PRACTICAL SPECTRUM SENSING WITH FREQUENCY-DOMAIN PROCESSING IN COGNITIVE RADIO

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ABSTRACT

The imperfections of practical receivers such as nonwhite noise floor, noise power uncertainty and spurs normally have large influence on the performance and stability of spectrum sensing. However, in most of the literatures about spectrum sensing, part of or all of theses imperfections are ignored and only addictive white Gaussian noise (AWGN) is assumed in many cases. In this paper, two spectrum sensing algorithms are proposed using frequency-domain processing. The first one is a blind detection and the second one takes the known power spectrum density (PSD) of the target signal as a template. Analysis and simulations show that they are equivalent or similar to some reported spectrum sensing algorithms. The advantage of the proposed ones is that the imperfections can be easily mitigated at nearly no cost of extra complexities, which makes it more feasible to low-cost implementations.

***Index Terms*—** spectrum sensing, receiver imperfections, practical issues, PSD estimation, cognitive radio

1. INTRODUCTION

In cognitive radio, one of the key functionalities is the sensing for available spectrum resources. The most basic and important type of spectrum sensing is detecting the presence of licensed primary user (PU)'s signal preformed by the cognitive secondary user (SU). Particularly, there are specific requirements on the probabilities of missed detection (PMD) at certain signal-to-noise ratio (SNR) levels with desired probability of false alarm (PFA) and observation time. The PMD means the effectiveness in detecting the target signal, while the PFA should be sufficiently low and constant in order to guarantee good utilization of available spectrum resources and stable quality of service (QoS).

Generally, there are three types of signal detection methods: energy detection, whiteness detection and feature detection.

- Energy detection tests only the change of received energy, which is a blind detection requiring no *a priori* information about the target signal. It suffers greatly from the noise power uncertainty (NPU) problem which prevents the reduction of PMD through increasing observation time [1][2].
- The "*whiteness*" of signal can be characterized in time domain by the autocorrelation of Dirac delta function and in frequency domain by the constant power spectrum density (PSD). Wireless signals normally exhibit nonwhiteness due to limited bandwidth, pulse shaping and the non-stationary properties caused by repeated preamble and pilot structures. The whiteness detections [3, 4] utilize this property to differentiate the signal from white noise, which is also blind.
- Various features of wireless signals can be used in detection, such as known preamble, pilot, cyclostationarity caused by periodicity in modulation scheme and the templates from pulse shaping, PSD and covariance matrix. The techniques such as matched filtering (MF), autocorrelation and spectral analysis, etc. can be used to exploit these features for detection.

The comprehensive studies on various types of spectrum sensing methods can be found in [5] and [6].

The imperfections of practical receivers usually have large influence on the accuracy and stability of spectrum sensing. These imperfections mainly include NPU, nonwhite noise floor caused by the responses of filters in receiver, spurs caused by harmonics from mixer, local oscillator (LO) leakage, DC offset, etc. However, in most of the literatures about spectrum sensing, part of or all of these imperfections are ignored and only additive white Gaussian noise (AWGN) is assumed in many studies.

In this paper, we propose two signal detection algorithms in which these imperfections of receiver can be easily mitigated at almost no cost of extra complexities. The first one is a blind detection method based on testing the received signal's whiteness, which is similar to the covariance matrix based eigenvalue detection [3][4], however, the test is performed in frequency domain on estimated power spectrum density (PSD). The second one takes the known PSD of the target signal as a template, which is proven to be equivalent to the matched-filtered energy detection and the covariance matrix based likelihood ratio test (LRT) detection [7]. In addition to the advantage of insensitiveness to the receiver imperfections, the detection performances of the proposed methods are comparable to the reported equivalent or similar ones, which is confirmed by computer simulations.

2. THE PROPOSED SIGNAL DETECTION

2.1. Signal Model

The signal detection in spectrum sensing can be modeled into a binary hypothesis test problem on the received signal $y[n]$:

$$\begin{aligned} \mathcal{H}_0 &: y[n] = w[n] + s[n] \\ \mathcal{H}_1 &: y[n] = x[n] + w[n] + s[n], \end{aligned} \tag{1}$$

in which $x[n]$ is the target signal with channel fading, $w[n]$ and $s[n]$ are the noise and spur signals in receiver respectively. The $\mathcal{H}_1$ and $\mathcal{H}_0$ denote the two hypotheses when the target signal is present or absent. A detection metric Λ is defined for distinguishing $\mathcal{H}_1$ from $\mathcal{H}_0$, which is the result of a processing on $y[n]$. Then the metric is compared to a threshold γ for decision:

$$\Lambda = \mathcal{F}\{y[n]\} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtreqless}} \gamma. \tag{2}$$

The PMD and PFA are defined by

$$P_{MD} = Pr\Big\{\Lambda < \gamma \Big| \mathcal{H}_1\Big\}, \quad P_{FA} = Pr\Big\{\Lambda \geq \gamma \Big| \mathcal{H}_0\Big\}. \tag{3}$$

In practice, the PFA is given first based on the SU's requirements on spectrum utilization and QoS. Then the threshold γ is calculated according to the prerequisite PFA.

2.2. PSD Estimation and Removal of Receiver Imperfections

The proposed detection methods are based on the PSD of $y[n]$, which can be estimated using Welch's method [8]. First, $y[n]$ are divided into K segments for discrete Fourier transform (DFT) with length N_{DFT}. Each segment is shifted from the previous one by step D:

$$Y_i[m] = \sum_{n=0}^{N_{DFT}-1} y[iD+n]v[n]e^{-j2\pi \frac{nm}{N_{DFT}}} \quad m = 0, 1, ..., N_{DFT}-1 \quad , \tag{4}$$

in which $v[n]$ is a smoothing window of length N_{DFT}. Then the PSD of $y[n]$ can be approximated by

$$\hat{Y}[m] = \frac{\sum_{i=0}^{K-1} \Big|Y_i[m]\Big|^2}{K \sum_{n=0}^{N_{DFT}-1} v^2[n]}. \tag{5}$$

From (4) it can be inferred that each segment has $N_{DFT} - D$ samples overlapped with neighboring segments. When hypothesis $\mathcal{H}_0$ holds, $\hat{Y}[m]$ is the average of $2K$ gaussian variables, hence it can be modeled by Chi-squared distribution. In [8] it was illustrated the overlapping can increase $\hat{Y}[n]$'s equivalent degree of freedom (EDF), thus decrease the variance of the estimates resulting in better detection performance. Based on the analysis in [8],

$$Var\Big\{\hat{Y}[m]\Big\} = \frac{Var\Big\{\Big|Y_i[m]\Big|^2\Big\}}{K}\left\{1 + 2\sum_{k=1}^{K-1}\frac{K-k}{K}\rho[k]\right\}, \tag{6}$$

in which

$$\rho[k] = \left\{\frac{\sum_{n=0}^{N_{DFT}-1} v[n]v[n+kD]}{\sum_{n=0}^{N_{DFT}-1} v^2[n]}\right\}^2. \tag{7}$$

Based on (5) and (6), $\hat{Y}[n]$ has the EDF of

$$\zeta = \frac{2Var\Big\{\Big|Y_i[m]\Big|^2\Big\}}{Var\Big\{\hat{Y}[m]\Big\}} = \frac{2K}{1 + 2\sum_{k=1}^{K-1}\frac{K-k}{K}\rho[k]}. \tag{8}$$

The multiplication by two in (8) is because $y_i[n]$ are complex values with independent real and imaginary parts.

Two receiver imperfections need to be addressed. First, in order to eliminate the destruction of spurs, the frequency components belonging to them should be excluded in detection. We define $\mathbf{S}$ to be the index set of frequencies without spurs. The other imperfections is that the noise in receiver is not perfectly white due to the response of filters, thus, the noise floor is not flat. The shape of the noise floor $\hat{W}[m]$ can be accurately estimated using (4) and (5) by averaging large number of segments, which can be then used to equalize the estimated PSD $\hat{Y}[m]$:

$$\hat{Y}_{eq.}[m] = \frac{\hat{Y}[m]}{\hat{W}[m]}. \tag{9}$$

2.3. PSD Whiteness Detection

In hypothesis $\mathcal{H}_0$, $\hat{Y}_{eq.}[m]$ converges to a constant when $K \to \infty$. Since the target signal is not white, the $\hat{Y}_{eq.}[m]$ is not a constant in hypothesis $\mathcal{H}_1$, thus, $\hat{Y}_{eq.}[m]$'s geometric mean is larger than its arithmetic mean. Hence, the whiteness detection can be performed by testing their quotient:

$$\Lambda_{PSD-AG} = \frac{\sum_{m\in\mathbf{S}} \hat{Y}_{eq.}[m]}{N_S\Big(\prod_{m\in\mathbf{S}} \hat{Y}_{eq.}[m]\Big)^{\frac{1}{N_S}}}, \tag{10}$$

in which N_S is the size of the set $\mathbf{S}$ and AG denotes arithmetic and geometric means. It should be noticed that the Λ_{PSD-AG} is a ratio value which is dimensionless, so it is not relevant to the magnitude of received signal's power. Then the calculation of threshold does not require the knowledge of noise power. Therefore, the PSD-AG is not affected by the NPU problem.

2.4. PSD Template Detection

When the knowledge of the target signal's PSD $\check{X}[m]$ is available, it can be used as a template, which is correlated with the estimated and equalized PSD of received signal, resulting in a second detection metric [7]:

$$\Lambda_{PSD-T} = \sum_{m\in\mathbf{S}} \hat{Y}_{eq.}[m]\check{X}[m]. \tag{11}$$

This detection metric has the same dimension as the signal power, hence it is influenced by the NPU problem. Here we propose a solution which can eliminate the NPU's influence on PFA and PMD, which is based on the cancelation of the detection metric's dimension:

$$\Lambda_{PSD-TC} = \frac{\sum_{m\in\mathbf{S}} \hat{Y}_{eq.}[m]\check{X}[m]}{\sum_{m\in\mathbf{S}} \hat{Y}_{eq.}[m]}. \tag{12}$$

2.5. Similar or Equivalent Detections in Time Domain

2.5.1. Whiteness Detection

In [3] and [4], it is proposed that the whiteness of signal can be also tested using the covariance matrix (CM) of received signal, which can be estimated statistically by

$$\hat{\mathbf{R}}_y = \frac{1}{N - N_c + 1} \sum_{i=1}^{N-N_c+1} \mathbf{y}_i \mathbf{y}_i^* \tag{13}$$

in which N is the total number of samples used in detection, * denotes the conjugate transposition of matrix and $\mathbf{y}_i$ is a vector consist of N_c successive samples:

$$\mathbf{y}_i = \left(y[i], y[i+1], ..., y[i+N_c-1]\right)^T, \tag{14}$$

with $(\cdot)^T$ denoting matrix transposition. The elements in $\hat{\mathbf{R}}_y$ are essentially time-domain autocorrelations of the received signal $y[n]$. Therefore, it can be inferred that

$$\begin{aligned} \mathcal{H}_0 &: \hat{\mathbf{R}}_y = \sigma_w^2 \mathbf{I} \\ \mathcal{H}_1 &: \hat{\mathbf{R}}_y = \mathbf{R}_x + \sigma_w^2 \mathbf{I}, \end{aligned} \tag{15}$$

in which $\mathbf{R}_x$ is the covariance matrix of the target signal, σ_w^2 is the power of noise and $\mathbf{I}$ denotes identity matrix.

One method of whiteness detection is to test the ratio of maximum and minimum eigenvalues (MME) [4]. First, eigenvalue decomposition is performed on $\hat{\mathbf{R}}_y$:

$$\begin{aligned} \hat{\mathbf{R}}_y &= \Phi_y \Psi_y \Phi_y^T \\ \Psi_y &= diag(\psi_{y,1}, \psi_{y,2}, ..., \psi_{y,N_c}) \end{aligned} \tag{16}$$

in which $\psi_{y,n}$ are the eigenvalues satisfying $\psi_{y,1} \geq \psi_{y,2} \geq ... \geq \psi_{y,N_c}$. Then the detection metric becomes

$$\Lambda_{CM-MME} = \frac{\psi_{y,1}}{\psi_{y,N_c}}. \tag{17}$$

In [3], an alternative method called covariance absolute value (CAV) detection is proposed and implemented. It also shows that this detection method shows a performance very close to that of MME detection. The detection metric of CAV detector is

$$\Lambda_{CM-CAV} = \frac{\sum_{m=1}^{N_c} \sum_{n=1}^{N_c} |r_{mn}|}{\sum_{m=1}^{N_c} |r_{mm}|}, \tag{18}$$

in which r_{mn} are the elements in $\hat{\mathbf{R}}_y$. The advantage of CAV detection over MME detection is that the computational costly eigenvalue decomposition is not needed. Since the detection metrics of CM-MME and CM-CAV are ratio values without dimension, they are not affected by NPU problem which is similar to the proposed PSD-AG detection.

2.5.2. Template Based Detection

In [9] and [7], it was shown that the known CM of the target signal $\mathbf{R}_x$ can be taken as a template in detection. If the noise power σ_w^2 is also known to the detector, an optimal estimator-correlator (EC) detection is derived in [10]:

$$\Lambda_{CM-EC} = \frac{1}{N - N_c + 1} \sum_{i=1}^{N-N_c+1} \mathbf{y}_i^* \mathbf{R}_x (\mathbf{R}_x + \sigma_w^2 \mathbf{I})^{-1} \mathbf{y}_i. \tag{19}$$

Notice that the diagonal components of $\mathbf{R}_x$ are actually equal to the signal power σ_x^2, thus, the SNR should also be known to the detector. When SNR is low, $\mathbf{R}_x + \sigma_w^2 \mathbf{I} \approx \sigma_w^2 \mathbf{I}$, then the EC detection in (19) is reduced to the CM based likelihood ratio test (LRT) detection at low SNR [7].

$$\Lambda_{CM-LRT} = \frac{1}{N - N_c + 1} \sum_{i=1}^{N-N_c+1} \mathbf{y}_i^* \mathbf{R}_x \mathbf{y}_i. \tag{20}$$

Alternatively, when the pulse shaping filter of the target signal's PSD $g[m]$ is known, it can be used as template in detection. The matched filter of $g[m]$ is formulated by

$$f[m] = g^\dagger[M - 1 - m], \tag{21}$$

in which † denotes conjugate operation. The detection metric can be defined by the energy of matched-filtered signal:

$$\Lambda_{MF-EG} = \sum_{n=0}^{N-1} \left| \sum_{m=0}^{M-1} f[m] y[n-m] \right|^2. \tag{22}$$

The next section will prove analytically that the PSD-T detection is actually equivalent to CM-LRT and MF-EG.

2.5.3. The Equivalence of PSD-T, CM-LRT and MF-EG

a) The Equivalence of MF-EG and CM-LRT

The convolution operation of matched filtering in (22) can be represented in matrix form

$$\mathbf{y}_f = \mathbf{F}^* \mathbf{y}, \tag{23}$$

in which $\mathbf{y} = (y[0], y[1], ..., y[N-1])^T$ is the vector of the received signal's samples and

$$\mathbf{F} = \begin{pmatrix} f[0] & f[1] & \dots & f[M-1] & 0 & \dots & 0 \\ 0 & f[0] & f[1] & \dots & f[M-1] & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & f[0] & f[1] & \dots & f[M-1] \end{pmatrix}, \tag{24}$$

in which $f[m]$ is the impulse response of the matched filter. The detection metric of MF-EG can be then rewritten as

$$\Lambda_{MF-EG} = E(\mathbf{y}^* \mathbf{F} \mathbf{F}^* \mathbf{y}). \tag{25}$$

The transmitted signal $\mathbf{x}$ can be modeled as an independent and identically distributed process $\mathbf{z}$ filtered by the pulse shaping filter. Since the detection focuses only on the spectrum template, the non-random structures in the target signal, such as preambles, pilot tones and cyclic prefix are ignored. The pulse shaping can be then denoted by

$$\mathbf{x} = \mathbf{F} \mathbf{z}^\dagger. \tag{26}$$

Assume the variance of $\mathbf{z}$ is normalized to 1, based on (13), the covariance matrix of $\mathbf{z}$ becomes identity matrix:

$$E(\mathbf{z}\mathbf{z}^*) = E(\mathbf{z}^\dagger \mathbf{z}^T) = \mathbf{I}. \tag{27}$$

Then the covariance matrix of the target signal $\mathbf{x}$ becomes

$$\mathbf{R}_x = E(\mathbf{x}\mathbf{x}^*) = E(\mathbf{F}\mathbf{z}^\dagger \mathbf{z}^T \mathbf{F}^*) = \mathbf{F} E(\mathbf{z}^\dagger \mathbf{z}^T) \mathbf{F}^* = \mathbf{F}\mathbf{F}^*. \tag{28}$$

Recalling the detection metric of MFD-EG in (25),

$$\Lambda_{MF-EG} = E(\mathbf{y}^* \mathbf{F}\mathbf{F}^* \mathbf{y}) = E(\mathbf{y}^* \mathbf{R}_x \mathbf{y}) = \Lambda_{CM-LRT}. \tag{29}$$

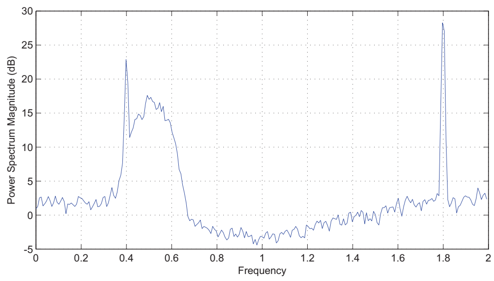

Fig. 1. The PSD of the target WM signal with nonwhite noise floor and two spurs, bandwidth: 200 kHz, sampling rate: 500 k/s

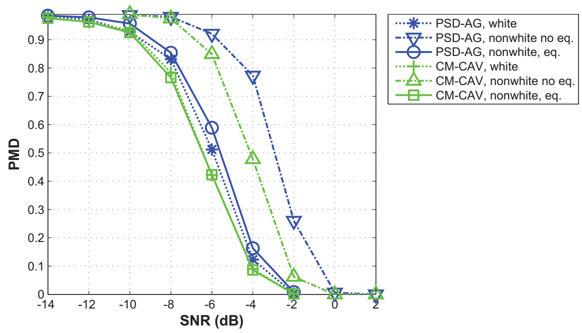

Fig. 2. PMD of PSD-AG and CM-CAV on WM signal with nonwhite noise floor, bandwidth: 200kHz, sampling rate: 500k/s, N_{DFT} = 64, D = 32, CM size:32, observation time:0.2ms, PFA:0.01

This ends the proof of MF-EG and CM-LRT's equivalence.

b) The Equivalence of CM-LRT and PSD-T

The DFT operation for estimating the PSD of the target signal using Welch's method can be written as

$$\mathbf{x}_F = \mathbf{W}\mathbf{x}, \tag{30}$$

in which $\mathbf{W}$ is the DFT matrix defined by

$$\mathbf{W} = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{N_{DFT}-1} \\ 1 & w^2 & w^4 & \dots & w^{2(N_{DFT}-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & w^{N_{DFT}-1} & w^{2(N_{DFT}-1)} & \dots & w^{(N_{DFT}-1)(N_{DFT}-1)} \end{pmatrix} \tag{31}$$

with $w = e^{-j2\pi/N_{DFT}}$ and $\mathbf{W}\mathbf{W}^* = \mathbf{I}$. Define another matrix

$$\begin{aligned} \mathbf{S}_x &= \mathbf{W}\mathbf{R}_x\mathbf{W}^* = \mathbf{W}E(\mathbf{x}\mathbf{x}^*)\mathbf{W}^* = E(\mathbf{W}\mathbf{x}\mathbf{x}^*\mathbf{W}^*) \\ &= E(\mathbf{x}_F\mathbf{x}_F^*) = diag(\check{X}[0], \check{X}[1], ..., \check{X}[N_{DFT}-1]), \end{aligned} \tag{32}$$

in which $\check{X}[m]$ is the PSD template of the target signal $\mathbf{x}$. The detection metric of PSD-T can then be formulated by

$$\begin{aligned} \Lambda_{PSD-T} &= E(\mathbf{y}_F^*\mathbf{S}_x\mathbf{y}_F) = E((\mathbf{W}\mathbf{y})^*\mathbf{S}_x(\mathbf{W}\mathbf{y})) = E(\mathbf{y}^*\mathbf{W}^*\mathbf{W}\mathbf{R}_x\mathbf{W}^*\mathbf{W}\mathbf{y}) \\ &= E(\mathbf{y}^*\mathbf{R}_x\mathbf{y}) = \Lambda_{CM-LRT}. \end{aligned} \tag{33}$$

This ends the proof of CM-LRT and PSD-T's equivalence.

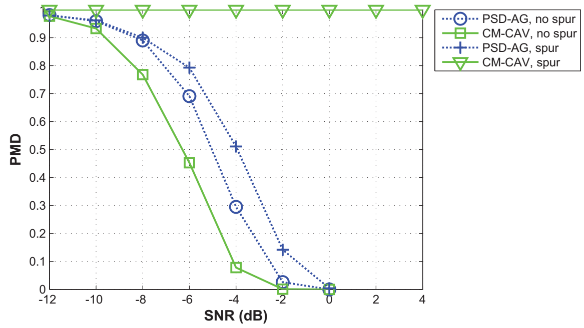

Fig. 3. PMD of PSD-AG and CM-CAV on WM signal with spurs, bandwidth: 200kHz, sampling rate: 500k/s, N_{DFT} = 64, D = 32, CM size:32, observation time:0.2ms, PFA:0.01

3. SIMULATION RESULTS

3.1. Noise Floor Equalization and Spur Removal

We first simulate the performance of the proposed PSD-AG detection with nonwhite noise floor and spurs. Fig. 1 presents the PSD of received wireless microphone (WM) signal with unflat noise floor and two spurs.

It is illustrated in (9) that the unflat noise floor can be e-qualized in the proposed PSD based detections. Fig. 2 shows that in PSD-AG and CM-CAV detections, the equalization can reduce the PMD notably to about the same level as the case when the noise floor is white.

In Section 2.2, it is illustrated that the spur's destruction can be straightforwardly removed in PSD based detections. Fig. 3 shows that, when the spurs are removed, the PSD-AG's detection performance is only slightly worse than the case when there is no spur. However, when the spurs are presented, the CM-CAV detection fails completely.

3.2. Comparison of PSD and CM based Detections

It is shown in Fig. 4 that reducing the shifting step D to $0.5N_{DFT}$ in PSD estimation can improve the detection performance to about the same level as the CM based detection. This result confirms the equivalence between PSD-T and CM-LRT detections. The performance improvement when $D = 0.5N_{DFT}$ can be explained by the increase of the EDF in PSD estimation by about 34%, which is calculated using (8).

3.3. PSD based Detections with Noise Power Uncertainty

The NPU is modeled using the *robust statistic* method presented in [2]. Particularly, the upper limit of noise power is used to calculate PFA while the lower limit is used to calculate PMD, which considers the worst case in NPU. In [2], the N-PU of 1 dB is concluded for typical receivers. Fig. 5 shows the "SNR wall" for PSD-T that the detection performance cannot be improved by increasing observation time when the NPU

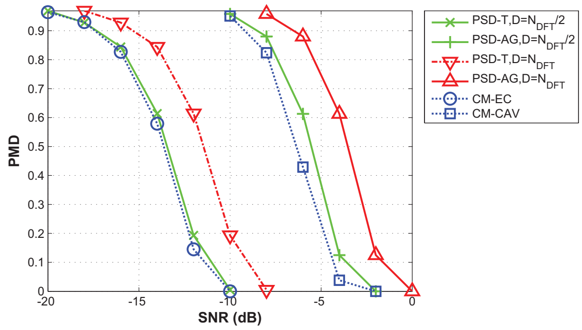

Fig. 4. Comparison of PSD and CM based detections on ATSC signal, bandwidth: 6 MHz, $N_{DFT} = 64$, $D = 32$, CM size: 32, observation time: 0.4 ms, PFA: 0.01, flat fading

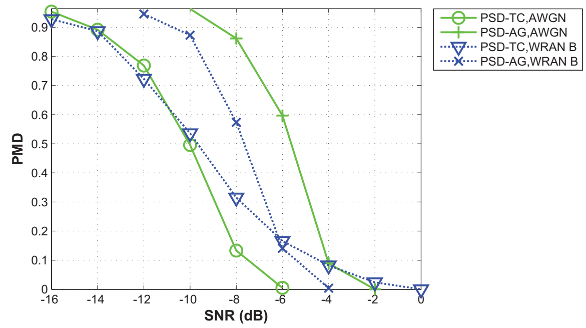

Fig. 6. PSD based detections on ATSC signal in multipath channel [11], bandwidth: 6 MHz, $N_{DFT} = 64$, $D = 32$, PFA: 0.01

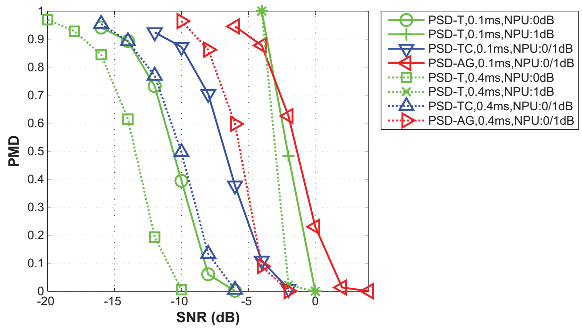

Fig. 5. PSD based detections on ATSC signal with NPU, bandwidth: 6 MHz, $N_{DFT} = 64$, $D = 32$, PFA: 0.01

is present. However, the proposed modified detection PSD-TC is not sensitive to NPU at the cost of some performance loss. The blind PSD-AG detection is inherently not sensitive to NPU, but its performance is worse than the PSD-TC.

3.4. PSD based Detections with Multipath Channel

It is shown in Fig. 6 that the blind PSWD-AG detection can perform better in multipath channel (WRAN Type B channel [11]) than in flat fading channel. This is because the non-whiteness of the target signal is actually strengthened by the frequency selectivity of the multipath channel.

4. CONCLUSION

Two signal detection methods based on frequency-domain processing are proposed in this paper, both are insensitive to receiver imperfections, such as nonwhite noise floor, NPU and spurs. One method named PSD-AG is blind detection based on testing the whiteness of the received signal using estimated PSD. The other one named PSD-TC takes the known PSD of the target signal as a template for detection and the solution for eliminating the effect of the NPU problem for this detection is also proposed. It's proven analytically that the later one is equivalent to reported robust covariance matrix based LRT detection and matched-filtered energy detection. Apart from the advantage of removing receiver imperfections, computer simulations show that the proposed detections has comparable performances to their equivalent or similar reported detections. It also shows the blind PSD-AG detection can performs better in multipath channel than in flat-fading channel. The proposed signal detection methods are well suited to implementations of spectrum sensing on low-cost hardware with strong receiver imperfections.

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