DISTRIBUTED COOPERATIVE SPECTRUM SENSING WITH SELECTIVE UPDATING

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ABSTRACT

This paper investigates the use of adaptive filtering with selective updating in a distributed spectrum sensing network. Through a structure similar to a set-membership filter, coefficient updating is not performed at every single input data, reducing misadjustment, computational complexity and power requirements at each sensing node. Results, presented in terms of convergence behavior and complementary receiver operating characteristic curves, show a substantial reduction in the number of updates performed by the adaptive filter without degrading the detection performance.

***Index Terms*—** Cognitive radio, spectrum sensing, distributed detection, set-membership filtering, least mean square algorithms

1. INTRODUCTION

Cognitive radio (CR) has emerged as the most promising technology for dynamic spectrum management. A secondary user (SU) employing CR technology is capable of collecting data from the environment and, with the help of such information, identifying spectral holes for opportunistic communication. This capability, referred to as *spectrum sensing*, optimizes the use of the transmission band even if it was previously assigned to a primary user (PU) [1]. In order to avoid interference, the SU needs to decide reliably if the the PU is present or not.

Independently of which detection method is used by CR for spectrum sensing - energy detector, matched filter or feature detector -, sensing performance can be improved by spatial cooperation among SUs, for a combination of their contributions can produce a more reliable decision. Most approaches in *cooperative spectrum sensing* consider the use of a fusion center, which collects the individual sensing information, fuse them and make the decision. However, gathering all information makes the whole network susceptible to link failures and increases complexity of the fusion center to process a very large amount of data, specially if the number of nodes is large. Furthermore, increasing distance between radios require them to use more power of transmission, reducing autonomy [2].

A distributed approach emerges as a good alternative: the final decision is made within each of several small *neighborhoods* [2]. In [3], we proposed a distributed spectrum sensing network featuring two-step data sharing among neighboring nodes, with good performance and relative simplicity. However, this type of cooperation usually requires more communication bandwidth due to continuous exchange of information during the sensing period. Moreover, signal processing is now performed locally, which may increase the complexity and energy consumption at each radio.

In order to reduce node complexity and consumption, we introduce the concept of *selective updating* to spectrum sensing networks using an approach similar to set-membership adaptive filtering (SMAF) in this paper. Selective updating means that adaptation of filter coefficients is performed only if the input data - neighboring contributions - are *jointly informative*. Since this does not occur for all incoming data, the number of coefficient updates using the proposed structure can be substantially reduced, saving local computational resources. Moreover, reducing processing makes the system more energy-efficient. Such features are important for distributed networks in which complexity and power requirements at each node are desired to be as small as possible.

For cognitive radio networks, it is specially important to guarantee reliable sensing performance. Thus, in this work, we evaluate the proposed selective updating structure both in terms of coefficient behavior and detection performance through complementary receiver operating characteristic (C-ROC) curves, considering uncorrelated and correlated node contributions.

2. DISTRIBUTED COGNITIVE NETWORK

The distributed spectrum sensing network considered in this work corresponds, for example, to that proposed in [3]: M spatially distributed secondary users (nodes) employ energy detectors [4] and simultaneously sense the environment under hypothesis $\mathcal{H}_0$ (absence of primary signal), or hypothesis $\mathcal{H}_1$ (presence of primary signal). Each node, say node k,

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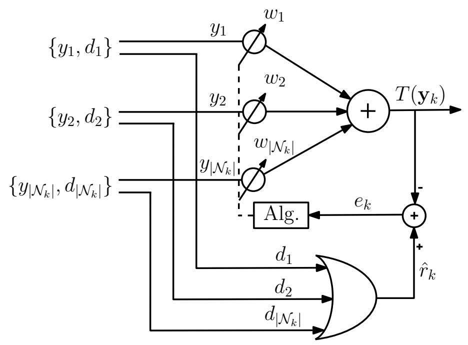

Fig. 1. Adaptive soft combiner at node k [3].

produces a local energy estimate, y_k, and shares it within its neighborhood, $\mathcal{N}_k$, defined as the set of nodes, including itself, linked to it, within a transmission radius [5]. Distributed detection is then performed in two steps: *soft combining step* and *hard combining step*. Both procedures are detailed in the following sections.

2.1. Soft Combining Step

Let us consider the neighborhood of node k with node degree $|\mathcal{N}_k|$. The general idea of the soft combining step is summarized in (1).

$$T(\mathbf{y}_k) = \sum_{i \in \mathcal{N}_k} w_i y_i = \mathbf{w}_k^{\mathrm{T}} \mathbf{y}_k \underset{u_k=0(\mathcal{H}_0)}{\overset{u_k=1(\mathcal{H}_1)}{\gtrless}} \gamma_k \tag{1}$$

First, the node k collects the vector of neighboring estimates, $\mathbf{y}_k = [y_1, y_2, \ldots, y_{|\mathcal{N}_k|}]^{\mathrm{T}}$, and combines such data using the coefficient vector, $\mathbf{w}_k = [w_1, w_2, \ldots, w_{|\mathcal{N}_k|}]^{\mathrm{T}}$. Following, the resulting test statistic $T(\mathbf{y}_k)$ is compared to a local threshold γ_k to yield a local binary decision, u_k.

According to the algorithm proposed in [3], such soft combination is done with the coefficient vector $\mathbf{w}_k$ that minimizes the following objective function:

$$E[e_k^2] = E[(r_k - T(\mathbf{y}_k))^2] \tag{2}$$

that is, the mean squared error (MSE) between the local test statistic and a reference signal, given as

$$r_k = \begin{cases} \mathbf{1}^{\mathrm{T}} \boldsymbol{\mu}_{k,0}, & \text{for } \mathcal{H}_0 \\ \mathbf{1}^{\mathrm{T}} \boldsymbol{\mu}_{k,1}, & \text{for } \mathcal{H}_1 \end{cases} \tag{3}$$

where $\boldsymbol{\mu}_{k,0}(\boldsymbol{\mu}_{k,1})$ is the mean vector of $\mathbf{y}_k$ under hypothesis $\mathcal{H}_0(\mathcal{H}_1)$, as defined in [6], and $\mathbf{1}$ is the $|\mathcal{N}_k| \times 1$ vector of all elements equal to 1. Although an ideal training signal, as modeled in (3), is not available in practice [3], achieving such optimal coefficients can be iteratively done by using the least-mean square (LMS) algorithm and a sufficiently accurate estimate of the reference, $\hat{r}_k$, according to the structure detailed in [3] and depicted in Fig. 1.

The test statistic $T(\mathbf{y}_k)$ is a linear combination of the individual estimates y_i, $i \in \mathcal{N}_k$. If we assume that every node uses a sufficiently large number of samples for computing y_i, such estimates may be considered Gaussian variables under each hypothesis [4], as well as $T(\mathbf{y}_k)$ [7]:

$$T(\mathbf{y}_k) \sim \begin{cases} \mathcal{N}\left(\mathbf{w}_k^{\mathrm{T}} \boldsymbol{\mu}_{k,0}, \mathbf{w}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{k,0} \mathbf{w}_k\right), & \text{for } \mathcal{H}_0 \\ \mathcal{N}\left(\mathbf{w}_k^{\mathrm{T}} \boldsymbol{\mu}_{k,1}, \mathbf{w}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{k,1} \mathbf{w}_k\right), & \text{for } \mathcal{H}_1 \end{cases} \tag{4}$$

where $\boldsymbol{\Sigma}_{k,0}(\boldsymbol{\Sigma}_{k,1})$ is the covariance matrix of $\mathbf{y}_k$ under hypothesis $\mathcal{H}_0(\mathcal{H}_1)$, as defined in [6]. This allows us to evaluate the probabilities of false alarm ($P_{f,k,1}$) and detection ($P_{d,k,1}$) after the soft combining step using the *complementary cumulative distribution* function, $Q(\cdot)$:

$$P_{f,k,1} = P\left(T(\mathbf{y}_k) \geq \gamma_k | \mathcal{H}_0\right) = Q\left(\frac{\gamma_k - \mathbf{w}_k^{\mathrm{T}} \boldsymbol{\mu}_{k,0}}{\sqrt{\mathbf{w}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{k,0} \mathbf{w}_k}}\right) \tag{5a}$$

$$P_{d,k,1} = P\left(T(\mathbf{y}_k) \geq \gamma_k | \mathcal{H}_1\right) = Q\left(\frac{\gamma_k - \mathbf{w}_k^{\mathrm{T}} \boldsymbol{\mu}_{k,1}}{\sqrt{\mathbf{w}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{k,1} \mathbf{w}_k}}\right) \tag{5b}$$

2.2. Hard Combining Step

The objective of the hard combining step is to combine the local neighboring decisions taken at the first step in order to obtain a local consensus decision. Specifically, the node k receives the binary decisions u_i, $i \in \mathcal{N}_k$, and uses conventional hard combining (OR-fusion rule) through which it decides $\mathcal{H}_1$ if at least one of the $|\mathcal{N}_k|$ nodes has suggested $\mathcal{H}_1$ [8].

We note that this second step is not only a fusion of information among nodes, but primarily among neighborhoods. In fact, each node decision u_i carries information from nodes within its own neighborhood $\mathcal{N}_i$ (see Section 2.1). This potentially improves the performance of the OR-fusion rule compared to its original configuration (without the first step), but also increases the correlation among nodes' decisions if their respective neighborhoods share nodes in common.

The effect of the correlation among nodes (or neighborhoods) to the final detection performance can be evaluated with help of the Bahadur-Lazarsfeld expansion [9]. Let $\mathbf{u}_k \in \{0,1\}^{|\mathcal{N}_k|}$ be the local decisions of the neighborhood $\mathcal{N}_k$. The expressions of $P_{f,k,2}$ and $P_{d,k,2}$ for the OR-fusion rule are

$$P_{f,k,2} = 1 - P(\mathbf{u}_k = [0, 0, \ldots, 0] | \mathcal{H}_0) \tag{6a}$$

$$P_{d,k,2} = 1 - P(\mathbf{u}_k = [0, 0, \ldots, 0] | \mathcal{H}_1) \tag{6b}$$

where a more general expression for $P(\mathbf{u}_k | \mathcal{H}_h)$, according to Bahadur-Lazarsfeld, is given by [9]

$$P\left(\mathbf{u}_k | \mathcal{H}_h\right) = \prod_{i,j,l,\ldots \in \mathcal{N}_k} P\left(u_i | \mathcal{H}_h\right) \left[1 + \sum_{i<j} \rho_{ij}^h z_i^h z_j^h + \sum_{i<j<l} \rho_{ijl}^h z_i^h z_j^h z_l^h + \ldots + \rho_{12\ldots|\mathcal{N}_k|}^h z_1^h z_2^h \ldots z_{|\mathcal{N}_k|}^h\right] \tag{7}$$

Note in (7) that $P(\mathbf{u}_k|\mathcal{H}_h)$ is a function of the correlation coefficients, ρ^h, of the neighbors' decisions conditioned on hypothesis $\mathcal{H}_h$. In its turn, z_i^h correspond to the binary random variable u_i normalized conditioned on $\mathcal{H}_h$. More details about the Bahadur-Lazarsfeld expansion can be found in [3, 9].

3. SELECTIVE UPDATING SCHEME

To obtain the coefficient updates during the soft combining step, we propose to use a concept of selective updating similar to that found in SMAF approaches [10]. First, we define the *constraint set* of node k, $\Theta_k[n]$, as the set of the coefficient vectors $\mathbf{w}_k$ that make the output error, at filter k and instant n, upper bounded in magnitude by $\bar{\gamma}_k[n]$ [10]:

$$\Theta_k[n] = \left\{ \mathbf{w}_k \in \mathbb{R}^{|\mathcal{N}_k|} : \left|\hat{r}_k[n] - \mathbf{w}_k^{\mathrm{T}}\mathbf{y}_k[n]\right| \leq \bar{\gamma}_k[n] \right\} \quad (8)$$

The contributions within neighborhood $\mathcal{N}_k$ at instant n are then said to be jointly informative if the constraint set $\Theta_k[n]$ associated to the input data pair $(\hat{r}_k[n], \mathbf{y}_k[n])$ does not contain the current coefficient vector $\mathbf{w}_k[n]$, according to (8). In this case, a new coefficient vector, $\mathbf{w}_k[n+1]$, is formed. Otherwise, the neighboring information is discarded and no coefficient updating is performed.

The choice of the bound $\bar{\gamma}_k[n]$ is crucial for the good performance of the proposed selective updating scheme. SMAF applications usually relate this threshold to the variance of the observation noise at input of the adaptive filter [10]. In this work, we consider the disturbance at output of the filter,

$$\nu_k = r_k - \mathbf{w_o}_k^{\mathrm{T}}\mathbf{y}_k \quad (9)$$

where $\mathbf{w_o}_k$ is the Wiener solution for the node k. Both mean μ_{ν_k} and variance $\sigma^2_{\nu_k}$ of the disturbance ν_k can be calculated using the first and second-order statistical information of neighborhood $\mathcal{N}_k$ available at node k, according to the following expressions:

$$\mu_{\nu_k} = \pi_0 \left((1 - \mathbf{w_o}_k)^{\mathrm{T}}\boldsymbol{\mu}_{k,0}\right) + \pi_1 \left((1 - \mathbf{w_o}_k)^{\mathrm{T}}\boldsymbol{\mu}_{k,1}\right) \quad (10)$$

$$\sigma^2_{\nu_k} = \pi_0 \left[\left((1 - \mathbf{w_o}_k)^{\mathrm{T}}\boldsymbol{\mu}_{k,0} - \mu_{\nu_k}\right)^2 + \mathbf{w_o}_k^{\mathrm{T}}\boldsymbol{\Sigma}_{k,0}\mathbf{w_o}_k\right] + \pi_1 \left[\left((1 - \mathbf{w_o}_k)^{\mathrm{T}}\boldsymbol{\mu}_{k,1} - \mu_{\nu_k}\right)^2 + \mathbf{w_o}_k^{\mathrm{T}}\boldsymbol{\Sigma}_{k,1}\mathbf{w_o}_k\right] \quad (11)$$

where $\pi_0(\pi_1)$ is the *a priori* probability of occurrence of $\mathcal{H}_0(\mathcal{H}_1)$, assumed known.

Furthermore, instantaneous estimates $\hat{\mu}_{\nu_k}[n+1]$ and $\hat{\sigma}^2_{\nu_k}[n+1]$ can be obtained from (10) and (11), respectively, by applying the updated coefficient vector $\mathbf{w}_k[n+1]$ instead of $\mathbf{w_o}_k$. By doing so, it is possible to use a time-varying threshold according to

$$\bar{\gamma}_k[n+1] = \alpha_k\bar{\gamma}_k[n] + (1 - \alpha_k)\sqrt{\beta_k\hat{\sigma}^2_{\nu_k}[n+1]} \quad (12)$$

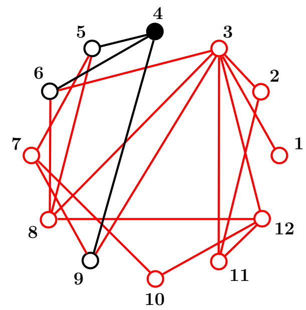

Fig. 2. Distributed topology with $M = 12$ nodes [12]. Neighborhood $\mathcal{N}_4$ is highlighted.

where α_k is a local forgetting factor and β_k is a constant commonly set to 5 [10]. However, simulations showed that the local parameter β_k needs to be adjusted to ensure good convergence for low signal-to-noise (SNR) ratio input data $\mathbf{y}_k$, in which the amount of error in the estimate $\hat{r}_k$ increases. Even so, this proposed scheme offers significant reduction of coefficient updates needed, as will be seen in Section 4.

Finally, the LMS algorithm featuring selective updating at node k can be implemented as the following ($\bar{\gamma}_k[1] = 0$):

$$e_k[n] = \hat{r}_k[n] - \mathbf{w}_k^{\mathrm{T}}[n]\mathbf{y}_k[n] \quad (13)$$

If $|e_k[n]| \geq \bar{\gamma}_k[n]$

$$\begin{aligned} \mathbf{w}_k[n+1] &= \mathbf{w}_k[n] + 2\mu_k e_k[n]\mathbf{y}_k[n] \\ \bar{\gamma}_k[n+1] &= \alpha_k\bar{\gamma}_k[n] + (1 - \alpha_k)\sqrt{\beta_k\hat{\sigma}^2_{\nu_k}[n+1]} \end{aligned} \quad (14)$$

else

$$\begin{aligned} \mathbf{w}_k[n+1] &= \mathbf{w}_k[n] \\ \bar{\gamma}_k[n+1] &= \bar{\gamma}_k[n] \end{aligned} \quad (15)$$

Note that this proposed algorithm is similar to the set-membership normalized LMS (SM-NLMS) algorithm [11]. The difference from the SMAF approach is that the step size μ_k in (14) is not adjustable by the gap between the instantaneous output error $e_k[n]$ and the current bound $\bar{\gamma}_k[n]$. Although this does not guarantee a new estimate $\mathbf{w}_k[n+1]$ on the boundary of the constraint set $\Theta_k[n]$ [11], it is important here to maintain a fixed step size to minimize the influence of very large error magnitudes in the event of a wrong estimate of the instantaneous reference $\hat{r}_k[n]$ in (13).

4. RESULTS AND DISCUSSION

In this section, we illustrate the features of the proposed selective updating algorithm in cooperative spectrum sensing through simulations with uncorrelated and correlated node contributions. The distributed network topology used in the simulations is depicted in Fig. 2. Each node k generates a total of 10^5 energy estimates under equal occurrence of $\mathcal{H}_0$ and $\mathcal{H}_1$ ($\pi_0 = \pi_1 = 0.5$).

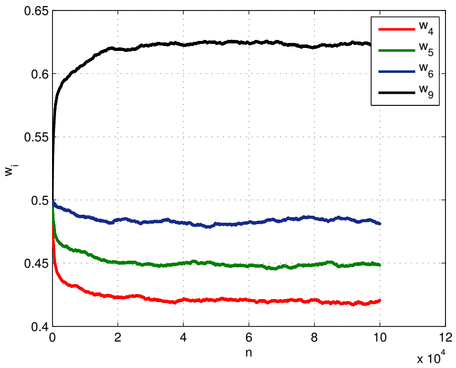

Fig. 3. Convergence behavior at node 4 using selective updating LMS: uncorrelated case.

Table 1. Percentage of updates at node 4 using selective updating LMS: uncorrelated case.

Interval	Updates
Transient (first 5% iterations)	26.46%
Steady state	17.00%

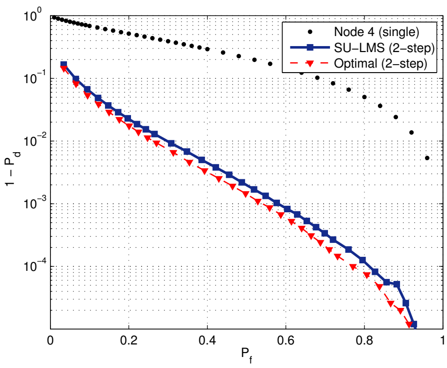

Fig. 4. C-ROC performance at node 4 employing single detection and distributed two-step cooperation within neighborhood $\mathcal{N}_4$: uncorrelated case.

We consider the statistical model of y_k proposed in [6]: for the uncorrelated simulation, the covariance matrices $\mathbf{\Sigma}_{k,0}$ and $\mathbf{\Sigma}_{k,1}$ are identity matrices for all k. For the correlated simulation, $E[(y_i - \mu_{i,h})(y_j - \mu_{j,h})] = 0.5$, where i and j are index of spatially adjacent nodes (not necessarily belonging to a same neighborhood), and $\mu_{i,h}$ and $\mu_{j,h}$ are, respectively, the means of the random variables y_i and y_j under hypothesis $\mathcal{H}_h$, $h \in \{0, 1\}$.

Following, we present the results obtained at node 4 after employing two-step distributed cooperation with the neighborhood $\mathcal{N}_4$. As illustrated in Fig. 2, the neighborhood $\mathcal{N}_4$ is composed of nodes 4, 5, 6 and 9. The SNR ratios of the individual estimates in $\mathcal{N}_4$ are equal to -1.94 dB, 0 dB, 1.58 dB and 5.1 dB, respectively (according to the expression for individual SNR in [7]). During the soft combining step, node

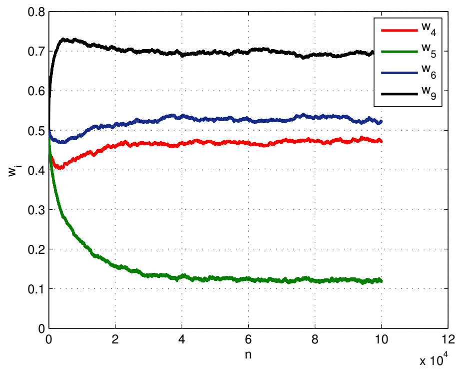

Fig. 5. Convergence behavior at node 4 using selective updating LMS: correlated case.

Table 2. Percentage of updates at node 4 using selective updating LMS: correlated case.

Interval	Updates
Transient (first 5% iterations)	20.00%
Steady state	13.73%

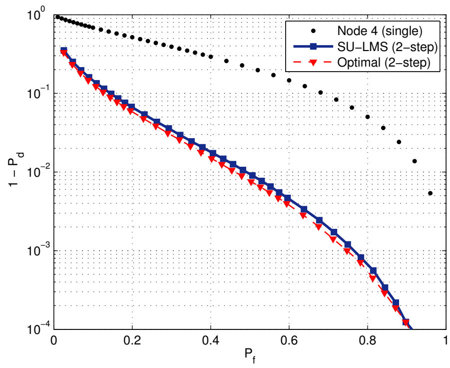

Fig. 6. C-ROC performance at node 4 employing single detection and distributed two-step cooperation within neighborhood $\mathcal{N}_4$: correlated case.

4 runs the proposed selective updating LMS (SU-LMS) algorithm with the following parameters: $\alpha_4 = 0.995$, $\beta_4 = 3$, $\mu_4 = 1 \times 10^{-4}$ for the uncorrelated case, and $\mu_4 = 5 \times 10^{-4}$ for the correlated case. Such parameters were chosen to obtain equal quality of convergence for both simulations.

Fig. 3 shows the convergence behavior (after 20 independent runs) of the adaptive filter at node 4 with uncorrelated input data. The coefficient curves are normalized. As desired, the coefficients converge such that they enhance the most reliable node contributions, i.e., with higher SNR. The associated percentage of coefficient updates taken during transient and steady state are shown in Table 1 (the choice of the value 5% is based on the transient of $\bar{\gamma}_4[n]$). Note the considerable reduction in the number of updates provided by the proposed SU-LMS algorithm.

Fig. 4 shows the C-ROC curves for node 4 after employing single detection and distributed two-step cooperation with its uncorrelated neighbors. For comparison purposes, we also consider the optimal linear fusion proposed in [7] for the soft combining step. As expected, the distributed structure outperforms the single detector. Furthermore, using the proposed SU-LMS as the first step in the distributed approach continues to offer good detection performance compared to that achieved by using optimal linear fusion as the first step, as already shown in [3] for the conventional LMS algorithm. This indicates that employing selective updating does not interfere in the detection performance.

The convergence behavior (after 20 independent runs) observed at node 4 for the correlated simulation is presented in Fig. 5. In this new scenario, estimates from node 5 are highly correlated with those from its adjacent nodes 4 and 6. Note, by the curves in Fig. 5, that the convergence occurs in such a way that the filter minimizes the influence of the correlated node. On the other hand, the percentage of updates shown in Table 2 indicates again the good capability of the proposed selective algorithm in reducing node processing.

The correspondent detection performance of node 4 for the correlated simulation is shown in Fig. 6. We note a general performance degradation (compared to Fig. 4) due to the correlation among neighbors. However, again the selective updating feature of the proposed scheme does not affect the detection performance, and the C-ROC curve correspondent to the distributed detection featuring SU-LMS at the first step approaches that correspondent to the optimal linear fusion.

5. CONCLUSION

In this paper, we investigated selective updating features for adaptive combining in cooperative spectrum sensing networks. Such features are specially useful in distributed networks in which signal processing is performed at each node. In this sense, we proposed an LMS algorithm employing the concept of constraint sets similar to set-membership filtering: although not ensuring every update on the boundary of the constraint set, such algorithm leads the adaptive filter to a solution within the set. By doing so, the node only performs a coefficient adaptation if its neighbors offer jointly relevant information. Through simulations with uncorrelated and correlated neighbors' contributions, results showed that the proposed selective updating algorithm can reduce processing and thereby complexity at node level without affecting the sensing performance.

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