AUDIO SIGNAL PROCESSING BY FILTERS SELF-ADJUSTABLE TO SPECTRAL POWER DISTRIBUTION

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ABSTRACT

This paper presents two-channel and three-channel complementary filter banks self-adjusting to the power spectral distribution of its input signal. The filter banks are based on EMQF or Butterworth double-complementary filter pairs. The adjustment of the 3dB-crossover frequency is achieved by a simple tuning algorithm that is suitable for the EMQF and Butterworth filters. A new algorithm for changing the 3dB-crossover frequencies as a function of the power spectral distribution of the input signal is proposed. The algorithm is based on the well-known binary search algorithm and is simple to implement in non-real time applications.

Index Terms— audio signal, complementary filter banks, self-adjusting, power spectral distribution

1. INTRODUCTION

Filter banks are used for the analysis and processing of audio signals in many different ways. They find applications as digital equalizers [1], crossovers [2], in hearing aid devices [3], [4], during recording and playing back of audio signals. Analysis filter banks for audio signals are often designed either as octave-band or third-octave-band filter banks [5]. However, in many applications it is necessary to define a filter bank that splits the signal into subband signals based on a criterion deduced from signal spectrum or defined by the application of the filter bank. Subsequently, it is required to design a filter bank that can split its input signal into subband signals, where the crossover frequencies are controlled by the aforementioned criterion. There are different applications, e.g. hearing aid devices, where variable filter banks are required. The variable filter bank is subsequently designed to provide the possibility of changing the 3dB-crossover frequencies or the gain of the channels independently.

The filter bank designed in this paper is based on the tunable, nonsubsampled, non-uniform filter bank first introduced in [4]. As stated in [4], this filter bank is suitable for applications that require the same sampling frequencies for all analysis filter bank channels. In addition, the proposed filter bank is double-complementary (all-pass and power-complementary). The all-pass complementarity property is useful because some of the channel data streams can be regrouped (summed up) into a single signal [4], [6]. Based on this property of the filter bank, a tool is developed that enables the user to choose the crossover frequencies of the filter bank as a function of the spectrum of the audio signal [6]. The power-complementarity property was used in [6], [7] for calculation of the power distribution of the subband signals among the channels defined by the specific spectral characteristics of the church bell sound.

In this paper, we use a similar filter bank structure and tuning procedure of the 3dB-crossover frequency as in [8] in order to design the filter bank that is self-adjustable to the power spectral distribution of the input signal. It means that the 3dB-crossover frequencies are changed automatically by the proposed self-adjusting algorithm. The proposed design can be used for the self-calibration of audio systems or devices or for the off-line analysis of the signals.

This paper is organized as follows: in section 2 the filter bank structure and the self-adjusting procedure are explained, in section 3 the results of the experiment are presented, and section 4 concludes the paper.

2. SELF-ADJUSTABLE FILTER BANK

The structure of the complementary filter pair (two-channel filter bank) is based on double-complementary filter pairs (being all-pass and power-complementary) realized as a parallel connection of two all-pass transfer functions $A_0(z)$ and $A_1(z)$ [8]–[10], Fig. 1,

$$G_{LP}(z) = [A_0(z) + A_1(z)]/2 \tag{1}$$

$$G_{HP}(z) = [A_0(z) - A_1(z)]/2\,. \tag{2}$$

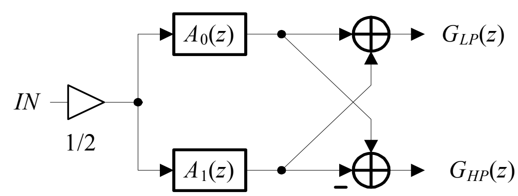

Fig. 1. Double-complementary filter pair.

This work was partially supported by the Ministry of Education and Science of Serbia under Grants TR-36026 and TR-32023.

2.1. Filter bank structure

The use of a variable multichannel filter bank to audio applications was introduced in [4] where the filter bank was designed as a connection of several complementary filter pairs and all-pass sections. A software tool based on the adjustable filter bank and applied to the analysis of sound signals of different kinds has been presented in [6].

Fig. 2 shows the three-channel filter bank constructed as a connection of two two-channel complementary filter banks with different crossover frequencies $\left(\omega_{3\text{dB}}^{1}, \omega_{3\text{dB}}^{2}\right)$ and one all-pass section.

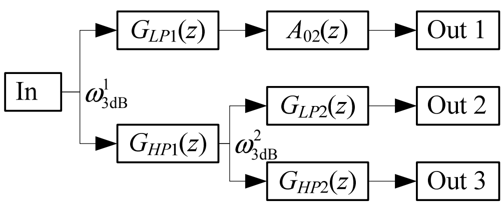

Fig. 2. Three channel filter bank.

The all pass section $A_{02}(z)$ in Fig 2 guarantees that the resulting three-channel filter bank is double-complementary due to the all-pass complementarity property of the lowpass/highpass filter pair:

$$G_{LP2}\left(e^{j\omega}\right)+G_{HP2}\left(e^{j\omega}\right)=A_{02}\left(e^{j\omega}\right). \tag{3}$$

EMQF or Butterworth filters are used as building elements of the filter pair and multichannel filter bank, respectively. EMQF and Butterworth filters are chosen because these filter types are suitable for the design of variable filter pairs (filter banks) [8]–[10]. In the case of EMQF and Butterworth filter pairs, the transfer functions of the all-pass branches $A_0(z)$ and $A_1(z)$ can be expressed as:

$$A_0(z)=\prod_{l=2,4,\dots}^{(N+1)/2}\frac{\beta_l+\alpha(1+\beta_l)z^{-1}+z^{-2}}{1+\alpha(1+\beta_l)z^{-1}+\beta_l z^{-2}}, \tag{4}$$

$$A_1(z)=\frac{\alpha_1+z^{-1}}{1+\alpha_1 z^{-1}}\cdot\prod_{l=3,5,\dots}^{(N+1)/2}\frac{\beta_l+\alpha(1+\beta_l)z^{-1}+z^{-2}}{1+\alpha(1+\beta_l)z^{-1}+\beta_l z^{-2}}, \tag{5}$$

where N is the filter order (an odd number), α_1 depends on the value of the single real pole, whereas the constants β_l, l=2, 3,..., $(N-1)/2$, are determined by the radii r_l of the conjugate-complex pole pairs:

$$\beta_l=(r_l)^2,\ \beta_l<\beta_{l+1}<1. \tag{6}$$

Note that the constant α is of the same value for all second-order sections.

In [6], the M-channel filter bank is designed by making use of the variable filter bank structure, Fig. 2. The user can define a set of the $M-1$ crossover frequencies by observing the spectrum of the analyzed signal. Once designed, the filter bank remains unchanged during the processing of the currently analyzed signal.

The M-channel filter bank consists of a connection of $M-1$ filter pairs, where all of them are designed starting from the same half-band filter pair [8]–[10]. Using a simple tuning procedure [8]–[10], a half-band filter pair can be transformed into a new filter pair with a desired crossover frequency. The resulting filter pair retains the characteristics of the start-up half-band filter pair, i.e. filter order, pass-band ripple, stop-band attenuation and the double-complementarity property [10].

In this paper, a slightly modified procedure of the original tuning algorithm is used [8]. As shown in [8], the start-up filter pair is not necessarily a half-band filter pair, and moreover any EMQF or Butterworth filter pair could be chosen instead. From the start-up filter pair $[G_{LP}^{S}(z)\ G_{HP}^{S}(z)]$ the resulting filter pair $[G_{LP}^{R}(z)\ G_{HP}^{R}(z)]$ with the changed 3dB-crossover frequency $\omega_{3\text{dB}}$ can be obtained. For the given start-up filter parameters α^S, α_1^S and β_l^S, l=2, 3, …, $(N+1)/2$, the parameters of the resulting filter pair α^R, α_1^R and β_l^R, l=2, 3, …, $(N+1)/2$ can be computed using the following simple procedure [8]. Firstly, we compute α^R and α_1^R

$$\alpha^R=-\cos(\omega_{3\text{dB}}), \tag{7}$$

$$\alpha_1^R=-\frac{\cos(\omega_{3\text{dB}})}{1+\sin(\omega_{3\text{dB}})}. \tag{8}$$

It should be observed that α^R and α_1^R depend on the 3dB-crossover frequency only. In the next step, the new values of the parameters β_l^R are calculated based on the values β_l^S of the corresponding second order section of the start-up filter pair [8]:

$$\beta_l^R=\left(\beta_l^S+\lambda\right)/\left(\beta_l^S\lambda+1\right),\ l=2,3,\dots,(N+1)/2, \tag{9}$$

where

$$\lambda=\left(\left(\alpha_1^R\right)^2-\left(\alpha_1^S\right)^2\right)/\left(1-\left(\alpha_1^S\right)^2\left(\alpha_1^R\right)^2\right). \tag{10}$$

Based on (6) it is obvious that for all parameters β_l^S,

$$\beta_l^S<1. \tag{11}$$

Furthermore, it follows from (8) that for $0<\omega_{3\text{dB}}<\pi$:

$$\left|\alpha_1^{S,R}\right|<1. \tag{12}$$

It is straightforward to prove that under conditions (10)–(12) the value of the parameter λ is restricted to:

$$|\lambda|<1. \tag{13}$$

As a result, it can be concluded that for $\beta_l^S<1$ and $|\lambda|<1$ the values of β_l^R, l=2, 3, …, $(N+1)/2$, of the resulting filter pair (9) should satisfy the condition:

$$\beta_l^R<1. \tag{14}$$

Thereby, from (12) and (14) one verifies that the procedure given by (7)–(10) always leads to the stable filter (filter pair) solution.

2.2. Self-adjusting procedure

In this paper, we start from the structure of the filter pair (filter bank) described in section 2.1 (Fig. 2) to design a system that is self-adjustable to the power distribution of the observed signal. The proposed algorithm is based on the fact that the filter pairs (filter banks) are power-complementary:

$$\left|G_{LP}\left(e^{j\omega}\right)\right|^2+\left|G_{HP}\left(e^{j\omega}\right)\right|^2=1\,. \tag{15}$$

Therefore, it is possible to adjust the 3dB-crossover frequency of the filter pair in such a manner that the powers/energies being transferred via the LP branch and HP branch stand in some predefined ratio. We propose a simple adjusting strategy based on the tuning algorithm (7)–(10).

We apply the well-known binary search algorithm [11] to develop a procedure for self-adjusting the channel bandwidths of the nonuniform filter bank as a function of the power spectral distribution of the input signal. To this end, we define the power distribution ratios denoted as p_{ri}, $i = 1, 2, \ldots, M$, for the filter bank output sequences $\{x_i(n)\}$, $i = 1, 2, \ldots, M$:

$$p_{ri}=100\frac{P_i}{\sum_{j=1}^{M}P_j}\ [\%],\ i=1,2,\ldots,M, \tag{16}$$

where P_i is the average power of the i-th channel output signal $\{x_i(n)\}$ of length L:

$$P_i=\frac{1}{L}\sum_{n=0}^{L-1}\left|x_i(n)\right|^2,\ i=1,2,\ldots,M. \tag{17}$$

The algorithm will be explained by the example of a two-channel filter bank, i.e. $M = 2$ in (16), (17), using the structure of Fig. 1 and the tuning procedure (7)–(10).

The goal is to find a 3dB-crossover frequency of the $[G_{LP}(z)\ G_{HP}(z)]$ filter pair under the condition:

$$DV-EV\le p_{ri}\le DV+EV\ , \tag{18}$$

where DV [%] is the desired value of p_{ri}, and EV [%] is a tolerable error value.

Starting values for the minimum $\omega_{3\mathrm{dBmin}}$ and maximum $\omega_{3\mathrm{dBmax}}$ values of the 3dB-crossover frequencies can be set by the user, or the minimum and maximum theoretical values ($\omega_{3\mathrm{dBmin}}=0$ and $\omega_{3\mathrm{dBmax}}=2\pi f_s/2$, f_s-sampling frequency) can be assumed. At the beginning, the 3dB-crossover frequency $\omega_{3\mathrm{dB}}(1)$ is set to the midpoint of the $[\omega_{3\mathrm{dBmin}}(1),\ \omega_{3\mathrm{dBmax}}(1)]$ interval. A segment of the input signal of the length L is filtered by the filter pair and the values p_{r1} and p_{r2} are calculated.

After that, the new values of the frequencies $\omega_{3\mathrm{dB}}$, $\omega_{3\mathrm{dBmin}}$, $\omega_{3\mathrm{dBmax}}$ are obtained as follows:

i) If $p_{r1}<DV-EV$ then $\omega_{3\mathrm{dBmin}}(2)=\omega_{3\mathrm{dB}}(1)$.

ii) If $p_{r1}>DV+EV$ then $\omega_{3\mathrm{dBmax}}(2)=\omega_{3\mathrm{dB}}(1)$.

iii) $\omega_{3\mathrm{dB}}(2)$ is set to the midpoint of the $[\omega_{3\mathrm{dBmin}}(2),\ \omega_{3\mathrm{dBmax}}(2)]$ interval.

iv) Filter bank is changed by tuning algorithm (7)–(10).

v) The same segment of the signal is filtered again, by the changed filter bank.

The procedure is repeated until condition (17) is fulfilled.

The k-th iteration of the self-adjusting algorithm is defined as:

i) If $p_{r1}<DV-EV$ then $\omega_{3\mathrm{dBmin}}(k)=\omega_{3\mathrm{dB}}(k-1)$.

ii) If $p_{r1}>DV+EV$ then $\omega_{3\mathrm{dBmax}}(k)=\omega_{3\mathrm{dB}}(k-1)$.

iii) $\omega_{3\mathrm{dB}}(k)$ is set to the midpoint of the $[\omega_{3\mathrm{dBmin}}(k),\ \omega_{3\mathrm{dBmax}}(k)]$ interval.

iv) Filter bank is changed by tuning algorithm (7)–(10).

v) The same segment of the signal is filtered again, by the changed filter bank.

Figs 3 and 4 illustrate the self-adjustment process realized for a single segment (frame) of a musical signal with DV=75%. The start-up frequencies $f_{3\mathrm{dBmin}}(1)$, $f_{3\mathrm{dBmax}}(1)$ of the 9th order EMQF filters are set to 1000 Hz and 4000 Hz, the sampling frequency is f_s=44100 Hz. Fig. 3 shows the gain responses of the filter pairs obtained for 3 iterations, and Fig. 4 shows the z-plane poles positions. Obviously, the positions of the filter poles are changed by the adjustment procedure. All poles that correspond to the same iteration lie on a circle in the z plane [9], [10]. This means that the filter pairs of all iterations are EMQF complementary filters [10]. In the presented case, 3 iterations were needed to satisfy condition (17). In Fig. 3, tick line corresponds to the final iteration. In Fig. 4, marker size increases with the iteration index.

Fig. 5 illustrates the case when the start-up frequencies $f_{3\mathrm{dBmin}}(1)$, $f_{3\mathrm{dBmax}}(1)$ are set to 0 Hz and $f_s/2$. As shown in Fig. 5, the starting position represents a half-band filter, all poles lie on the imaginary axes of the z plane. In this case, four iterations were needed to satisfy condition (17).

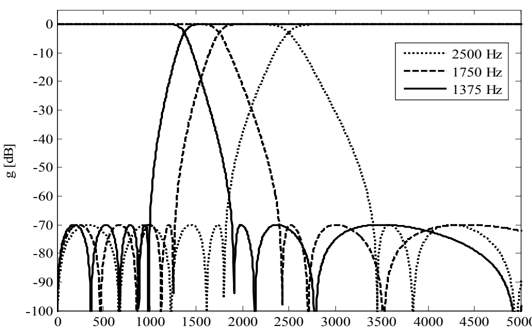

Fig. 3. Gain responses in dB of the filter pairs for all three iterations (1-2500 Hz, 2-1750 Hz, 3-1375 Hz).

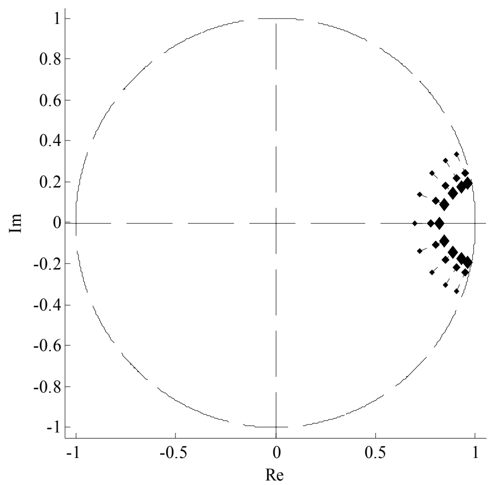

Fig. 4. Pole placement of the filter pairs for all iterations of the self-adjusting procedure.

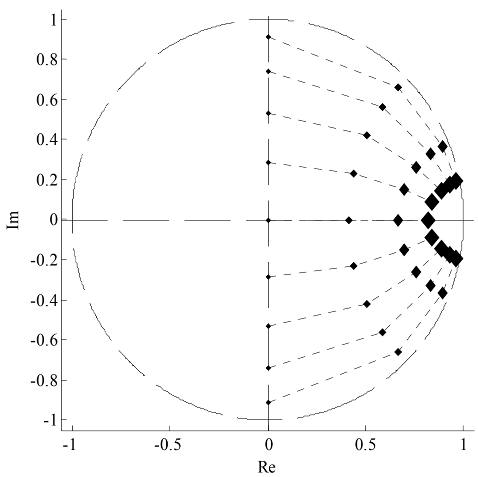

Fig. 5. Pole placement of the filter pairs for all iterations of the self-adjusting procedure (half-band start-up filter pair).

The number of iterations in the self-adjusting procedure depends on the starting values of the frequencies f_{3dBmin}, f_{3dBmax}, selectivity of the filter, and on the value of the acceptable tolerance.

The procedure explained above can be used to analyze the entire signal. In this case, the input signal is divided into subframes (segments) of the same length L, and the procedure is repeated for all the frames. The goal is to calculate the 3dB-frequencies for each frame in compliance with condition (17). This approach is suitable for the tracking of the frequency being consistent with the given power/energy distribution. We use this approach to analyze sound bell signals. Example will be presented in the next section.

3. EXAMPLE

The self-adjustable filter bank described in section 2 was designed to serve as a tool in research concerned with analysis and characterization of a bell sound. The spectrum of the bell sound consists of discrete components called partials. The ratio of the first five partial frequencies of a tuned, or so called "ideal" bell are 1 : 2 : 2.4 : 3 : 4. These five partials dominantly influence the sound of a bell and its strike note [7]. The research presented in [7] reveals significant correlations between subjective evaluations of quality of the bell sound and power/energy distribution of the first five dominant frequency components. It was shown that bell sounds ranked in subjective tests as "good" had most of the energy (approximately more than 70%) in the frequency band defined by the first five spectral components.

The subjective tests and the analysis explained above were performed by using the fixed interval of bell sound (first 1 s), but a demand has occurred to analyze the change of bell sound energy in time. The partials in the bell sound commonly have different decay rate in time. Therefore, its power spectral distribution changes in time. The three-channel self-adjustable filter bank composed of 9th order EMQF filter pairs was designed for such analysis. The structure of the bank was that of Fig. 2, and the self-adjustment was performed by using the algorithm described in subsection 2.2. The signal frame length was 0.5 s and successive frames overlapped for 75 %.

The following conditions were appointed for the self-adjusting procedure:

(i) Power distribution ratio for the first channel, p_{r1}:

$$60 - EV \leq p_{r1} \leq 60 + EV \,, \qquad (19)$$

(ii) Power distribution ratio for the both of first two channels, $p_{r1}+p_{r2}$:

$$90 - EV \leq p_{r1} + p_{r2} \leq 90 + EV \,. \qquad (20)$$

Recoil that p_{ri} is defined by (16).

Figs. 6 and 7 present the results of such an analysis for two different bells. The results show information about the time dependence of the bell sound power spectral distribution (the variation of 60% and 90% of bell sound energy in time). The frames at the beginning of the signal are important for the determination of the strike note of a bell [12]. Fig. 6 presents that in the frames at the beginning of the bell sound subjectively ranked as "good" about 90% of the sound power is placed in the frequency range that corresponds to the first five partials. All other components of that sound carry only 10% of bell sound energy. In the sound of the bell subjectively ranked as "bad" 90% range covers not only the first five partials, but also some additional components above fifth partial, as shown in Fig. 7. Thus, the results of analysis by self-adjustable filter bank give an explanation of the results obtained in subjective testing [7].

One expects that the results obtained from the presented experiment can be used for the appropriate selection of the sound recordings and preparation of the sound samples for further investigation of the strike note of a bell [12].

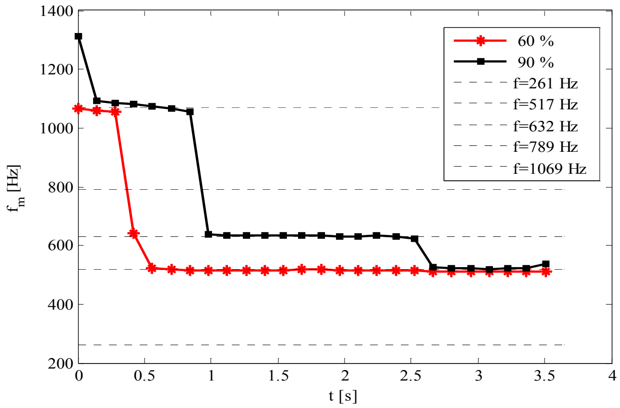

Fig. 6. The change of the power spectral distribution in time for the bell subjectively ranked as "good": variation of 60% and 90% of sound energy (frequencies of first five partials are marked).

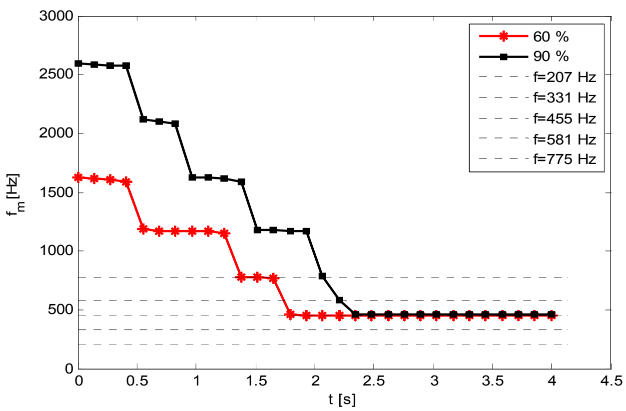

Fig. 7. The change of the power spectral distribution in time for the bell subjectively ranked as "bad": variation of 60% and 90% of sound energy (frequencies of first five partials are marked).

4. CONCLUSION

In this paper filter bank structures that are self adjustable as a function of the spectral power distribution of the input signal are presented. The self tuning procedure applies on the well-known binary search algorithm. The filter bank is based on complementary filter pairs and therefore it can be used for the processing of audio signals. The presented filter bank can be used for self-calibration of audio instruments and devices or for the off-line analysis of sound signals. Future research will be directed to developing similar filter banks suitable for real-time applications.

5. REFERENCES

[1] P. Eastty, "Accurate IIR Equalization to an Arbitrary Frequency Response, with Low Delay and Low Noise Real-Time Adjustment," *Audio Engineering Society Convention 125*, October, 2008, Paper No 7639.

[2] I. Salom, D. Todorović, L Milić, "The Influence of Impulse Response Length and Transition Bandwidth of Magnitude Complementary Crossovers on Perceived Sound Quality," *Journal of Audio Engineering Society*, Vol. 56, No. 11, November, 2007, pp. 941 - 954.

[3] T.B. Deng, "Three-channel variable filter-bank for digital hearing aids," *Intelligent Signal Processing and Communications Systems*, 2008. ISPACS 2008, February, 2009, pp.1-4.

[4] R.J. Cassidy, J.O. Smith, "A tunable, nonsubsampled, non-uniform filter bank for multi-band audition and level modification of audio signals," *Conf. Rec. 2004 Thirty-Eight Asilomar Conference on Signals, Systems and Computers*, Vol. 2, 2228-2232.

[5] U. Zölzer, Digital Audio Signal Processing, John Wiley and Sons, New York, 2nd ed. 2008.

[6] D. Šumarac Pavlović, I. Salom, J. Ćertić, "Nonuniform complementary filter bank for analysis of audio signals," *Forum Acusticum 2011*, Aalborg, Denmark, June 27–July 1, 2011

[7] I. Salom, B. Despot, J. Ćertić, M. Mijić, D. Šumarac Pavlović, "Subjective analysis of bell sound quality," (in Serbian) *ETRAN 2011*, AK2.2-1-4.

[8] J.D. Ćertić, L.D. Milić, "Investigation of computationally efficient complementary IIR filter pairs with tunable crossover frequency," *AEU - International Journal of Electronics and Communications*, vol. 65 (2011), pp. 419-428.

[9] L.D. Milic, M.D. Lutovac, "Efficient algorithm for the design of high-speed elliptic IIR Filters," *AEU - International Journal of Electronics and Communications*, vol. 57 (2003), pp. 255-262.

[10] L. Milić, T. Saramäki, Three classes of IIR complementary filter pairs with an adjustable crossover frequency, *Proc. (2003) Int. Symp. on Circuits and Systems*, vol. 4, pp. IV-145-148.

[11] D. Knuth, *The Art of Computer Programming*, Volume 3, Sorting and Searching (3rd edition ed.). Addison-Wesley. pp. 409–426, 2007.

[12] I.M. Salom, M.M. Mijić, B.D. Despot, "Aural determination of the strike note of a bell," *Proc. 2010 Inter-Noise, The 39 International Congress and Exposition on Noise Control Engineering*, Lisbon, Portugal, 13-16 June 2010