the vector of parameters, two features of $\mathbf{W}[n]$, which prevent from using typical models like those described in [3], must be taken into account: 1) $\mathbf{W}[n]$ can be non stationary and 2) its elements can be dependent. Bearing in mind these characteristics, the model proposed here, whose block diagram is presented in Fig. 4, consists of two stages:

1) Decorrelation: The purpose of this stage is to provide a new vector, $\mathbf{Y}[n]$, whose components would be independent in order to be individually modeled later on. To this end, the PCA (Principal Component Analysis) transformation [7] has been used. Hence, $\mathbf{Y}[n] = H \cdot \mathbf{W}[n]$, where the $H \in \mathcal{M}_{M\times M}$ matrix was estimated over the whole $\mathbf{W}[n]$. This approach finds its methodological justification on the hypothesis that the structure of the covariance matrix of $\mathbf{W}[n]$ keeps constant in time.

2) 1-D Temporal modelling: The elements of $\mathbf{Y}[n]$, which are non stationary, are to be modeled at this stage. As literature states, physiological signals in general and cardiovascular ones in particular are modulated by the ANS (Autonomous Nervous System) [1]; hence, variabilities introduced by the sympathetic (long-term variability) and parasympathetic (short-term variability) systems result in non stationary signals. Striving to model stationary series, these variabilities have been separated in two stationary series through a low pass filter (30 samples moving average) —$h_{MA(30)}[n]$— as follows[1]:

$$\mathbf{Y}_{j,T}[n] = \mathbf{Y}_j[n] * h_{MA(30)}[n] \quad \text{(Trend)}, \tag{3}$$
$$\mathbf{Y}_{j,R}[n] = \mathbf{Y}_j[n] - \mathbf{Y}_{j,T}[n] \quad \text{(Residue)}, \tag{4}$$

where $j = 1, \cdots, M$ denotes the component of the vector series. Hereafter, each element of $\mathbf{Y}_{j,T}[n]$ and $\mathbf{Y}_{j,R}[n]$ can be

	ECG
$\mathbf{W}[n]$	Time and amplitude labels resulting from the complete delineation of the ECG —onset, peak and end of all the waves (P, Q, R, S and T)—. Additionally, one more label has been located in the middle of every slope. Temporal labels are relatives to the time of the R peak, whilst this one is relative to the time of the previous R peak. These labels are marked in Fig. 3-(a) as ℓ_k.
$\Gamma(\cdot\,, t)$	Piecewise Hermite's interpolation [6] over the ℓ_k labels.

	PPG
$\mathbf{W}[n]$	t_1 and t_2 parameters are the length of the intervals onset-maximum and maximum-end respectively. m_0 and m_1 parameters are devoted to model the linear trend defined by the first and the last point of each beat. a_1, b_1, c_1, a_2, b_2 and c_2 parameters define two Gaussian curves. All these parameters can be seen in Fig. 3-(b).
$\Gamma(\cdot\,, t)$	This curve is formed by a sum of two Gaussian curves and a linear trend, $\gamma(t) = a_1 \exp\left[-\frac{(t-b_1)^2}{c_1^2}\right] + a_2 \exp\left[-\frac{(t-b_2)^2}{c_2^2}\right] + m_0 + m_1 t$, (2) which is time-warped in such a way that onset-maximum and maximum-end durations would take always their average value (computed over all the beats): η_{t_1} and η_{t_2}.

Table 1. Shape modelling indications for both sorts of cardiovascular signals.

[1] This residue should not be confused with the residue of the equation (1), denoted as $r(t)$.

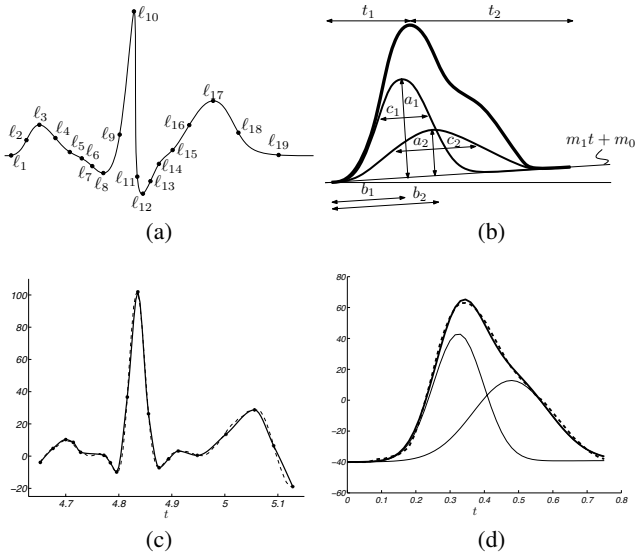

Fig. 3. Schemes for the parametrization of ECG (a) and PPG (b) pieces of one beat. Real examples of the shape model fitting for ECG (c) and PPG (d) pieces of one beat. Continuous line is the given by the model. Dashed line represents the original signal.

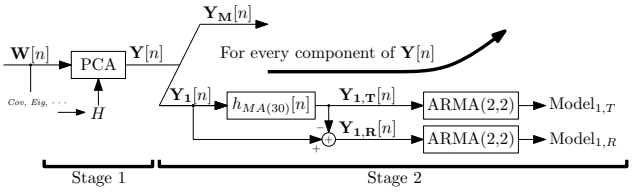

Fig. 4. Block diagram of the temporal model. Two stages are identified.

represented by means of ARMA models, whose expression for any signal $s[n]$ is

$$s[n] = \sum_{i=1}^{p} a_i s[n-i] + \epsilon[n] + \sum_{k=1}^{q} b_k \epsilon[n+k], \quad (5)$$

where a_i and b_k are the coefficients of the model, and $\epsilon[n]$ is the exciting noise —white Gaussian noise $\mathcal{N}(0, \sigma_\epsilon^2)$—. The order of the model has been fixed to $(p, q) = (2, 2)$ through some experiments based on spectra similarity assessment.

2.1.2. Estimation of the Model from the Corrupted Signal

Given a signal $x(t)$, acquired for a temporal interval $[0, \tau]$, a corrupted piece is visually detected from t_o to t_e. So, signal can be split in three pieces:

$$x_{pre}(t) = x(t),\ 0 \le t < t_o, \quad (6)$$
$$x_{corr}(t) = x(t),\ t_o \le t \le t_e, \quad (7)$$
$$x_{post}(t) = x(t),\ t_e < t \le \tau. \quad (8)$$

To estimate the model, the following steps must be followed:

1. The largest uncorrupted piece of $x(t)$ must be used as reference to estimate the model. That is, $x_{ref}(t) = x_{pre}(t)$ if $t_o > \tau - t_e$; $x_{ref}(t) = x_{post}(t)$, otherwise.
2. $x_{ref}(t)$ is preprocessed to achieve $\{x_n^{ref}(t)\}_{n=1}^N$.
3. Vector series $\mathbf{W}^{ref}[n],\ n = 1, \cdots, N$ must be computed from $\{x_n^{ref}(t)\}_{n=1}^N$ according to the shape models proposed in the previous section.
4. Parameters of the theoretical model are estimated from $\mathbf{W}^{ref}[n],\ n = 1, \cdots, N$: the PCA matrix (H), and those parameters of the ARMA models $(a_i, b_k, \sigma_\epsilon^2)$ for $\mathbf{Y}_{1,T}^{ref}[n], \mathbf{Y}_{1,R}^{ref}[n], \cdots, \mathbf{Y}_{M,T}^{ref}[n]$ and $\mathbf{Y}_{M,R}^{ref}[n]$.

2.2. Reconstruction (Synthesis)

The reconstruction stage brings a synthetic piece of signal whose properties —statistical, spectral, some nonlinear and some clinical (heart rate, for instance) properties at least— are similar to those of the corrupted piece without the effects of the degrading procedure. To this end, the model (previously estimated) is simulated quite a few times and, posteriorly, the goodness of each simulation is evaluated to reconstruct the signal with the most appropriate simulation. In summary, these are the steps to reconstruct the signal:

1. The model was designed to simulate series of parameters, that is, to generate series lasted a specific number of beats; however, the exact number of beats that must be simulated to reconstruct the signal cannot be known, since only the temporal duration of the corrupted piece is available. Therefore, the number of beats to be simulated is computed according to:

$$N_b = 2 \times \left\lceil \frac{t_e - t_o}{\overline{T_b}} \right\rceil, \quad (9)$$

where $\overline{T_b}$ is the average duration of beats from $x_{ref}(t)$ and $\lceil \cdot \rceil$ is the integer part of the argument. We are aware that N_b is close to the double of the necessary number of beats, but thanks to this, we can ensure that simulated pieces are not going to be shorter than $t_e - t_o$. Hence, simulations of the model must generate N_b beats and use the vectors $\mathbf{Y}_{j,T/R}^{pre}[-1]$ and $\mathbf{Y}_{j,T/R}^{pre}[0]$ as initial conditions for the ARMA filters (remember the order of the model was set to $(p, q) = (2, 2)$). Regarding the number of simulations to carry out, a good choice is 10000, since satisfactory reconstructions can be achieved, and the procedure does not take more than 2 minutes.

2. Every simulation must be evaluated according to the following criteria:

   - Temporal error (ε_{t_R}): it can be expressed as

$$\varepsilon_{t_R} = \frac{1}{4} \sum_{k=1}^{4} \left\| t_{post}^M[k] - t_{sim}^M[k] \right\|, \quad (10)$$

     where $t_{post}^M[k]$ and $t_{sim}^M[k]$ denote the time at which the maxima of the k-th beat (from the original and the simulated piece respectively) after t_e take place (see Fig. 5).

   - Amplitude error (ε_x): mean absolute error of $x_{sim}(t)$ and $x_{post}(t)$ during four beats after t_e (see Fig. 5). Mathematically, the expression is

$$\varepsilon_x = \frac{1}{t_{4B}} \int_{t_e}^{t_e + t_{4B}} \| x_{sim}(t) - x_{post}(t) \|\, dt \quad (11)$$

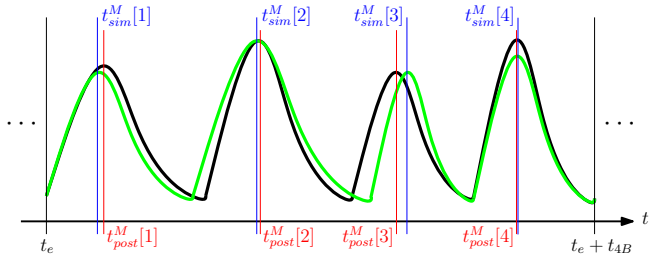

Fig. 5. Diagram of the methodology for the computation of the temporal error. The original piece —$x_{post}(t)$— is plotted in black, whereas the green line represents the synthetic piece —$x_{sim}(t)$—.

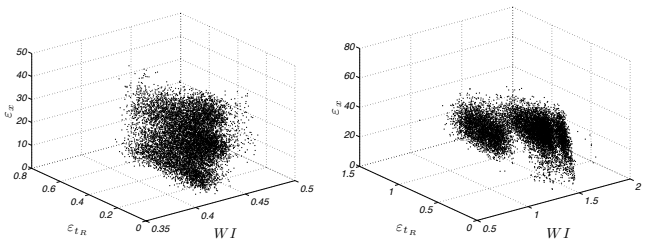

Fig. 6. Evaluation of all the simulations for (a) ECG and (b) PPG according to the temporal error (ε_{t_R}), the amplitude error (ε_x) and the statistical similarity (WI).

- Statistical similarity (WI): for a set of r raters, the Williams' Index provides a measurement of how one rater agrees with the other $(r-1)$ raters in comparison with how they agree with each other [8]. In this work, we have used the covariance matrix of $\mathbf{W}^{sim}[n]$ and those of the uncorrupted series ($\mathbf{W}^{pre}[n]$ and $\mathbf{W}^{post}[n]$) as raters, and the inverse of the sum of the absolute mean error (element by element) between raters as the agreement magnitude.

Good solutions yield low values of ε_{t_R} and ε_x, and high values of WI. In next section we give some details about how to choose the right simulation based on the measurements proposed above.

3. Finally, the reconstructed signal must be computed as follows:

$$x_{Rec}(t) \simeq \Gamma(\mathbf{W}^{Rec}[n], t) \quad (12)$$

$$\mathbf{W}^{Rec}[n] = \begin{cases} \mathbf{W}^{pre}[n] & \text{if } 1 \leq n < n(t_o) \\ \mathbf{W}^{sim}[n] & \text{if } n(t_o) \leq n \leq n(t_e) \\ \mathbf{W}^{post}[n] & \text{if } n(t_e) < n \leq n(\tau), \end{cases}$$

where $n(t_i)$ is the cardinal of the beat that happens at t_i time[2].

3. EXPERIMENTS AND DISCUSSION

For the experiments presented here, two signals (one ECG and one PPG) have been used. A 25 years old healthy patient (under resting conditions) has undertaken signal acquisition using an Omicrom FT Surveyor device (RGB Medical Devices), during 5 minutes, using 250 Hz and 66.67 Hz as sampling rate for ECG and PPG, respectively. These signals which are corruption-free, have been modified by omitting a piece at random. This has been done with the purpose of comparing the reconstructed and the omitted pieces. Thus, ECG signal has been corrupted from $t_o = 245.5$ s to $t_e = 261$ s, while for PPG, the interval was from $t_o = 236.5$ s to $t_e = 253$ s. For both signals, models were estimated from the largest piece, i.e., the piece placed before the beginning of the corruption. Then, the models were simulated 10000 times, and each simulation was evaluated according to the criteria presented in section 2.2. Results from the evaluation are drawn in Fig. 6 where the axes are the three proposed goodness-of-fit measurements. Among the three criteria,

[2]In Eq. (12) the residue $r(t)$ (see Eq. (1)) has been ignored. It can be considered for the $x_{pre}(t)$ and $x_{post}(t)$ if needed, but for the $x_{sim}(t)$ is not possible for now, so we have omitted it all over the signal.

ECG				PPG			
Sim.	ε_{t_R}	ε_x	WI	Sim.	ε_{t_R}	ε_x	WI
6302	0.0041	30.4187	0.4427	7648	0.0150	21.3536	1.4750
6950	**0.0048**	**18.4077**	**0.4407**	**8428**	**0.0150**	**15.6317**	**1.3540**
8268	0.0056	31.1043	0.4483	8135	0.0225	10.3966	1.4723
1719	0.0089	35.2778	0.4312	6987	0.0225	9.3603	1.4739
9900	0.0090	21.2593	0.4396	9913	0.0262	11.5589	1.2681
1720	0.0093	5.9946	0.4130	4702	0.0300	8.8356	1.4715

Table 2. Best 6 simulations, in terms of the temporal error, for both signals. Optimal simulations have been written in bold face.

temporal and amplitude error are the most important, since they determine how the simulation is in agreement with $x_{post}(t)$ (boundary condition). From the signal processing point of view, we have not found any evidence to give more importance to any error than to the rest; however, from the clinical standpoint, the temporal error is more significant, since temporal information is used to generate the HRV (Heart Rate Variability) signal [1]. Under this argument (ε_{t_R} criterium), the best 6 simulations have been summarized in Table 2. Since the temporal error of the first two rows is quite similar, we have chosen the simulation of the second row as the optimal one due to their lower values of ε_x.

Reconstructions performed with the optimal simulations are shown in Fig. 7. Reconstructed pieces do not seem anomalous; long-term and short-term variabilities are still present in the reconstructed signals. The WI tell us how atypical the reconstructions provided are. Henceforth, for the ECG signal, the WI of the original piece is 0.4181, while for the reconstructed one is 0.4407; for the PPG signal, the original piece yields 0.9391, whilst the reconstructed one does 1.3540. This results means that, for ECG, the reconstructed piece is as atypical as the omitted one; while for PPG, the reconstructed piece is more common.

Among many important aspects that deserve comments or criticisms, we would highlight the following ones:

- The validity of the model was tested by means of statistical, spectral and non-linear methods. This fact allows for using some parameters derived from these signals with clinical purposes. This is beyond the scope of this paper and is not presented here.
- Regarding the temporal model of the series $\mathbf{Y}_{j,T/R}[n]$, not all of their components have to be necessarily represented by an ARMA model. In virtue of the PCA transformation, components can be sorted by their variance in descending order, so last components are less significant and, consequently, can be simply modeled as white noise. After some experiments, we can state that the last three/four components hardly provide

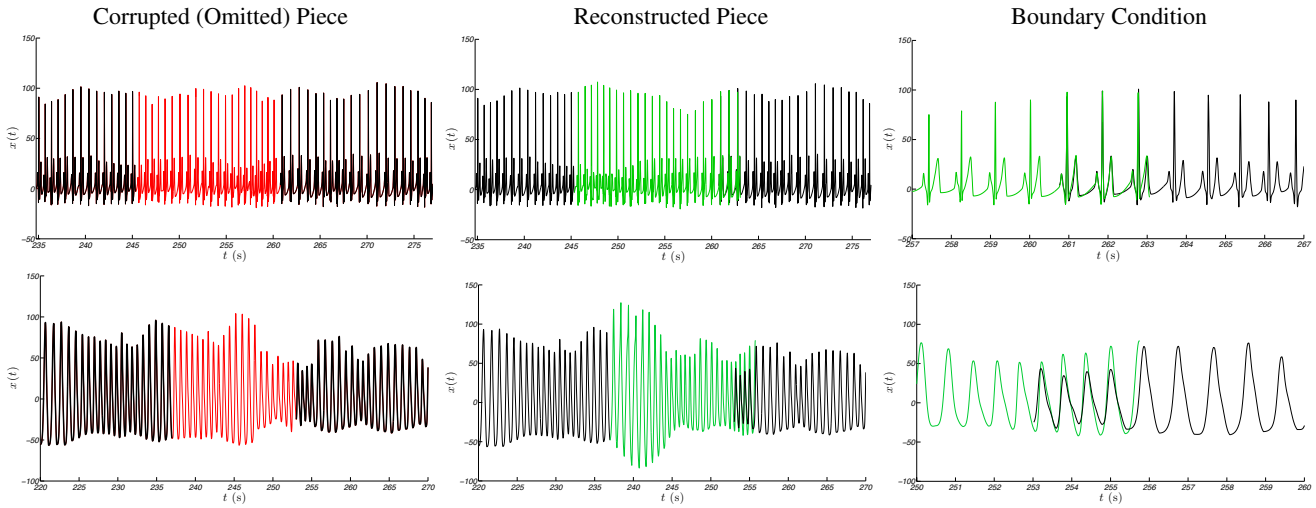

Fig. 7. Corrupted piece, reconstructed piece (with the optimal simulation), and zoom in on the boundary condition for ECG (top row) and PPG (bottom row).

valuable information, so they can be discarder.

- For the experiments performed in this study, the simulations of the model have been carried out using initial conditions, however, these are not present if the corrupted piece is located at the beginning of the signal. We have simulated this scene (using random initial conditions) in several experiments, yielding results similar to those presented here.
- The stochastic nature of the model ensures that if the 10000 simulations are performed several times, the optimal simulation will be (with high probability) different each time. This can be understood as a drawback, however, this behavior is coherent with reality.
- The sensitivity of the human eye is greater for amplitude errors than for temporal ones, hence the reader might consider as optimal any simulation with low amplitude error instead of the one we have chosen. However, the clinical usability of the model leads to prioritize the temporal error, since large part of clinical parameters are related to time instead of amplitude (heart rate, for instance).
- The human eye is not sensitive to the correlation between parameters either, however the clinical relevance of this aspect is considerable. Since the model has been designed to conserve these correlations, a criterium based on this fact has been introduced to choose the optimal simulation: the Williams' Index.
- This study focuses on the application of reconstructing signal, nevertheless, the model can be also used to expand signals in a similar way.

4. CONCLUSIONS

We have introduced a novel methodology for reconstructing cardiovascular signals based on modelling. This methodology consists of estimating the model from the largest uncorrupted piece of the signal, simulating the model using the initial and the boundary conditions provided by the uncorrupted pieces and selecting the optimal simulation according to some criteria of goodness. In spite of the main contribution of this paper being the methodology for reconstructing ECG and PPG signals, the own modelling is also a contribution of the authors. Experimental results over manually corrupted signals have shown that reconstructed pieces are not far from reality, since the main features (statistical, spectral, nonlinear and clinical ones) of the original signal are also present in the reconstructed piece. On the other hand and for the sake of reality, the stochastic nature of the model brings the user a wide range of valid solutions.

5. REFERENCES

[1] L. Sörnmo and P. Laguna, *Bioelectrical Signal Processing in Cardiac and Neurological Applications*, Elsevier Academic Press, Burlington, MA, USA, 2005.

[2] K. Chourdakis, *Encyclopedia of Quantitative Finance*, John Wiley & Sons, Ltd., Chichester, UK, 2010.

[3] G.E.P. Box, G.M. Jenkins, and G.C. Reinsel, *Time Series Analysis. Forecasting and Control*, Prentice Hall, Oakland, CA, USA, 1994.

[4] P. E. McSharry, G. D. Clifford, L. Tarasseko, and L. A. Smith, "A Dynamical Model for Generating Synthetic Electrocardiogram Signals," *IEEE Trans. Biomed. Eng.*, vol. 50, pp. 289–294, 2003.

[5] G.D. Clifford, A Shoeb, P. McSharry, and B. Janz, "Model-Based Filtering, Compression and Classification of the ECG," *Int. Journal on Bioelectromagn.*, vol. 7, pp. 158–161, 2005.

[6] G. Birkhoff, M.H. Schulz, and R.S. Varga, "Piecewise Hermite Interpolation in One and Two Variables with Applications to Partial Differential Equations," *Numer. Math.*, vol. 11, pp. 232–256, 1968.

[7] H. Abdi and L.J. Williams, "Principal Component Analysis," *Wiley Interdiscip. Rev. Comput. Stat.*, vol. 2, pp. 433–459, 2010.

[8] G.W. Williams, "Comparing the Joint Agreement of Several Raters with Another Rater," *Biometrics*, vol. 32, pp. 619–627, 1976.