OUTPUT RECONSTRUCTION IN AN OFB WITH INSTANTANEOUS ERASURE USING PARITY-CHECK MATRIX

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ABSTRACT

This paper introduces a new method based on parity-check matrix for reconstructing the output of an Oversampled Filter Bank (OFB) when sub-band vectors in different time instances are subject to erasures with arbitrary patterns (i.e. *instantaneous erasure*). Using the property of parity-check matrix, in this method, the erased samples of the sub-band vectors are recovered in the sub-band domain and then they are fed to the synthesis filters for output perfect reconstruction. In particular, we provide the theoretical sufficient conditions on OFB structure in order for this approach to be applicable for (maximum) erasure recovery. Furthermore, we modify the proposed reconstruction method in the presence of quantization noise and suggest an approach based on consistent reconstruction. We will also provide simulation results to evaluate the performance of this reconstruction method.

Index Terms— Instantaneous Erasure, Consistent Reconstruction, Oversampled Filter Bank, Parity-Check Matrix

1. INTRODUCTION

Oversampled Filter Banks (OFBs) can be considered as error/erasure correcting codes which act over the field of real numbers [1, 2, 3]. More specifically, this is because the process of injecting redundancy for potential error/erasure correcting purposes happens on real sequences before quantization which is in contrast with classic finite field and binary codes. Nevertheless, it has been shown that there exist several parallelisms between OFBs and binary codes where OFBs generalize different concepts such as syndrome and parity-check matrix from binary domain to the real field [2]. In this paper, we intend to exploit the properties of parity-check matrix in OFBs for output perfect reconstruction under the condition that samples of sub-band signals are subject to instantaneous erasure [4]. Instantaneous erasure accounts for a situation when samples of different sub-band vectors are erased based on different erasure patterns. Application of this type of erasure can be envisaged in a packet loss scenario when during packetization, sub-band samples are properly interleaved, or when erasures are introduced deliberately in the sub-band as part of a data rate reduction strategy similar to puncturing.

We have previously presented a two-step method for output reconstruction under the above conditions [4]: In the first step, synthesis filters are changed based on the current erasure pattern and output is reconstructed. In the second step, the newly reconstructed outputs are used to recover erased sub-band samples in the most recently received sub-band vector. Since on the synthesis side, output samples can not be reconstructed perfectly in presence of possible noises such as quantization, this method suffers from catastrophic degradation specially for large number of erasures [4]. Here, we pursue another approach where the erased samples in sub-band vectors are first recovered using parity-check matrix. Thus, the output can be reconstructed using the original synthesis filters. To achieve this goal, we utilize the orthogonality of parity-check matrix and analysis polyphase matrix. Under ideal circumstance (absence of any noise), we find the theoretical conditions that should be met by OFB in order to recover the erased samples and reconstruct the output. Next, by introducing quantization noise in the sub-band, we modify the proposed method such that it would result in the *consistent* reconstruction [5] of erased sub-band samples. The modification of the proposed method is based on the parity-check test of the reconstruction method described recently [6] in an error correction application of OFB. There, the authors propose an output reconstruction approach using a suboptimal (and thus less computationally intensive) ML estimator and the parity-check test is used to eliminate outputs which are not consistent. Using simulation results, we evaluate the performance of the reconstruction method based on parity-check matrix and show that it can improve the resilience of an OFB against erasure. The rest of this paper is organized as follows. In Section 2, we review the reconstruction method we have proposed before and then we lay out the new reconstruction method based on parity-check matrix. In Section 3, we find the conditions on OFB structure which will guarantee the applicability of the parity-check matrix for maximum erasure recovery. In Section 4, the parity-check matrix method is modified in the case of quantization noise in sub-band domain. Section 5 is dedicated to simulation results and finally conclusions are made in Section 6.

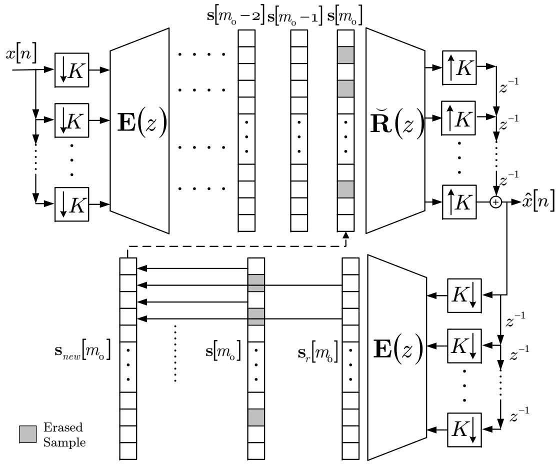

Fig. 1. [4] Illustration of the reconstruction based on synthesis filters change for an instantaneous erasure at time $n_0 = m_0K$. First, based on the current erasure pattern, the new synthesis filters are calculated and one K-sized block of output is reconstructed (First Step). Then the erased samples are recovered by passing the reconstructed output through the local analysis filters (Second Step) and a new version of sub-band vector is generated ($\mathbf{s}_{new}[m]$). This new version will replace $\mathbf{s}[m]$ for reconstructing the output signal in the future.

2. OUTPUT RECONSTRUCTION WITH INSTANTANEOUS ERASURE IN SUB-BAND DOMAIN

Suppose an OFB with N bands and downsampling rate $K < N$ where all filters have length $L = MK$ for an integer M. Under the instantaneous erasure condition, we assume a transmission scenario where the current sub-band vector at time $n_0 = m_0K$ is received with e samples erased ($0 \leq e \leq N-K$). Without loss of generality, we further assume that no erasure has happened before, so the current sub-band vector is the only one with erasure so far. Under this circumstance, the following two approaches can be utilized for output reconstruction.

2.1. Reconstruction Based on Synthesis Filters Change

This method has been depicted in Fig.1. In this figure, $\mathbf{E}(z)$ and $\mathbf{R}(z)$ refer to the analysis and synthesis polyphase matrices respectively. As it can be seen, in the first step, the new synthesis filters are calculated based on the current erasure pattern such that the new synthesis filters satisfy the instantaneous time-domain perfect reconstruction equations [4]. $\breve{\mathbf{R}}(z)$ refers to the polyphase matrix of these new synthesis filters. Next, by passing the newly constructed output samples through local analysis filters, the erased samples of current sub-band vector are recovered and will be used for reconstruction of output signal in future. It is obvious that in order for OFB to be able to reconstruct the required output samples before arriving a new sub-band vector, some constraints should be applied on OFB's delay. Additionally, some conditions should also be met for existence of synthesis filters. The following theorem provides such sufficient conditions on the OFB structure.

Theorem 1 [4]: A stable OFB with analysis polyphase matrix $\mathbf{E}(z) = \sum_{i=0}^{M-1} \mathbf{E}_i z^{-i}$ is robust to instantaneous erasure at the current time instance and the output can be reconstructed using the above method if both of the following conditions are satisfied:

(i) $\mathbf{E}_0$ remains full column rank after eliminating the rows which correspond to the erasure pattern.

(ii) $\bar{\mathbf{E}}(z)$ which is obtained by deleting the rows corresponding to the erasure pattern at time n_0 from $\mathbf{E}(z)$, has a stable inverse.

Condition (i) ensures that for any erasure pattern, $\mathbf{E}(z)$ has an inverse and a set of synthesis filters exits and condition (ii) guarantees the stability of such synthesis filters.

2.2. Reconstruction Based on Parity-Check Matrix

Suppose there exists an $(N-K) \times N$ matrix $\mathbf{C}(z) = \sum_{i=0}^{M'} \mathbf{C}_i z^{-i}$ such that $\mathbf{C}(z)\mathbf{E}(z) = \mathbf{0}$. In absence of any noise, this results in $\mathbf{C}(z)\mathbf{s}(z) = \mathbf{0}$. Here, $\mathbf{s}(z)$ indicates the z-transform of sub-band vector $\mathbf{s}[n]$ (defined as $\mathbf{s}[n] = [s_1[n], s_2[n], \cdots, s_N[n]]^T$) and is related to the input signal by $\mathbf{s}(z) = \mathbf{E}(z)\mathbf{u}(z)$. Furthermore, $\mathbf{u}(z) = [X_1(z), \cdots, X_K(z)]^T$ where $X_i(z) = \sum_{m=-\infty}^{\infty} x[mK + i - 1]z^{-m}$. The relation between parity-check matrix $\mathbf{C}(z)$ and sub-band vector in time-domain can be written as

$$\sum_{i=0}^{M'} \mathbf{C}_i\mathbf{s}[n-i] = \mathbf{C}_0\mathbf{s}[n] + \sum_{i=1}^{M'} \mathbf{C}_i\mathbf{s}[n-i] = 0. \quad (1)$$

Under the conditions we assumed at the beginning of this section, if there is any lost samples in $\mathbf{s}[n]$ (the current sub-band vector received on the synthesis side), the erased samples can be recovered by finding the solution to the following system of equations

$$\mathbf{C}_0\mathbf{s}[n] = -\sum_{i=1}^{M'} \mathbf{C}_i\mathbf{s}[n-i], \quad (2)$$

where erased samples appear as unknowns. Depending on the number of erasures, the above system might become an over-determined one. However, since the erased samples are removed from a sub-band vector which satisfies (1), the resulting over-determined system will always have a solution. Here again, it is realized that for maximum erasure recovery (when $e = N - K$) using (2), some conditions should be assumed on the OFB structure. These conditions can be summarized as follows.

c1: For the given OFB, there should exist a causal and FIR parity-check matrix $\mathbf{C}(z) = \sum_{i=0}^{M'} \mathbf{C}_i z^{-i}$. This requirement results directly from (2).

c2: Any $(N-K)\times(N-K)$ sub-matrix of $\mathbf{C}_0$ in $\mathbf{C}(z)$ should be non-singular. This requirement again results from (2) and the maximum erasures recovery.

In the next section, we find the conditions on OFB structure which will guarantee the existence of the above requirements.

3. CONDITIONS ON OFB FOR RECONSTRUCTION BASED ON PARITY-CHECK MATRIX

We start off by showing that *c1* is true for any OFB. Our proposition is based on Proposition 1 in [2] where the authors proved existence of an FIR parity-check matrix for any FIR OFB. Although such parity-check matrix can always be made causal by proper number of shifts to the right, here by a slight modification in the proof, we show the causality can be achieved directly too. This will prove instrumental in the results which we will present later on.

Proposition 1: For any causal, FIR OFB with $N\times K$ analysis polyphase matrix $\mathbf{E}(z)$ with normal rank K, there exists an $(N-K)\times N$ FIR and causal parity-check matrix $\mathbf{C}(z)$ such that $\mathbf{C}(z)\mathbf{E}(z)=\mathbf{0}$.

Proof: Let us assume $\mathbf{E}(z)$ is a polynomial matrix in z. As a result, it admits a Smith decomposition form [7] based on which we can write

$$\mathbf{E}(z)=\mathbf{V}(z)\mathbf{D}(z)\mathbf{W}(z)=\mathbf{V}(z)\left[\begin{array}{c}\mathbf{\Lambda}(z)\\ \mathbf{0}\end{array}\right]\mathbf{W}(z), \quad (3)$$

where in above $\mathbf{V}(z)$ and $\mathbf{W}(z)$ are unimodualr matrices and $\mathbf{\Lambda}(z)$ is a $K\times K$ diagonal matrix. In addition, all polynomial matrices on both sides of equality are written based on z. Now since Smith form is an equality, it should be true for any values of z thus if we change z to z^{-1}, the equality still holds (except maybe for $z=0$). As a consequence, we can claim for any causal FIR polynomial matrix, the Smith form can be written in terms of z^{-1}. Because $\mathbf{V}(z)$ is unimodular, causal and FIR, its inverse is unimodular, causal and FIR too. So, one can partition $\mathbf{U}(z)=\mathbf{V}^{-1}(z)=\left[\begin{array}{c}\mathbf{U}_1(z)\\ \mathbf{C}(z)\end{array}\right]$, where $\mathbf{U}_1(z)$ and $\mathbf{C}(z)$ are of size $K\times N$ and $N-K\times N$. Besides, $\mathbf{C}(z)\mathbf{E}(z)=\mathbf{0}$, thus the result follows. ■

Corollary 1: In Proposition 1, $\mathbf{C}_0$ in $\mathbf{C}(z)$ has full row rank.

Proof: This results from unimodularity of $\mathbf{U}(z)$ and the fact that for any square FIR unimodular matrix such as $\mathbf{U}(z)=\sum_{i=0}^{L'}\mathbf{U}_i z^{-i}$, $\mathbf{U}_0$ is non-singular [8]. So since $\mathbf{C}_0$ is a sub-matrix of $\mathbf{U}_0$, it will have full row rank accordingly.

Using Lemma 1 in [9], the order of $\mathbf{C}(z)$ in Proposition 1 can also be upper bounded by the following proposition. In what follows, Order$(\cdot)$ indicates the highest power of z^{-1} in the polynomial matrix/vector that appears between the parentheses.

Proposition 2: For a causal, FIR and full column rank $\mathbf{E}(z)$, let $r_i\in[1,N]$ for $i=1,\cdots,N$ and $c_j\in[1,K]$ for $j=1,\cdots,K$ be two series of integers such that

$$\begin{array}{c}\text{Order}(\text{row}_{r_1}\mathbf{E}(z))\geq\cdots\geq\text{Order}(\text{row}_{r_N}\mathbf{E}(z))\\ \text{Order}(\text{col}_{c_1}\mathbf{E}(z))\geq\cdots\geq\text{Order}(\text{col}_{c_K}\mathbf{E}(z)).\end{array}$$

For the parity-check matrix $\mathbf{C}(z)=\sum_{i=0}^{L'}\mathbf{C}_i z^{-i}$ defined based on Proposition 1, we have

$$L'\leq\min\left\{\sum_{i=1}^{K-1}\text{Order}(\text{row}_{r_i}\mathbf{E}(z)),\sum_{j=1}^{K-1}\text{Order}(\text{col}_{c_j}\mathbf{E}(z))\right\}. \quad (4)$$

Proof: From (3), defining unimodular matrix $\mathbf{U}(z)=\mathbf{V}^{-1}(z)$ yields $\mathbf{U}(z)\mathbf{E}(z)=\left[(\mathbf{\Lambda}(z)\mathbf{W}(z))^T,\mathbf{0}\right]^T$. Without loss of generality, let us assume $\mathbf{E}(z)$ can be partitioned as $\mathbf{E}(z)=\left[\begin{array}{c}\mathbf{E}_a(z)\\ \mathbf{E}_b(z)\end{array}\right]$ where $\mathbf{E}_a(z)$ is square $K\times K$ and non-singular. Define

$$\mathbf{A}(z)=\left[\begin{array}{cc}\mathbf{E}_a(z) & \mathbf{0}_{K\times N-K}\\ \mathbf{E}_b(z) & \mathbf{I}_{N-K}\end{array}\right]. \quad (5)$$

$\mathbf{A}(z)$ is a square $N\times N$ matrix which is non-singular and det$(\mathbf{A}(z))$ = det$(\mathbf{E}_a(z))$. We can also write $\mathbf{U}(z)\mathbf{A}(z)=\mathbf{T}(z)$ where $\mathbf{T}(z)$ is non-singular too. Since $\mathbf{U}(z)$ is unimodular, Order(det$(\mathbf{T}(z))$) = Order(det$(\mathbf{A}(z))$) = Order(det$(\mathbf{E}_a(z))$). Now using the same reasoning as Lemma 1 in [9] which we omit for the sake of brevity, one can show Order$(\mathbf{U}(z))$ is upper bouned as (4). Since $\mathbf{C}(z)$ is a sub-matrix of $\mathbf{U}(z)$, it will be upper bounded as (4) too. ■

Based on Proposition 1 and Corollary 1, it is concluded that condition *c1* is satisfied for all perfect reconstruction FIR OFBs. In the the following we provide sufficient conditions for satisfying *c2*.

Lemma 1: If $\mathbf{E}_0$ in $\mathbf{E}(z)$ remains full column rank after removing a row, then the corresponding column of $\mathbf{C}_0$ in $\mathbf{C}(z)$ from Proposition 1 can not be zero.

Proof: Since $\mathbf{C}(z)\mathbf{E}(z)=\mathbf{0}$, hence $\mathbf{C}_0\mathbf{E}_0=\mathbf{0}$. Now if there is any zero column in $\mathbf{C}_0$, one gets $\mathbf{C'}_0\mathbf{E'}_0=\mathbf{0}$ where $\mathbf{C'}_0$ and $\mathbf{E'}_0$ are obtained by removing the zero column from $\mathbf{C}_0$ and corresponding row from $\mathbf{E}_0$ respectively. Because $\mathbf{E'}_0$ has full column rank, removing a row from $\mathbf{E}_0$ will decrease the dimension of its orthogonal complement subspace by one. So from $\mathbf{C'}_0\mathbf{E'}_0=\mathbf{0}$, it is concluded that $\mathbf{C'}_0$ is not full row rank. Thus there exists an $(N-K)\times 1$ vector $\mathbf{a}\neq\mathbf{0}$ such that $\mathbf{a}^T\mathbf{C'}_0=\mathbf{0}_{1\times(N-1)}$. However, from last equality on can also get $\mathbf{a}^T\mathbf{C}_0=\mathbf{0}_{1\times N}$. So $\mathbf{C}_0$ is not full row rank which is a contradiction according to Corollary 1.■

Proposition 3: If $K\times K$ sub-matrices of $\mathbf{E}_0$ in $\mathbf{E}(z)$ are non-singular, then condition *c2* will be satisfied for $\mathbf{C}_0$ in $\mathbf{C}(z)$ from Proposition 1.

Proof: Let us assume there is a sub-matrix of $\mathbf{C}_0$ which is singular. Without loss of generality let $\mathbf{C}_0=[\mathbf{C}_{0,a}|\mathbf{C}_{0,b}]$ where $\mathbf{C}_{0,a}$ is $(N-K)\times(N-K)$ and singular. Let $\mathbf{E}_0=\left[\mathbf{E}_{0,a}^T|\mathbf{E}_{0,b}^T\right]^T$ where $\mathbf{E}_{0,b}$ is $K\times K$ and non-singular. Since

$\mathbf{C}_{0,a}$ is singular, using elementary column operations (and corresponding row operations on $\mathbf{E}_{0,a}$ such that $\mathbf{C}_0\mathbf{E}_0 = \mathbf{0}$ still holds), it is possible to make one of its columns zero. However, this is a contradiction based on Lemma 1, because $\mathbf{E}_{0,b}$ is non-singular and after removing the corresponding row from $\mathbf{E}_{0,a}$, $\mathbf{E}_0$ will still have full column rank. ■

Finally, the result of this section can be summarized in the following theorem.

Theorem 2: A stable OFB with analysis polyphase matrix $\mathbf{E}(z) = \sum_{i=0}^{M-1} \mathbf{E}_i z^{-i}$ is maximally robust to instantaneous erasure and the output can be reconstructed using the parity-check matrix method if $\mathbf{E}_0$ remains full column rank after eliminating the rows corresponding to the erasure pattern.

4. INTRODUCING QUANTIZATION NOISE

In any transmission scenario, there is at least one source of noise due to quantization. In the presence of quantization modeled as an additive noise, the relation between parity-check matrix and sub-band vector will be $\mathbf{C}(z)\mathbf{s}_q(z) = \mathbf{C}(z)\mathbf{q}(z)$ where $\mathbf{s}_q(z) = \mathbf{s}(z) + \mathbf{q}(z)$. Furthermore, $\mathbf{q}(z)$ is the z-transform of $\mathbf{q}[n] = [q_1[n], \cdots, q_N[n]]^T$ where $q_i[n]$ represents the quantization noise component added to each sub-band sample. In time-domain, one gets

$$\mathbf{C}_0\mathbf{s}_q[n] + \sum_{i=1}^{M'} \mathbf{C}_i\mathbf{s}_q[n-i] = \sum_{i=0}^{M'} \mathbf{C}_i\mathbf{q}[n-i]. \tag{6}$$

It is fairly easy to see that in case of any erasure in $\mathbf{s}_q[n]$, the previous method of solving the system of equations can not be applied here. So, in order to be able to use the parity-check matrix method in practical situations, it is imperative to modify the method properly. To that end, we suggest a modified method as we describe in the following.

Let us define a vector of intervals $[\mathbf{s}_{q_{min}}[n], \mathbf{s}_{q_{max}}[n]]$ such that for each sub-band sample $s_i[n]$, we have $s_{i,q_{min}}[n] \leq s_i[n] \leq s_{i,q_{max}}[n]$ where $s_{i,q_{min}}[n]$ and $s_{i,q_{max}}[n]$ refer to the lower and upper limit of the quantization interval to which $s_i[n]$ belongs. We assume interval vectors containing perviously received sub-band vectors are available on the synthesis side. For example for a quantizer that outputs midpoint values, the interval vector can be easily found for a newly received sub-band vector by adding and subtracting half of the quantizer's step-size to each sample. Since $\mathbf{C}(z)\mathbf{s}(z) = \mathbf{0}$, we must have [6]

$$\mathbf{0} \in \sum_{i=0}^{M'} \mathbf{C}_i \left[\mathbf{s}_{q_{min}}[n-i], \mathbf{s}_{q_{max}}[n-i]\right]. \tag{7}$$

For each element c_{ij}^k of $\mathbf{C}_k$ in $\mathbf{C}(z)$ and interval $[a, b]$, define

$$f_{max}(c_{ij}^k, [a,b]) = \begin{cases} a & \text{if } c_{ij}^k < 0 \\ b & \text{if } c_{ij}^k \geq 0 \end{cases}, \tag{8}$$

and

$$f_{min}(c_{ij}^k, [a,b]) = \begin{cases} b & \text{if } c_{ij}^k < 0 \\ a & \text{if } c_{ij}^k \geq 0 \end{cases}. \tag{9}$$

For any $i = 1, \cdots, N-K$, from (7), it yields

$$\sum_{k=0}^{M'} \sum_{j=1}^{N} c_{ij}^k f_{max}(c_{ij}^k, [s_{j,q_{min}}[n-k], s_{j,q_{max}}[n-k]]) \geq 0 \tag{10}$$

$$\sum_{k=0}^{M'} \sum_{j=1}^{N} c_{ij}^k f_{min}(c_{ij}^k, [s_{j,q_{min}}[n-k], s_{j,q_{max}}[n-k]]) \leq 0. \tag{11}$$

Now, using interval arithmetic, for each erased value, one can find an interval which satisfies (10) and (11). Obviously, the desired erased quantized value will reside in this interval and thus can be recovered. In case when the resulting interval from system of inequalities of (10) and (11) encompasses more than one possible quantized values, the average of all quantized values would be announced as the recovered sample which could be considered as the best answer in average. Also notice that when there is more than one erasure, instead of solving the inequalities (10) and (11) directly for the original bank, one might consider a reduced system for which the number of erasures is one. More precisely, if there are e erasures in the current sub-band vector, we can consider an OFB with analysis polyphase matrix $\mathbf{E}_e(z)$. $\mathbf{E}_e(z)$ is an $(N-e+1) \times K$ matrix obtained by keeping the $N-e$ channels which are erasure-free and one of the channels that has been received with erasure. Under this circumstance, the system at hand can be treated as if there is only one erasure. Finally, this can be repeated e times to recover all the erased samples. During simulations, we adopted this method as it gave better results.

5. SIMULATION RESULTS

To validate the modified reconstruction method based on parity-check matrix, we considered a scenario where a Gauss-Markov source with variance 1 and correlation coefficient 0.95 is being encoded and decoded by analysis and synthesis banks successively in the presence of quantization noise. The OFB used was a Unimodular bank with $N = 16$, $K = 9$ and $L = 18$ in which $\mathbf{E}_0$ of analysis polyphase matrix is a real-valued DFT matrix [10]. Each sub-band sample is quantized with a 4-bit uniform quantizer. Then each sub-band vector is subject to an erasure pattern for a specified erasure number. Each erasure pattern is also selected randomly with uniform distribution among all possible erasure patterns. To reconstruct the output, first the parity-check matrix was calculated. To find the parity-check matrix, we used a method based on QR factorization of Sylvester matrix of analysis filters [11]. Then, the system of inequalities of (10) and (11) was solved for the erased samples using a Matlab ToolBox called INTLAB [12]. To reduce the reconstruction error, if

Table 1. SNR (dB) Resulting from Parity-Check matrix, Synthesis Filter Change and Linear Prediction Reconstruction Methods for Different Number of Erasures

Number of Erasures	$N = 16, K = 9, L = 18$		
	Synthesis Filters Change [4]	Parity-Check Matrix	Linear Prediction
0	$\mathbf{10.34}$	$\mathbf{10.34}$	$\mathbf{10.34}$
1	$\mathbf{9.21}$	8.06	8.04
2	$\mathbf{6.71}$	6.58	6.57
3	2.57	5.49	$\mathbf{5.55}$
4	-2.12	4.56	$\mathbf{4.66}$
5	-5.94	3.94	$\mathbf{3.98}$
6	-8.01	3.2	$\mathbf{3.39}$
7	-9.08	2.7	$\mathbf{2.83}$

the resulting recovered interval was out of possible bounds of sub-band samples (i.e. interval $[-3, 3]$), the recovered sub-band samples would be announced as zero. At the end, the output was reconstructed by passing the sub-band vectors through the synthesis bank.

The performance of the parity-check matrix reconstruction method has been displayed in Table 1 in terms of the SNR values of the reconstructed output signal versus different erasure numbers. These values have been obtained by calculating the error per-sample of an input signal with length $10,000$ for the given erasure number and then repeating the experiment 100 times and finally averaging over all iterations. The performance of the reconstruction method based on changing the synthesis filters has also been tabulated in Table 1. For comparison, this table also shows the performance of a reconstruction scheme where the erased samples are recovered by using an FIR Linear Prediction (LP) filter with order 10 applied on previously received sub-band samples. As it can be seen, the performance of three methods are almost the same for small number of erasures. However, for large number of erasures, the parity-check matrix gives better results with respect to synthesis filter change method and its performance is comparable to linear prediction scheme. It should be noted that the reason that LP scheme and parity-check method give similar results lies behind the fact that the nature of both methods is almost the same where a filter is applied on previously received sub-band samples to recover an erasure. In addition, LP method has an advantage over parity-check matrix because the coefficients of LP filter are optimized based on received data. So in their calculation, existence of quantization noise is incorporated whereas parity-check matrix method is optimum when no noise is present. This could be considered as the reason behind the slightly better performance of LP over parity-check matrix.

6. CONCLUSION

In this paper, we proposed a new reconstruction method for recovering erased samples of sub-band vector under instantaneous erasure premise based on parity-check matrix calculation. We obtained the sufficient condition on OFB structure for maximum erasure recovery. Then by introducing the quantization noise in the sub-band domain, we modified the reconstruction method which would result in consistent recovery of erased samples. The simulation results showed that parity-check matrix method can improve the performance of output reconstruction of OFB with instantaneous erasure specially when the number of erasures is large.

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