OPTIMAL LOCAL DETECTION FOR SENSOR FUSION BY LARGE DEVIATION ANALYSIS

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ABSTRACT

Fusion is widely used to improve the overall detection performance in applications such as radar, wireless sensor networks, wireless communications, spectrum sensing and so on. While the optimum fusion strategy for any preset local decision performance can be easily obtained by the Neyman-Pearson lemma, the selection of the local detection strategy that optimizes the global performance is intractable due to its complexity and the limited global information at local detectors. In this paper, we use large deviation analysis to determine a local decision rule to optimize the asymptotic global performance. Some interesting properties of the decision rule are observed. Numerical results show that our proposed strategy approximates the optimal performance very well even with a small number of local detectors.

***Index Terms*—** sensor fusion, optimal local detection strategy, large deviation analysis, asymptotic performance, global performance.

1. INTRODUCTION

Signal detection is a common problem in applications including radar, wireless sensor networks, wireless communication systems, cognitive radio spectrum sensing, and so on. To enhance performance, a fusion center collects information from multiple local detectors and makes a global decision. Due to the bandwidth constraint, the local detectors often make decisions first and transmit the one bit decisions to the fusion center. Accordingly, the entire process is called detection fusion or decision fusion [1].

In the pioneering work of Tsitsiklis on this problem [2], it has been shown that while the fusion strategy can be easily obtained by the Neyman-Pearson (NP) lemma, the selection of a local decision rule to optimize the global performance is mathematically intractable. In the current literature, some work fixes the fusion rule and then obtains the optimal local decision rule [3]; whereas others compare the fusion detection performance for various local decision rules, including the locally optimal minimized average error probability [4], the maximum decision output entropy [5] and the largest divergence between the statistical distribution under different hypotheses [6, 7]. None of these detectors is optimum.

Recently, some asymptotic analyses for detection fusion have been reported in literature. For example, [8] develops a fusion rule for channel distorted decisions using a Chernoff exponent bound analysis. Similar analysis is followed in [9] to obtain an asymptotically optimum fusion rule for an M-hypothesis testing problem, and in [10] for non-centralized distributed fusion. However, these papers focus only on designing the *fusion* rule, while the optimum local decision strategy remains an open problem. In this paper, our goal is to find an optimal *local* decision strategy that optimizes the asymptotic global performance.

We will deal with a parallel fusion structure [11] and work with a binary hypothesis testing problem. By large deviation analysis, we will optimize the local thresholds to obtain the best global performance, asymptotically in the number of local detectors. Compared with existing work in the literature, our method has a lower complexity and guarantees the global optimal performance, asymptotically. Some interesting properties of the optimal strategy will also be discussed. Then, with a specific example of cooperative energy sensing, we will demonstrate the optimality of our proposed algorithm.

This paper is organized as follows: we first present the general signal model for detection fusion in Section 2 and formulate the joint optimization problem in Section 3. Then, we will present the error exponent expressions in Section 4 and develop the asymptotically optimized local detection strategy accordingly in Section 4. Finally, we present a case study to compare performance under various local decision strate-

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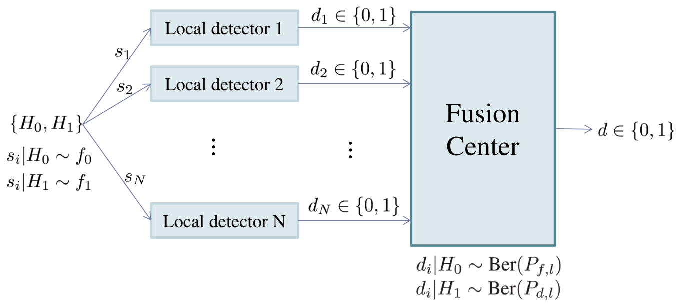

Fig. 1. System diagram for detection fusion.

gies in Section 6 and give concluding remarks in Section 7. Throughout the paper, $X \sim \mathcal{CN}(\mu, \sigma^2)$ denotes a random variable X following a proper complex Gaussian distribution with mean μ and variance σ^2; $d \sim \mathrm{Ber}(p)$ denotes a Bernoulli random variable; $X \sim \mathrm{Bin}(N, p_1)$ denotes a random variable X following a binomial distribution; $f(x) \sim g(x)$ means that $\lim_{x\to+\infty} \frac{f(x)}{g(x)} = c$ where c is a constant.

2. SYSTEM MODEL

The diagram for a detection fusion system is shown in Fig. 1. As depicted in this figure, there is a common random signal source which follows either distribution f_0 under hypothesis H_0, or distribution f_1 under hypothesis H_1, where $P(H_0 \text{ true}) = \pi_0$ and $P(H_1 \text{ is true}) = \pi_1$ are the a priori probabilities of the hypotheses. Each local detector will make its own local decision $d_i \in \{0, 1\}$ based on its own observed signal s_i. Then, a fusion center will collect all local decisions d_is and make a global decision $d \in \{0, 1\}$ accordingly.

It has been shown that in the case that the signals at local decisions are dependent, the solution for optimal detection fusion is non-deterministic polynomial-time hard [12]. Therefore, in our analysis, we assume that the signals at different detectors are independent, which is true in many real applications. Then, in [2], it is proved that to obtain asymptotically optimal performance, all local detectors should follow the same decision rule. Under this strategy, the d_is are independently identically distributed.

To describe the distributions of the d_is at the fusion center, we denote $P_{f,l} = P(d_i = 1|H_0)$ as the local false alarm probability and $P_{d,l} = P(d_i = 1|H_1)$ as the local detection probability. Then, $(P_{f,l}, P_{d,l}) \in [0, 1] \times [0, 1]$ is called the receiver operating characteristic (ROC) curve. The local decision d_i follows a Bernoulli distribution with $P_{f,l}$ and $P_{d,l}$ under hypothesis H_0 and H_1, respectively. At the fusion center:

$$
\begin{aligned}
P(d_1, d_2, \ldots, d_N|H_0) &= P_{f,l}^{\sum_{i=1}^N d_i} (1 - P_{f,l})^{N-\sum_{i=1}^N d_i} \ , \\
P(d_1, d_2, \ldots, d_N|H_1) &= P_{d,l}^{\sum_{i=1}^N d_i} (1 - P_{d,l})^{N-\sum_{i=1}^N d_i} \ .
\end{aligned} \tag{1}
$$

Accordingly, $d_s = \sum_{i=1}^{N} d_i$ is the sufficient statistics and it follows a binomial distribution under each hypothesis.

3. OPTIMUM LOCAL AND FUSION DECISIONS

In this paper, we adopt the global average error probability as the performance metric, i.e., $P_e = \pi_0 P(d = 1|H_0) + \pi_1 P(d = 0|H_1)$. To obtain the best performance, we want to find a local threshold and a corresponding fusion rule that minimizes P_e.

The Bayesian detector will minimize P_e by implementing the likelihood ratio test [13]

$$
\frac{\pi_1 P_{d,l}^{d_s}(1 - P_{d,l})^{N-d_s}}{\pi_0 P_{f,l}^{d_s}(1 - P_{f,l})^{N-d_s}} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \ , \tag{2}
$$

and the corresponding minimized P_e can be calculated. Notice that as long as $(P_{f,l}, P_{d,l})$ is known to the fusion center, the optimal fusion rule can be easily obtained according to Eq. (2).

From Eq. (2), it is easy to verify that for any given local false alarm probability $P_{f,l}$, the larger the local detection probability $P_{d,l}$ is, the smaller the global average error probability P_e will be. Therefore, at local detectors, the NP detector or equivalently the maximum likelihood (ML) detector [13] should be adopted to achieve the best performance:

$$
\frac{f_1(s_i)}{f_0(s_i)} \underset{H_0}{\overset{H_1}{\gtrless}} L \ . \tag{3}
$$

However, this will only give an ROC curve $(P_{f,l}, P_{d,l})$ for the local detectors. How to select the optimal point $(P_{f,l}^o, P_{d,l}^o)$ on the ROC of the NP detector according to Eqs. (2) and (3) is usually a non-convex and mathematically intractable problem. In addition, the optimization process involves the number of local detectors N, which is not always available to local detectors.

In this paper, we will use large deviation analysis to obtain the optimal local decision strategy, i.e. $(P_{f,l}^o, P_{d,l}^o)$ to minimize the global average error probability P_e, asymptotically in N.

4. ERROR EXPONENT EXPRESSIONS

As introduced in Section 2, the sufficient statistic at the fusion center $d_s = \sum_{i=1}^N d_i$ follows a binomial distribution:

$$
\begin{aligned}
H_0 &: d_s \sim \mathrm{Bin}(N, P_{f,l}) \ , \\
H_1 &: d_s \sim \mathrm{Bin}(N, P_{d,l}) \ .
\end{aligned} \tag{4}
$$

Let the fusion threshold be $P_{f,l}N < \eta_f = \theta_F N < P_{d,l}N$. Then by large deviation analysis, the global error probabilities are asymptotically [14]:

$$\begin{aligned} P_f = P(d_s \geq \theta_F N | H_0) & \sim e^{-NE_0} , \\ P_{md} = P(d_s < \theta_F N | H_1) & \sim e^{-NE_1} , \end{aligned} \tag{5}$$

where

$$\begin{aligned} E_0 &= \theta_F \log \frac{\theta_F}{P_{f,l}} + (1-\theta_F) \log \frac{1-\theta_F}{1-P_{f,l}} = D_{KL}(\theta_F || P_{f,l}) , \\ E_1 &= \theta_F \log \frac{\theta_F}{P_{d,l}} + (1-\theta_F) \log \frac{1-\theta_F}{1-P_{d,l}} = D_{KL}(\theta_F || P_{d,l}) , \end{aligned} \tag{6}$$

and $D_{KL}(\cdot)$ denotes the Kullback-Leibler divergence [15]. Accordingly, the overall probability of error is given by:

$$P_e = \pi_0 P_f + \pi_1 P_{md} \sim \pi_0 e^{-NE_0} + \pi_1 e^{-NE_1} \sim e^{-N \min(E_0, E_1)} \tag{7}$$

5. ASYMPTOTICALLY OPTIMAL LOCAL DECISION

To minimize this global average error probability asymptotically, we need to maximize $\min(E_0, E_1)$. Hence, the problem becomes:

$$\max_{P_{f,l}, P_{d,l}, \theta_F} \min(E_0, E_1) \tag{8}$$

It should be noticed that when $P_{f,l} < \theta_F < P_{d,l}$, $E_0(\theta_F)$ is an increasing function of θ_F and $E_1(\theta_F)$ is a decreasing function of θ_F. As a result, the maximum value of $\min(E_0(\theta_F), E_1(\theta_F))$ is achieved when $E_0(\theta_F) = E_1(\theta_F)$. According to Eq. (6),

$$\theta_F^o = \frac{\log \frac{1-P_{d,l}}{1-P_{f,l}}}{\log \frac{P_{f,l}}{P_{d,l}} + \log \frac{1-P_{d,l}}{1-P_{f,l}}} . \tag{9}$$

According to Eqs. (8) and (9), the optimal local decision rule can be obtained as follows:

$$(P_{f,l}^o, P_{d,l}^o) = \arg \max_{(P_{f,l}, P_{d,l})} D_{KL}(\theta_F^o || P_{f,l}) , \tag{10}$$

where θ_f^o is parameterized by $(P_{f,l}, P_{d,l})$ according to Eq. (9).

Recall that for local detectors, we already have an NP detector ROC curve which can represent $P_{d,l}$ as a function of $P_{f,l}$. So, Eq. (10) can be interpreted as a search over the ROC curve to find a point which leads to the maximum error exponent. Although $(P_{f,l}, P_{d,l})$ is two-dimensional, it only has a one-dimensional degree of freedom, namely the local threshold. This renders the optimization a one dimensional problem. In fact, under many signal models, the NP local detectors are in the form of a scalar sufficient statistic compared to a single threshold and in this case $P_{f,l}$ and $P_{d,l}$ can often be represented by this threshold analytically in closed form. Therefore, the global average error exponent can be rewritten as a single-variable function. The objective function $D_{KL}(\theta_F^o || P_{f,l})$ is uni-modal in many scenarios and hence can be easily optimized by line search techniques such as those in [16, Chapter 7].

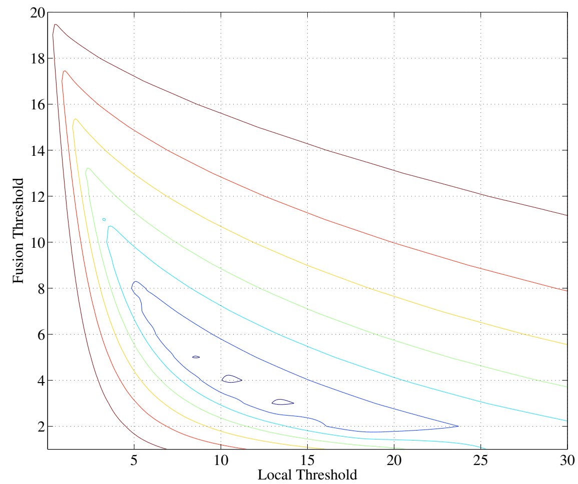

Fig. 2. The performance surface under local and fusion thresholds with $N = 20$.

Note that although Eq. (9) gives an asymptotically optimal fusion threshold, the fusion center always uses an NP detector according to Eq. (2) to obtain the best fusion performance.

Remarks:

1. Asymptotically, the optimal local decision strategy is *independent* of the total number of sensors N, but only dependent on the signal model s_i under the original hypotheses. This enables the global optimization even when the local distributed detectors do not know the network size N. In fact, if the sensors have sufficient computing resources, the local thresholds could be periodically recomputed locally if the distribution of s_i changes over time.

2. Asymptotically, the optimal local decision strategy is independent of the a priori probabilities. This is due to the fact that when N approaches infinity, the π_0 and π_1 terms in Eq. (2) will contribute very little to the likelihood ratio.

6. EXAMPLE: ENERGY SENSING

To illustrate our solution for the asymptotically optimum detection fusion, we adopt the specific signal model for a cooperative energy sensing problem as an example and show the performance comparisons.

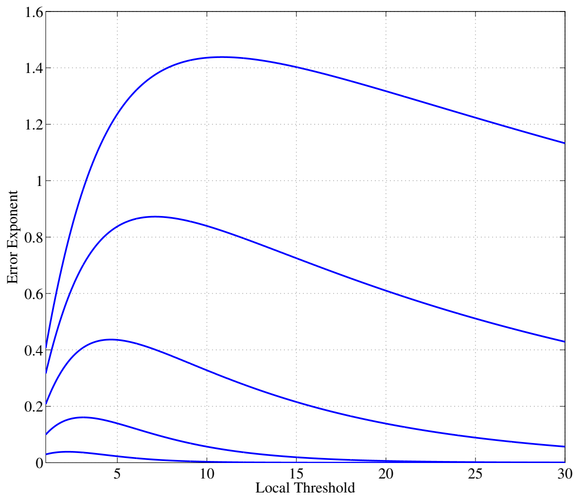

Fig. 3. Error exponent under different local thresholds. From bottom to top, the per sensor SNR is $\gamma = 0, 5, 10, 15, 20$ dB.

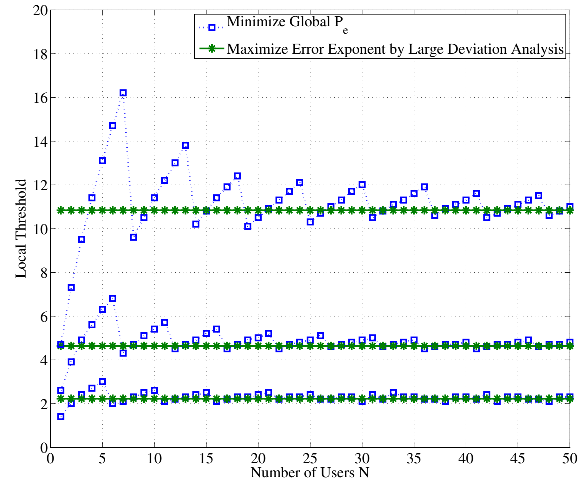

Fig. 4. Local decision thresholds under joint optimization by exhaustive search and large deviation analysis. From bottom to top, the per sensor SNR is $\gamma = 0, 10, 20$ dB.

6.1. Signal Model

In the energy sensing problem, the task is to determine whether there is a signal transmitted over a certain channel (H_1) or not (H_0). Under Rayleigh fading and additive white Gaussian noise, the normalized signal model for local detectors is [4]:

$$\begin{aligned} s_i|H_0 &= n \sim \mathcal{CN}(0,1) \\ s_i|H_1 &= hx + n \sim \mathcal{CN}(0,\gamma+1) \end{aligned} \tag{11}$$

where n is white Gaussian noise, h is a Rayleigh fading channel, x is the transmitted signal and γ is the average signal to noise ratio (SNR). Under this signal model, the NP detector is the energy detector:

$$\|s_i\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta \; . \tag{12}$$

Correspondingly, the local false alarm and detection probabilities are

$$\begin{aligned} P_{f,l} &= e^{-\eta} \; , \\ P_{d,l} &= e^{-\frac{\eta}{\gamma+1}} \; . \end{aligned} \tag{13}$$

6.2. Numerical Results

To gain a better understanding of the detection fusion optimization problem, we first plot in Fig. 2 the performance (P_e) surface vs. the local and fusion thresholds. In this figure, the number of local detectors is $N = 20$. Evidently, there are 4 local minima. This verifies our discussions of the non-convexity in Section 3. In addition, the number of local minima will increase with N.

For large deviation analysis, we plot the error exponent under different local decision thresholds in Fig. 3. In this figure, it can be observed that with the energy sensing signal model, the error exponent is a uni-modal function of the local threshold. Therefore, the optimal local threshold can be easily found using a one-dimensional line search algorithm.

The local thresholds for joint optimization by exhaustive search, and the thresholds by large deviation analysis for several different per sensor SNR values are plotted vs. N in Fig. 4. It can be verified that as the number of local detectors increases, the local thresholds obtained by the joint optimization will converge to the threshold given in our large deviation analysis.

In Fig. 5, we compare the performance of the large deviation solution with existing ones, including the local average error probability minimization $\min(P_{f,l} + 1 - P_{d,l})$ [4], the decision output entropy maximization [5] with $P_{f,l} = P_{d,l}$ (or equivalently the balanced detector [13]) and the mutual information maximization between decision and hypothesis with $\max(I(H, d_i))$ [7]. We also present the performance limit by optimizing the local thresholds via exhaustive search. Note that in all cases, the fusion threshold is obtained accordingly to Eq. (2). In Fig. 5, we plot the global average error probability at per sensor SNR $\gamma = 15$ dB as a function of the number of local detectors N. It can be observed that the average error probability does decay exponentially with N as the large deviation analysis indicates. In addition, our proposed method approaches the optimized detection fusion by exhaustive search very well and actually does not require N to be very large to approach the optimal performance.

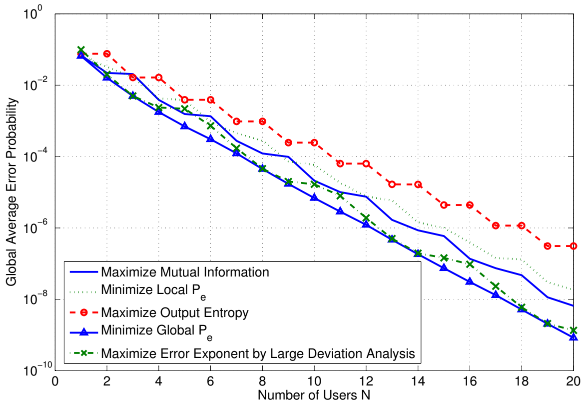

Fig. 5. Global average error probability under different local decision criteria at per detector SNR $\gamma = 15$ dB as a function of the number of local detectors N.

7. CONCLUSIONS

In this paper, large deviation analysis is used to derive the asymptotically optimal local detection strategy for detection fusion. Asymptotically, the joint optimization problem was simplified to a simple line search on an ROC curve. It was observed that the asymptotically optimal local decision rule is independent of the number of local detectors N and the a priori probabilities of the hypotheses. A cooperative energy sensing problem was considered to demonstrate our proposed approach. Numerical results verify that our proposed method approaches the optimal local detection strategy obtained by exhaustive search and has demonstrated better performance than all other reported local decision alternatives at small to moderate N values, with no additional information required at the local detectors.

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