WIRELESS POSITIONING USING ELLIPSOIDAL CONSTRAINTS

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ABSTRACT

Indoor location-based positioning systems have attracted notable interest during recent years for a wide range of personal and commercial applications. As is well known, the GPS systems does not allow for accurate positioning indoors, resulting in the development of various forms of indoor positioning techniques, mostly being based on radio frequency measurements. In this paper, we examine a novel way to reduce the positioning error by using the notion of separating ellipsoids in the context of received signal strength (RSS) fingerprinting. To avoid excessive computational complexity, the algorithm is paired with the A* algorithm, exploiting mapping information of the building of interest, to take into account obstacles such as walls. The proposed algorithm is evaluated on RSS measurements made in a shopping mall, and found to offer an improved positioning accuracy as compared to the Gaussian kernel approach.

***Index Terms*—** Indoor positioning, wireless systems, separating ellipsoids

1. INTRODUCTION

Positioning systems have since the advent of the GPS system become an integral part of our society, finding an ever growing number of applications in a wide range of fields. As is well known, the satellite-based GPS system is regrettably limited in the sense that it does not allow for accurate positioning in an indoor environment, and the development of systems that would allow for indoor positioning has during recent years become an active research field. Notably, several systems based on radio frequency (RF) measurements have been proposed, often exploiting a fingerprinting approach that matches the measured received signal strength (RSS) at the measurement location to a range of predetermined location fingerprints, estimating the likely location as the one minimizing some form of distance measure to these fingerprints (see, e.g., [1–12], and the references therein). Given the proliferation of wireless local area networks (WLANs) and since a power-sensing function is available in every WLAN device, RSS-based techniques based on the IEEE 802.11 standard has been found to be a frequent and cost-effective solution in many location-based systems. Several such wifi-based systems have been proposed, for example, neural networks, kernel-based techniques, and probabilistic approaches to form the distance measure between the RSS measurements from the available access points (AP) and the fingerprints (see, e.g., [8–11]). In this work, we propose a positioning algorithm that uses the notion of separating ellipsoids, introduced in [13] in the context of vowel recognition, to form the fingerprints and the distance measure, such that maximally separating ellipsoids are computed from the training data for each considered location. Any measurement inside such an ellipsoid is then mapped to that location, whereas any measurement which is outside all the ellipsoids is mapped to the location ellipsoid being closest to the measurement. Typically, the measure of closeness to the various fingerprints is then used to form an interpolated estimates, such that the user is allowed to also be positioned in between fingerprint locations. Given the computational complexity of forming separating ellipsoids for a large number of locations and training measurements, each typically resulting from a large number of AP, it is necessary to reduce the dimensionality of the problem sufficiently. Common ways to handle this is by using AP selection [14,15] or projection techniques [1,4,16]. Herein, we exploit the projection technique in [4], but then also further reduce the dimensionality of the resulting maximization by restricting the number of considered locations. This may be done by exploiting the previously found position, typically being formed using a basic positioning algorithm such as noted in the above references, and then subsequently only consider locations that may be physically accessible at the time of the new measurement. Here, using the popular A* algorithm introduced in [17], we exploit mapping information of the building of interest, taking into account obstacles such as walls, to find the set of possible locations given one current position. Clearly, the proposed dimensionality reduction will be vulnerable to poor initial positioning estimates and/or possible positioning error, potentially causing the position of interest to be excluded from the set of feasible locations. To reduce the risk for such an error, one needs to include an overarching safeguard algorithm. If the overall fit of the ellipsoid algorithm is poor, the positioning estimate shall instead be formed over the entire set of locations using a basic algorithm.

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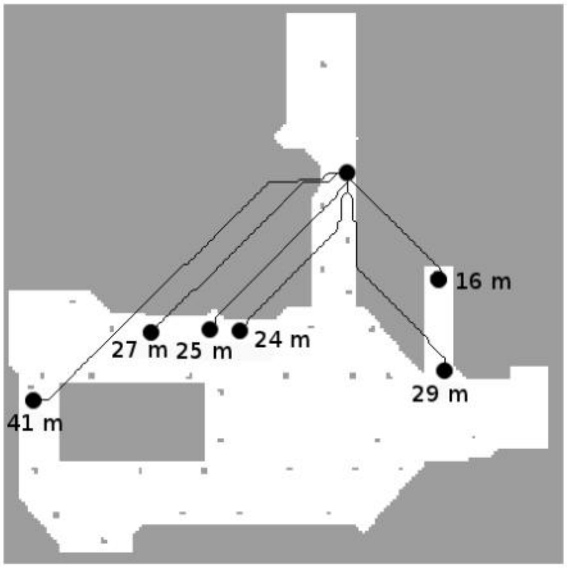

Fig. 1. The figure shows the measured distance and path between the nearby location without considering the geometry of the indoor environment. The black marks indicate the fingerprint locations.

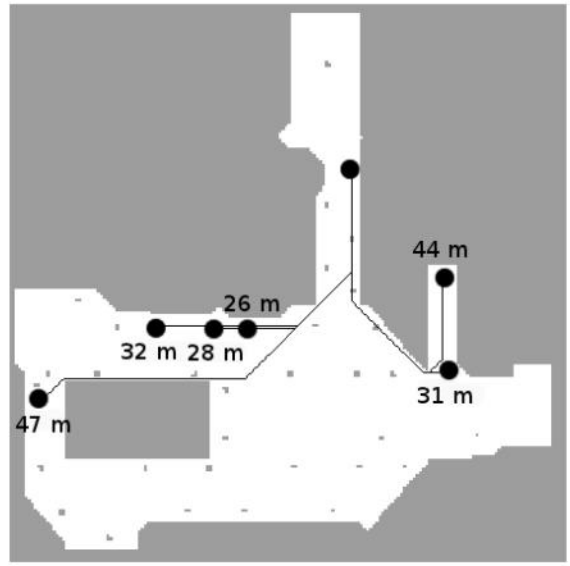

Fig. 2. The figure shows the measured distance and path between the nearby location taking the geometry of the indoor environment into account. The black marks indicate the fingerprint locations.

2. PROBLEM FORMULATION AND PROPOSED POSITIONING ALGORITHM

Consider a collection of N indoor positions with coordinates $\mathbf{p} = (x, y, z)$ with respect to some predefined reference point. For simplicity, it is assumed that for each location, M measurements of the RSSs measurable at the location have been obtained in a training phase. These measurements have been arranged such that $\mathbf{q}_{k,\ell}$ contains the RSSs for location k at time ℓ, ordered such that each vector contains a response from all AP measurable in the entire training set, with zeroes being inserted for those APs not measured at that location and/or time. Using the multiple discriminant analysis (MDA) based projection approach introduced in [4], one may then construct a projection matrix $\mathbf{A}$ that forms the most discriminant features of the between-class and within-class scattering matrices, such that the maximally discriminant components (DCs) may be formed as

$$\mathbf{z}_{k,\ell} = \mathbf{A}\mathbf{q}_{k,\ell} \tag{1}$$

for all locations k and measurement times ℓ, with the dimension of the projection matrix selected such that it restricts the dimensionality of the resulting DCs to only contain the D most relevant dimensions (see [4] for further details). As any ellipsoid in $\mathcal{R}^D$ may be viewed as the intersection of a homogeneous ellipsoid centered at the origin, and a hyperplane in $\mathcal{R}^{D+1}$ along $[\,\mathbf{z},\,1\,]$ with $\mathbf{z} \in \mathcal{R}^D$, we proceed to form the augmented and transformed measurement vectors (see [13] for further details on this reformulation)

$$\mathbf{x}_{k,\ell} = \begin{bmatrix} \mathbf{z}_{k,\ell} \\ 1 \end{bmatrix} \tag{2}$$

and compute the separating ellipsoids, Φ_k, as the semi-definite program (SDP) [13]

$$\begin{aligned} \min_{\Phi_k, \eta_{k,\ell}} \quad & \textstyle\sum_{k,\ell} \eta_{k,\ell} \\ \text{subj. to} \quad & \mathbf{x}_{k,\ell}^T \Phi_k \mathbf{x}_{k,\ell} \leq 1 + \eta_{k,\ell}, \\ & \mathbf{x}_{j,\ell}^T \Phi_k \mathbf{x}_{j,\ell} \geq \rho - \eta_{j,\ell}, \\ & \forall j = 1, \ldots, N \quad j \neq k \\ & \forall \ell = 1, \ldots, M \\ & \Phi_k \succeq 0, \quad \eta_{k,\ell} \geq 0 \end{aligned} \tag{3}$$

This minimization forms two separating ellipsoids that share the same center and axis directions. The second ellipsoid is larger than the first by a factor ρ, subsequently termed the separation ratio. Thus, the larger the separation ratio, the wider apart the measurement clusters from the different locations will be. The inner ellipsoid is constructed to enclose all measurements $\mathbf{x}_{k,\ell}$ from location k, for all the DCs, whereas all the measurements from the other locations, $\mathbf{x}_{j,\ell}$, for $j \neq k$, $\forall \ell$, are restricted to be outside the larger second ellipsoid. It is worth noting that the introduced slack variables, $\eta_{k,\ell}$, allow some marginal points of the class k to be outside the internal ellipsoid, with some points of the remaining classes to be

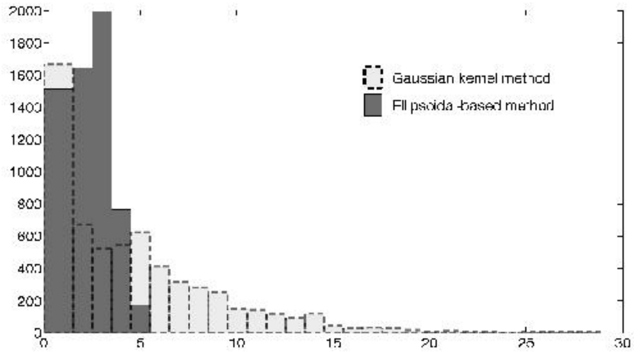

Fig. 3. Positioning errors for the Gaussian kernel method and the proposed ellipsoidal-based method.

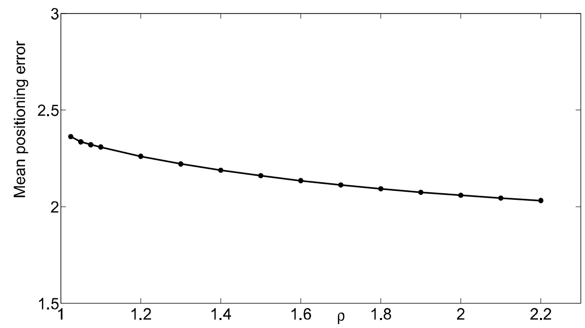

Fig. 4. Mean positioning errors for the proposed ellipsoidal-based method as a function of ρ.

inside the external ellipsoid. The minimization of the sum of these slack variables thus corresponds to the solution having the smallest number of outlier measurements. As a result, if the measurements are well separated, the sum of these slack variables will be zero. We refer the reader to [13, 18] for a discussion on how the resulting convex optimization problem in (3) can be solved efficiently, but note that as is clear from the formulation, the dimensionality of minimization in (3) grows quickly with a growing number of locations, training measurements and DCs. For realistic situations, this may result in a computationally prohibitive optimization problem, and there is therefore an urgent need to restrict the number of considered locations for each minimization to allow for a practically working solution. One way to achieve this is to compute the distance to nearby locations and only include those locations close to the earlier position, which in this sense is treated as being known. Clearly, this is only possible after first acquiring an initial location estimate, but allows for a notable computational simplification after that. Let $\mathbf{z} = [z^1, \ldots, z^D]^T$ denote the measured RSS vector that should be used for positioning, and $\mathbf{p}_k$ the coordinates for location k. Herein, considering all possible locations in forming the estimate, we compute such an initial, basic, location estimate using the Gaussian kernel method, which forms the estimated position using the interpolation formula (see, e.g., [4, 9])

$$\hat{\mathbf{p}} = \sum_{n=1}^{N} w_n \mathbf{p}_n, \tag{4}$$

where w_n denotes the weight for location n, with a larger w_n thereby indicating a likelier location, being formed as

$$w_n = \frac{c_n P(\mathbf{z}|\mathbf{p}_n)}{\sum_{k=1}^{N} c_k P(\mathbf{z}|\mathbf{p}_k)} \tag{5}$$

with the weights c_k, for now being set to one, and

$$P(\mathbf{z}|\mathbf{p}_k) = \sum_{d=1}^{D} \frac{1}{\sqrt{2\pi s_k^d}} \exp\left(-\frac{(z^d - u_k^d)^2}{2s_k^d}\right) \tag{6}$$

with u_k^d and s_k^d denoting respectively the mean and variance for the dth DC of the measured RSS vectors at the kth location, as computed using the training data. After such a location estimate has been established, we proceed to only consider the locations deemed feasible to reach from this location. These may be found using strategies such as the A* algorithm [17], which is commonly used in various forms of computer games for similar purposes. The algorithm exploits the geometry of the indoor environment to compute the closest allowed path between the fingerprint locations, without passing through the walls. With this information, the proposed ellipsoidal-based positioning estimate is formed with the weights in (4) instead being computed using the separating ellipsoids obtained from the training data using (3) as

$$w_n = \frac{c_n \cdot \dfrac{1}{\mathbf{z}^T \Phi_n \mathbf{z}}}{\sum_k c_k \cdot \dfrac{1}{\mathbf{z}^T \Phi_k \mathbf{z}}} \tag{7}$$

with the summation now being formed over only the reachable locations, and with the weights being set to zero for all other locations. To further penalize positions further away from the previous location (as a way to indicate that these are more unlikely than the closer ones), one may introduce a further weighting, given as c_k in (5) and (7), that in some sense reflects the distance to the location. We have found that a simple such scheme can be beneficial, and will therefore include the weighting

$$c_k = \begin{cases} 1 & \text{if } 0 < E_k < 5\,m \\ 0.8 & \text{if } 5 \leq E_k < 8\,m \\ 0.6 & \text{if } 8 \leq E_k \leq 12\,m \end{cases} \tag{8}$$

Method	Mean error	Max error
Gaussian method (All)	1.7	25.8
Gaussian method (Reachable)	1.4	11.0
Ellipsoid method (Reachable)	1.1	4.9
Combined method (Reachable)	1.3	10.9

Table 1. The positioning errors for the ground floor data set.

Method	Mean error	Max error
Gaussian method (All)	4.7	28.8
Gaussian method (Reachable)	3.2	12.3
Ellipsoid method (Reachable)	2.3	4.8
Combined method (Reachable)	2.4	7.5

Table 2. The positioning errors for the first floor data set.

where E_k denotes the Euclidean distance between the previous location and the examined location k. Clearly, the here used weighting is quite ad hoc, and could likely be improved by being more carefully designed,

3. RESULTS

In order to evaluate the proposed algorithm, we examine two data sets measured at the Hansa Mall in Malmö, Sweden. The first set, measured at the first floor of the mall, consists of 38 locations, with 160 wifi measurements for locations ℓ with $\ell = 1, \ldots, 35$, and 80 wifi measurements for locations $\ell = 36, 37, 38$. The second data set, measured at the ground floor, consists of 30 locations, each with 160 wifi measurements. Clearly, one may exploit the information in the z coordinate to allow for transitions between floors in the building, but will then also noticeably benefit from only allowing floor transitions at locations where such are appropriate. To ensure this requires a bit further care, for instance, the interpolation in (4) needs to be modified appropriately, and to simplify the evaluation, the floor level has here been assumed known, and the z coordinate is therefore not used in the following. Figure 1 illustrates the geometry of the first floor of the mall, as well as shows the Euclidean distances from an assumed location to several nearby locations, without considering the geometry of the indoor environment. As is clear from the figure, these distances are misleading. For instance, the distance to the location given as 16 m is clearly further away if one takes the walls into account. Applying the A* algorithm on this map allows for a notably more accurate distance measurement, as shown in Figure 2, where the distance to the mentioned location is now found to be 44 m. Clearly, this makes it difficult to move to this location with the time given between RSS measurements, and therefore this location, as well as other distant locations, should not be considered as possible locations to move to from the current location. Prior to computing the separating ellipsoids, for each location we must find the other locations within a physical distance, $d_{\max}$, that can be considered possible to move to from that location. The choice of $d_{\max}$ should be made considering that the current location estimate will contain some margin of error and should therefore be somewhat larger than one can assume reasonable to move to, even if running. Then, for each location, we find the separating ellipsoids for that location using (3), considering only the locations within a distance of $d_{\max}$ from the considered location. Herein, we use a maximal distance of $d_{\max} = 12$ m. Tables 1 and 2 present the resulting mean and maximum positioning errors in meters for the proposed method, using a separation ratio of $\rho = 1.05$, as compared to the Gaussian kernel method which uses all the possible locations as well as only the ones deemed reachable using the distances found from the A* algorithm. In our experience, the performance is robust to the choice of ρ, as is also illustrated in Figure 4, showing the mean positioning error of the proposed ellipsoid-based method as a function of ρ. The tables also show the performance of a combined method that forms its weights as the average weights from the Gaussian kernel and the ellipsoidal-based weights. As is clear from the tables, the Gaussian kernel method is able to form reasonably accurate estimate for both data sets, although it suffers from some very large positioning errors. This is illustrated in Figure 3 showing the histogram of the positioning errors for the first floor data set. The ground floor has a better radio environment due to smaller open areas, and have only a few large positioning errors. If only considering the reachable locations, the Gaussian kernel method is notably improved, essentially by the absence of positioning errors larger than $d_{\max}$. The proposed ellipsoidal-based approach is seen to further reduce the positioning errors. As also seen in Figure 3, the ellipsoidal-based method lacks large position errors, with the largest error being notably less than $d_{\max}$. Clearly, the technique allows for a very accurate positioning, without an increase of the computational complexity of the positioning algorithm. Finally, the combined approach is seen to be worse than the ellipsoidal-based method, which is mainly due to the occurrence of positioning errors as large as $d_{\max}$, resulting from the Gaussian kernel weights.

4. CONCLUSIONS

In this paper, we have introduced an indoor positioning algorithm based on the notion of separating ellipsoids, exploiting the map information of the current location to reduce the number of possible transitions. The resulting algorithm adapts the ellipsoidal separation algorithm presented by Xiao and Deng for vowel recognition for the problem of indoor positioning, while allowing for a reduced dimensionality of required minimization problem using a basic positioning algorithm to form an initial positioning. The algorithm is shown to yield notably lower mean and maximum positioning errors as compared to the commonly used Gaussian kernel approach when evaluated on data measured in a large shopping mall.

5. REFERENCES

[1] S.-H. Fang, T.-N. Lin, and P.-C. Lin, "Location Fingerprinting in a Decorrelated Space," *IEEE Trans. Knowledge and Data Engineering*, vol. 20, no. 5, pp. 685 –691, May 2008.

[2] S.-H. Fang, T.-N. Lin, and K.-C. Lee, "A novel algorithm for multipath fingerprinting in indoor wlan environments," *IEEE Trans. Wireless Communications*, vol. 7, no. 9, pp. 3579–3588, Sept. 2008.

[3] J. Yin, Q. Yang, and L. M. Ni, "Learning adaptive temporal radio maps for signal-strength-based location estimation," *IEEE Trans. Mobile Computing*, vol. 7, no. 7, pp. 869–883, July 2008.

[4] S.-H. Fang and T.-N. Lin, "Projection-Based Location System via Multiple Discriminant Analysis in Wireless Local Area Networks," *IEEE Trans on Vehicular Technology*, vol. 58, no. 9, pp. 5009–5019, Nov. 2009.

[5] C. Figuera, I. Mora-Jimenez, A. Guerrero-Curieses, J. L. Rojo-Alvarez, E. Everss, M. Wilby, and J. Ramos-Lopez, "Nonparametric Model Comparison and Uncertainty Evaluation for Signal Strength Indoor Location," *IEEE Trans. on Mobile Computing*, vol. 8, no. 9, pp. 1250–1264, Sept. 2009.

[6] S. Mazuelas, A. Bahillo, R. M. Lorenzo, P. Fernandez, F.. Lago, E. Garcia, J. Blas, and E. J. Abril, "Robust Indoor Positioning Provided by Real-Time RSSI Values in Unmodified WLAN Networks," *IEEE J. Sel. Topics in Signal Processing*, vol. 3, no. 5, pp. 821–831, Oct. 2009.

[7] S.-H. Fang and C.-H. Wang, "A Dynamic Hybrid Projection Approach for Improved Wi-Fi Location Fingerprinting," *IEEE Trans. on Vehicular Technology*, vol. 60, no. 3, pp. 1037 –1044, March 2011.

[8] C. Nerguizian, C. Despins, and S. Affes, "Geolocation in mines with an impulse response fingerprinting technique and neural networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 603611, March 2006.

[9] A. Kushki, K. N.Plataniotis, and A. N. Venetsanopoulos, "Kernel-Based Positioning in Wireless Local Area Networks," *IEEE Trans. on Mobile Computing*, vol. 6, no. 6, pp. 689 –705, June 2007.

[10] Y. Jin, W.-S. Soh, and W.-C. Wong, "Indoor localization with channel impulse response based fingerprint and nonparametric regression," *IEEE Trans. Wireless Commun.*, vol. 9, no. 3, pp. 11201127, March 2010.

[11] T. Roos, P. Myllymaki, H. Tirri, P. Misikangas, and J. Sievanen, "A probabilistic approach to WLAN user location estimation," *Int. J. Wireless Inf. Netw.*, vol. 9, no. 3, pp. 155164, July 2002.

[12] S. H. Fang and T. Lin, "Principal component localization in indoor wlan environments," *IEEE Trans. Mobile Computing*, vol. 11, no. 1, pp. 100–110, Jan. 2012.

[13] L. Xiao and L. Deng, "A Geometric Perspective of Large-Margin Training of Gaussian Models," *IEEE Signal Processing Magazine*, vol. 27, no. 6, pp. 118 –123, Nov. 2010.

[14] M. Youssef, A. Agrawala, and A. U. Shankar, "WLAN location determination via clustering and probability distributions," in *Proc. Pervasive Comput. Commun.*, 2003, p. 143150.

[15] A. Kushki, K. N.Plataniotis, and A. N. Venetsanopoulos, "Intelligent dynamic radio tracking in indoor wireless local area networks," *IEEE Trans. Mobile Comput.*, vol. 9, no. 3, pp. 405–419, March 2010.

[16] B. Denby, Y. Oussar, I. Ahriz, and G. Dreyfus, "High-performance indoor localization with full-band GSM fingerprints," in *Proc. IEEE Int. Conf. Commun.*, 2009.

[17] P. E. Hart, N. J. Nilsson, and B. Raphael, "A Formal Basis for the Heuristic Determination of Minimum Cost Paths," *IEEE Transactions on Systems Science and Cybernetics*, vol. 4, pp. 100 –107, July 1968.

[18] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, Cambridge, UK, 2004.