

SUBCARRIER ALLOCATION AND POWER LOADING STRATEGIES FOR MULTI-USER BROADCAST FBMC/OQAM SYSTEMS

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ABSTRACT

With the objective of maximizing the sum-rate this paper tackles the subcarrier and the power allocation problems in the context of multi-user FBMC. Under highly frequency selective channels the demodulated data is affected by ISI and ICI. As a solution the band is partitioned into three subsets so that there is no overlapping between the subcarriers of the same subset. Then we propose a low complexity heuristic to carry out the resource allocation over each subset. Simulation-based results demonstrate that at low noise regime, the proposed solution is able to outperform the conventional approach.

1. INTRODUCTION

Multicarrier modulations have been demonstrated to be a powerful technique to efficiently use the spectrum. The orthogonal frequency division multiplexing (OFDM) is the most widespread multicarrier scheme. However more advanced multicarrier solutions have been recently proposed. The improvement mainly consists in shaping the subcarrier signals with well-frequency localized waveforms. The other key feature is that no redundancy in the form of a cyclic prefix (CP) is transmitted. In the literature these schemes are known as filter bank based multicarrier (FBMC) modulations. Provided that bit streams on each subband are mapped according to the offset quadrature amplitude modulation (OQAM), a delayed version of the information can be recovered at the receive side. The perfect reconstruction property for FBMC systems employing OQAM, namely OFDM/OQAM or FBMC/OQAM, is derived in [1].

It is well-known that when the channel frequency response (CFR) is flat at each subchannel, the single tap equalizers used in OFDM are capable of restoring the orthogonality in FBMC/OQAM. With low coherence bandwidth channels,

The research leading to these results has received funding from the Spanish Ministry of Science and Innovation (Ministerio de Ciencia e Innovación) under projects TEC2011-29006-C03-02 (GRE3N-LINK-MAC) and the Catalan Government (2009SGR0891).

the flatness assumption is no longer true and the equalization stage in FBMC/OQAM has to deal with intercarrier interference (ICI) and intersymbol interference (ISI), [2, 3]. Focusing in this scenario, we tackle the user selection and the power allocation problems for the broadcast channel (BC). The objective function that governs the design in OFDM is the signal to noise ratio (SNR), [4]. By contrast, our solution is based on the signal to interference plus noise ratio (SINR). It is important to remark that in presence of ICI, the problem of jointly assigning carriers to users and allocating power to subbands is very complex. To remedy this, we devise an algorithm that separately addresses the subcarrier assignment and the power loading. The complexity is further reduced if we arrange the subcarrier indexes into three subsets and then we sequentially process each subset. Numerical results show that the proposed algorithm provides higher rates in comparison to the benchmark based on the technique of [4].

The rest of the paper is organized as follows. Section 2 reviews the multi-user system model. In Section 3 we pose the problems to find the subcarrier assignment and the power distribution. The simulation results are presented in Section 4 and the conclusions are drawn in Section 5.

2. MULTI-USER SYSTEM MODEL

In this section we focus on the downlink, where the base station provides service to U users. When the FBMC/OQAM scheme of [1] is implemented, the transmitted signal consists in frequency multiplexing M signals, which is expressed as

$$s[n] = \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} a_m[k] f_m[n - kM/2] \quad (1)$$

$$f_m[n] = p[n] e^{j \frac{2\pi}{M} m(n - (L-1)/2)} \quad (2)$$

where $a_m[k] = d_m[k] \theta_m[k]$ is the transmitted symbol on the k -th instant and m -th subcarrier. Let $d_m[k]$ be the real PAM symbol and $\theta_m[k]$ be the phase term written of the form

$$\theta_m[k] = \begin{cases} 1 & m+k \text{ even} \\ j & m+k \text{ odd} \end{cases}. \quad (3)$$

The signal $p[n]$ is the prototype pulse, which length is L . Due to its good time-frequency localization we have selected the design of [5] with an overlapping factor equal to four.

At the receive side each user obtains a degraded version of the transmitted signal, which is distorted by multipath fading and additive white Gaussian noise (AWGN). Therefore, the samples to be fed to the analysis filter bank (AFB) of the l -th user are expressed as $r_l[n] = h_l[n] * s[n] + w_l[n]$. Let $h_l[n], w_l[n]$ be the channel impulse response and the noise associated to the l -th user.

The PAM symbols $d_q[k]$ are estimated by filtering the received signal through a bank of filters and downsampling the outcomes by a factor $M/2$, which leads to the expression $y_q^l[k] = (r_l[k] * f_q^*[-k]) \downarrow_{M/2}$. Taking into account that sub-carrier signals only overlap with their adjacent neighbours the demodulated signals can be compactly formulated as follows

$$y_q^l[k] = \sum_{m=q-1}^{q+1} a_m[k] * g_{qm}^l[k] + w_q^l[k] \quad (4)$$

where $g_{qm}^l[k] = (f_q^*[-k] * h_l[k] * f_m[k]) \downarrow_{\frac{M}{2}}$ is different from zero for $-L_2 \leq k \leq L_1$. This highlights that $y_q^l[k]$ is affected by ISI and ICI. In this sense we propose to filter $y_q^l[k]$ with the broadband filters $b_{q,l}[k]$. Finally, compensating the phase term and extracting the real component we obtain a symbol estimate, which reads as $\hat{d}_q^l[k] = \Re(\theta_q^*[k] (b_{q,l}^*[k] * y_q^l[k])) = \Re(z_q^l[k])$. The estimates can be formulated in a matrix way by defining the diagonal matrix $\mathbf{D}_m[k] = \text{diag}(\theta_m[k + L_2 + L_b], \dots, \theta_m[k - L_1 - L_b])$ and the circulant matrix \mathbf{G}_{qm}^l , which first row is $[g_{qm}^l[-L_2], \dots, g_{qm}^l[L_1]0\dots0]$. Using these matrices it follows that

$$z_q^l[k] = \sum_{m=q-1}^{q+1} \mathbf{b}_{q,l}^H \bar{\mathbf{G}}_{qm}^l \mathbf{d}_m[k] + \mathbf{b}_{q,l}^H \mathbf{w}_q^l[k] \quad (5)$$

$$\mathbf{b}_{q,l} = [b_{q,l}[-L_b], \dots, b_{q,l}[L_b]]^T \quad (6)$$

$$\mathbf{d}_m[k] = [d_m[k + L_2 + L_b], \dots, d_m[k - L_1 - L_b]]^T \quad (7)$$

$$\mathbf{w}_q^l[k] = [\theta_q^*[k]w_q^l[k + L_b], \dots, \theta_q^*[k]w_q^l[k - L_b]]^T \quad (8)$$

where $\bar{\mathbf{G}}_{qm}^l = \mathbf{G}_{qm}^l \mathbf{D}_m[k] (\theta_q^*[k] \mathbf{I})$ and \mathbf{I} is the identity matrix. Let \mathbf{a} be either a vector or a matrix, its extended version is defined as $\mathbf{a}_e = [\Re(\mathbf{a}^T) \Im(\mathbf{a}^T)]^T$. This notation enables compactly formulating the estimated symbol as

$$\hat{d}_q^l[k] = \sum_{m=q-1}^{q+1} \mathbf{b}_{q,l,e}^T \bar{\mathbf{G}}_{qm,e}^l \mathbf{d}_m[k] + \mathbf{b}_{q,l,e}^T \mathbf{w}_{q,e}^l[k]. \quad (9)$$

It is important to remark that bands are exclusively assigned to users. Without loss of generality we assume that the information conveyed on the q -th subband is intended for the $u(q)$ -th user, $u(q) \in \{1, \dots, U\}$. Assuming that symbols transmitted

on different time instants and subbands are independent and denoting p_q the power of the symbol transmitted on the q -th subband, it can be checked that the SINR on the q -th subband is the same for $q + k$ even and $q + k$ odd. Thus, we can drop the index k and write the expression as follows

$$\text{SINR}_q = \frac{p_q \left\| \mathbf{b}_{q,u(q),e}^T \bar{\mathbf{G}}_{qq,e}^{u(q)} [k] \mathbf{e}_l \right\|^2}{i_q^{u(q)} + z_q^{u(q)}} = \frac{p_q h_q^{u(q)}}{i_q^{u(q)} + z_q^{u(q)}} \quad (10)$$

$$i_q^{u(q)} = \sum_{m=q-1, m \neq q}^{q+1} p_m \left\| \mathbf{b}_{q,u(q),e}^T \bar{\mathbf{G}}_{qm,e}^{u(q)} [k] \right\|^2 + p_q \left\| \mathbf{b}_{q,u(q),e}^T \bar{\mathbf{G}}_{qq,e}^{u(q)} [k] (\mathbf{I} - \mathbf{e}_l \mathbf{e}_l^T) \right\|^2 \quad (11)$$

$$i_q^{u(q)} = p_{q-1} \alpha_{qq-1}^{u(q)} + p_q \alpha_{qq}^{u(q)} + p_{q+1} \alpha_{qq+1}^{u(q)}. \quad (12)$$

Defining $N_0 = E\{|w_l[n]|^2\}$ and taking into consideration that the noise at the output of the AFB is colored, $z_q^{u(q)}$ is given by

$$z_q^{u(q)} = \frac{N_0}{2} \mathbf{b}_{q,u(q),e}^T \mathbf{C}_q \mathbf{b}_{q,u(q),e} \quad (13)$$

where the matrix \mathbf{C}_q can be easily computed off-line using the analytical expression provided in [2]. The vector \mathbf{e}_l is zero-valued except in the l -th position, thus it has the objective of selecting the desired data. For the sake of complexity we have considered that the equalizers are designed under the zero-forcing (ZF) criterion as the authors propose in [3].

3. SUBCARRIER ALLOCATION AND POWER LOADING STRATEGIES

Bearing in mind the multi-user system model, this section aims at devising the strategy to assign bands to users and distribute the power among subcarriers under the criterion of maximizing the sum-rate for a given power budget. From (10) it follows that the rate on the q -th subband is given by

$$r_q = \log_2 \left(1 + \frac{1}{\Gamma} \frac{p_q h_q^{u(q)}}{\sum_{m=q-1}^{q+1} p_m \alpha_{qm}^{u(q)} + z_q^{u(q)}} \right) \quad (14)$$

where the parameter Γ indicates the gap with respect to the theoretical capacity. Fixing the symbol error rate to 10^{-4} it takes the value 5.48 according the formula given in [6]: $\Gamma = \frac{1}{3} (Q^{-1}(\frac{SER}{4}))^2$. The index $u(q)$ may take any value within $\{1, \dots, U\}$ so that unambiguously indicates the user who has been selected to transmit on the q -th subband.

In the absence of ICI and ISI the optimal subcarrier allocation strategy in the FBMC/OQAM context coincides with OFDM. The authors in [4] have demonstrated that the optimal assignment on a given subband stems from choosing the

user who presents the highest SNR. Nevertheless, this work focuses on a more challenging scenario where ICI and ISI are not negligible. The original problem

$$P : \max_{\{p_q, u(q)\}} \sum_{q=0}^{M-1} r_q \quad (15)$$

$$s.t. \sum_{q=0}^{M-1} p_q \leq P_T, \quad p_q \geq 0, \quad u(q) \in \{1, \dots, U\},$$

which jointly designs p_q and $u(q)$, is not convex. In order to overcome this hurdle we propose a suboptimal strategy consisting in partitioning the band into three subsets, followed by a sequential optimization over these subsets. The key issue relies on the fact that subcarrier signals gathered in the same subset do not overlap in the frequency domain. A possible partitioning strategy is depicted in Fig. 1. The reason why we

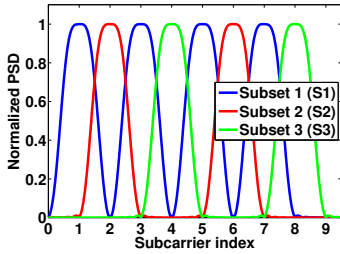


Fig. 1. Band partitioning scheme.

choose this scheme comes from the intuition that increasing the number of subproblems deviates us from the optimal solution. This suggests that selecting one subcarrier out of two is preferable to partitioning the band into three subsets. Later on in Subsection 3.2 we explain why subsets S2 and S3 have not been merged.

The problem P_i , which is associated to S_i , is given by

$$P_i : \max_{\{p_q, u(q)\}} \sum_{q \in S_i} r_q \quad (16)$$

$$s.t. \sum_{q \in S_i} p_q \leq P_{T_i}, \quad p_q \geq 0, \quad u(q) \in \{1, \dots, U\}.$$

Note that splitting the original problem into three subproblems also entails partitioning the power budget. Since problems are executed sequentially when solving P_i the interference that comes from the subcarriers belonging to S_j ($j < i$) are treated as noise and those from S_l ($l > i$) are nonexistent because these subsets have not been processed yet.

It is worth mentioning that (16) can be solved resorting to the dual optimization framework [7]. Aiming at reducing the complexity we decouple the subcarrier and the power allocation problems. In this sense, we first associate the subcarriers collected in S_l to users and then the power is distributed among S_l . Subsequently we proceed exactly in S_2 and finally in S_3 .

3.1. Subcarrier allocation algorithm

In this subsection we propose to find out the subcarrier allocation in the downlink, which maximizes the data throughput. Provided that we focus on subset S_i , the solution boils down to find the piecewise-maximum of all the SINRs in the corresponding subband as follows

$$u(q) = \operatorname{argmax}_{\{1 \leq j \leq U\}} \left(\frac{p_q h_q^j}{p_q \alpha_{qq}^j + B_q^j} \right), \quad q \in S_i \quad (17)$$

$$B_q^j = p_{q+1} \alpha_{q+1}^j + p_{q-1} \alpha_{q-1}^j + z_q^j. \quad (18)$$

As we have discussed earlier the partitioning strategy shown in Fig. 1 alleviates the complexity because the term B_q^j can be treated as noise. Note that the selection depends on the power to be allocated. This highlights that some assumptions have to be made as for the power coefficients. In this sense we assume that p_q will lie within this interval $[b_q, a_q]$. Then, the strategy consists in collecting the indexes that solve (17) for each value belonging to $[b_q, a_q]$. After checking the set that contains all the indexes we select the user, which SINR function is higher than the rest with the largest number of points. It is important to remark that the complexity can be dramatically reduced by thoroughly analysing the SINR expression. First note that the fraction $\frac{hp}{B+\alpha p}$ saturates at h/α and takes the value 0 at the origin. Besides it is monotonically increasing for $p > 0$ and concave since the second derivative is negative. Bearing this in mind and supposing that there are only two users ($U=2$), it can be easily verified that the SINR functions coincide at this point

$$t_q = \frac{B_q^2/h_q^2 - B_q^1/h_q^1}{\alpha_q^1/h_q^1 - \alpha_q^2/h_q^2}. \quad (19)$$

As a consequence, if $\alpha_q^1/h_q^1 < \alpha_q^2/h_q^2$ the user selection described above boils down to selecting the user 1 when $t_q < 0.5(a_q + b_q)$ or user 2 when $t_q \geq 0.5(a_q + b_q)$. Conversely, provided that $\alpha_q^1/h_q^1 > \alpha_q^2/h_q^2$ we choose the user 2 when $t_q < 0.5(a_q + b_q)$ or user 1 when $t_q \geq 0.5(a_q + b_q)$. This strategy saves us evaluating the SINRs in the interval $[b_q, a_q]$. Hence the midpoint plays a key role to reduce the complexity. The Algorithm 1 describes how the idea proposed in this section can be extended to the case where the number of users is higher than two.

In this paper we have assumed that the reduction in the sum-rate, as a consequence of distributing the power according to the uniform power allocation (UPA) strategy, is reasonably small. Hence, we have set the midpoint of $[b_q, a_q]$ to $P_{T_i}/|S_i|$, when the i -th subset is addressed. Alternatively, the midpoint may be calculated after setting a_q to any given spectral mask constraint and b_q to the minimum power required to transmit one bit on the q -th subband.

Algorithm 1 Subcarrier allocation.

```
1: Set the best user to be the first,  $BU=1$ 
2: for  $u = 2 : U$  do
3:   Compute the point where the SINRs of the  $BU$ -th user
   and the  $u$ -th user cross according to (19)
4:   if  $t_q < 0.5(a_q + b_q)$  then
5:      $BU = \operatorname{argmin} \{ \alpha_q^u / h_q^u, \alpha_q^{BU} / h_q^{BU} \}$ 
6:   else
7:      $BU = \operatorname{argmax} \{ \alpha_q^u / h_q^u, \alpha_q^{BU} / h_q^{BU} \}$ 
8:   end if
9: end for
```

3.2. Optimal and suboptimal power allocation

Once the subcarrier assignment is addressed, the power distribution is obtained by solving

$$\begin{aligned} \max_{\{p_q\}} \sum_{q \in S_i} \log_2 \left(1 + \frac{p_q h_q}{\Gamma p_q \alpha_{qq} + B_q} \right) \\ \text{s.t. } \sum_{q \in S_i} p_q \leq P_{T_i}, \quad p_q \geq 0. \end{aligned} \quad (20)$$

For simplicity we have adopted the same notation used in the single user scenario. Notice that (20) is a concave maximization problem, thus there exists a global optimal point that can be found by using for example the MATLAB OPTIMIZATION TOOLBOX. Alternatively, we may allocate the power according to the UPA strategy, i.e. $p_q = P_{T_i} / |S_i|$ ($q \in S_i$).

At this point we could have solved (16) via the dual formulation [7], or heuristically as we propose in this paper. In both cases the interferences that leak through the adjacent subbands prompt rate degradation since r_{q+1} and r_{q-1} are monotonically decreasing in p_q . Consequently, it might happen that the new rate r_q does not compensate for the rate reduction in bands $q-1$ and $q+1$. To prevent this from happening it is mandatory to evaluate the aggregate: $SR_q(p) = \sum_{m=q-1}^{q+1} r_m(p_q = p)$ before and after solving (16). Let p_q^* be the power that is computed after executing any resource allocation algorithm. If

$$SR_q(0) > SR_q(p_q^*), \quad (21)$$

then we have to compute the value $s_q \in [0, p_q^*]$ that maximizes $SR_q(s_q)$ and repeat the resource allocation in the rest of subcarriers. For practicality reasons instead of calculating the maximum, which may be extremely complex, we choose the optimal value from the finite discrete set $\{0, \Delta_q^1, \dots, \Delta_q^{N_q}\}$. The l -th element of the set, which corresponds to the power required to transmit l bits on the q -th subband, is given by

$$\Delta_q^l = \frac{\Gamma(2^l - 1) B_q}{h_q - \alpha_{qq} \Gamma(2^l - 1)}. \quad (22)$$

The parameter N_q is selected as $N_q = \left\lceil \log_2 \left(1 + \frac{h_q p_q^*}{B_q + \alpha_{qq} p_q^*} \right) \right\rceil$. The overall algorithm is summarized hereinafter.

Algorithm 2 Power allocation.

```
1: Execute the resource allocation algorithm
2: for  $q \in S_i$  do
3:   if  $SR_q(0) > SR_q(p_q^*)$  then compute
4:      $\Delta_q^l$  ( $l = 1, \dots, N_q$ )
5:      $p_q^* = \operatorname{argmax} \{ SR_q(0), \dots, SR_q(\Delta_q^{N_q}) \}$ 
            $\{0, \Delta_q^1, \dots, \Delta_q^{N_q}\}$ 
6:      $S_i \rightarrow S_i \setminus q$  and  $P_{T_i} \rightarrow P_{T_i} - p_q^*$ 
7:   end if
8: end for
9: if (21) has been satisfied at least once then go to 1
10: end if
```

One of the benefits of separating the power and the subcarrier allocation is that there is no need to repeat the user selection if (21) holds. Instead, we may solely repeat the power loading according to the UPA or OPA. It must be highlighted that if we merge $S2$ and $S3$ it is not possible to measure the damage of a power allocation to other subbands on a per-subcarrier basis. Alternatively, all the bands collected in $S2 \cup S3$ should be jointly processed in order to determine if a given subcarrier should be given less power.

4. SIMULATION RESULTS

In this section we evaluate the sum-rate in the BC when the FBMC/OQAM transmultiplexer is implemented. Once the bands are assigned to users as subsection 3.1 exposes, the power is distributed among subcarriers according to OPA or UPA. Regarding how the power budget is split among subsets we have set $P_{T1} = P_T/2$ and $P_{T2} = P_{T3} = P_T/4$. We have empirically observed that this configuration gives satisfactory results and outperforms other strategies that increase P_{T2} and P_{T3} to the detriment of P_{T1} .

The first benchmark to compare with is based on the proposed band partitioning strategy. However, (16) is not decoupled into two problems but it is solved in a Lagrange-dual way, [7]. The second benchmark consists in assigning bands exclusively to users by evaluating the SNR as in OFDM, [4]. Next the power coefficients are computed. Unlike OFDM, the power allocation problem in FBMC/OQAM is not convex due to ICI. However, the objective function can be interpreted as a difference of concave functions (DC), which enables using branch and bounds methods to obtain the global optimal solution, [8]. The suboptimal strategy presented in [9], which is identified in simulations as DC, enables achieving a large portion of the maximum sum-rate with a reduced complexity.

Regarding the system parameters there are $M=512$ carriers and the frequency sampling is $f_s = 11.2$ MHz. This leads to a subcarrier spacing of $\Delta_f = 21.88$ KHz. The propagation conditions obey the ITU-Vehicular B channel model.

In Fig. 2 we have plotted the average rate $\frac{1}{M} \sum_{q=0}^{M-1} r_q$

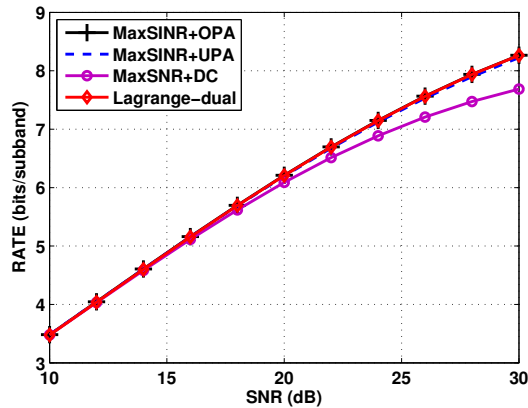


Fig. 2. Rate vs. SNR for $U=4$.

against the signal to noise ratio defined as $\frac{P_T}{MN_0}$. Note that the plots of the MaxSINR+UPA, MaxSINR+OPA and Lagrange-dual virtually coincide. However, the most interesting result is that the proposed solutions clearly outperform the MaxSNR+DC at the low noise regime.

In Fig. 3 we investigate the impact of the user diversity when $\text{SNR}=30\text{dB}$. As expected the more users are present in the coverage area the higher the rate is. Since the MaxSINR+UPA, MaxSINR+OPA and Lagrange-dual nearly give the same performance we can conclude that separating the subcarrier and the power allocation results in a marginal rate degradation. Furthermore, the strategy of equally distributing the power performs close to the OPA. It must be highlighted that when there are 9 users connected to the base station the gain may amount up to 0.9 bits/subband, when the comparison is made between our solution and the MaxSNR+DC.

5. CONCLUSIONS

This paper tackles the subcarrier allocation and the power loading problem in the FBMC/OQAM context. The numerical results confirm that the conventional approach based on the SNR figure is no longer optimal in presence of ICI and ISI. Since the problem in presence of crosstalk is really complex to be solved we have applied a relaxation based on dividing the original problem into three simpler problems. Simulation-based results allows us to conclude that the performance strongly relies on the subcarrier allocation as well as separating the subcarrier and the power allocation has a marginal impact on the rate. As a result the low-complexity solution of equally splitting the power among subcarriers becomes very attractive in FBMC/OQAM.

6. REFERENCES

[1] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank the-

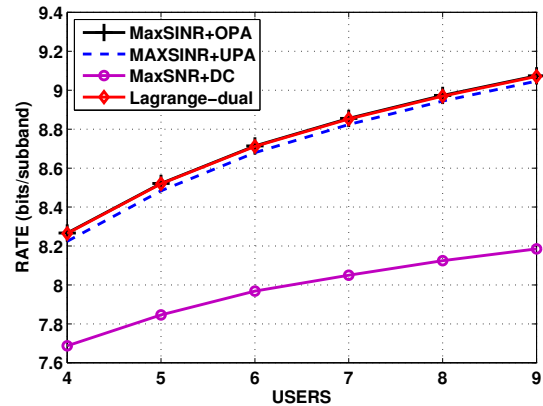


Fig. 3. Rate vs. the number of users for $\text{SNR}=30\text{dB}$.

ory," *Sig.l Proc., IEEE Trans.*, vol. 50, no. 5, pp. 1170–1183, May 2002.

- [2] D.S. Waldhauser, L.G. Baltar, and J.A. Nossek, "MMSE subcarrier equalization for filter bank based multicarrier systems," in *SPAWC*, July 2008, pp. 525–529.
- [3] T. Ihalainen, T. H. Stitz, M. Rinne, and M. Renfors, "Channel equalization in filter bank based multicarrier modulation for wireless communications," *EURASIP JASP*, vol. 2007, January 2007.
- [4] Jiho Jang and Kwang Bok Lee, "Transmit power adaptation for multiuser OFDM systems," *Selected Areas in Communications, IEEE Journal on*, vol. 21, no. 2, pp. 171–178, Feb 2003.
- [5] M.G. Bellanger, "Specification and design of a prototype filter for filter bank based multicarrier transmission," in *ICASSP*, 2001, pp. 2417–2420.
- [6] K. El Baamrani, V.P.G. Jimenez, A.G. Armada, and A.A. Ouahman, "Multiuser Subcarrier and Power Allocation Algorithm for OFDM/Offset-QAM," *Signal Processing Letters, IEEE*, vol. 17, no. 2, pp. 161–164, Feb 2010.
- [7] Jianwei Huang, V.G. Subramanian, R. Agrawal, and R.A. Berry, "Downlink scheduling and resource allocation for OFDM systems," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 1, pp. 288–296, Jan. 2009.
- [8] K. Eriksson, Shuying Shi, N. Vucic, M. Schubert, and E.G. Larsson, "Globally optimal resource allocation for achieving maximum weighted sum rate," in *GLOBECOM*, Dec. 2010, pp. 1–6.
- [9] Nikola Vucic, Shuying Shi, and Martin Schubert, "DC programming approach for resource allocation in wireless networks," in *The 6th Workshop on Resource Allocation in Wireless Networks*, 2010, pp. 380–386.