MINIMAL DISTORTION 3-D WATERMARKING USING STATISTICS OF GEODESIC DISTANCES

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ABSTRACT

This paper describes a statistical 3-D digital watermarking method using distributions of distances measured along mesh surfaces. The proposed method employs a mesh propagation procedure called the Fast Marching Method (FMM), which defines regions of equal geodesic distance width calculated with respect to a reference location on the mesh surface. The embedding is performed by statistically changing the normalized distribution of local geodesic distances. Vertices are moved onto the plane of triangles containing the current geodesic front line as generated by the FMM. Such vertex perturbations ensure that the resulting distortions in the 3-D graphics is minimal.

***Index Terms*—** 3-D watermarking, surface preservation, Fast Marching Method, geodesic distances.

1. INTRODUCTION

This paper describes an approach to blind digital watermarking of shapes represented as meshes. The aim of the proposed methodology is to introduce a minimal distortion in order to preserve the mesh surface following watermarking. Watermarking of graphics has been performed in the spatial domain as well as in the transform domains and can be characterized as deterministic or statistical based on the the way how the watermark embedding is performed. Usually, deterministic methods allow a higher capacity of information embedding, making them suitable for steganography, but achieve a lower robustness to attacks [1].

Statistical 3-D watermarking embedding was used in [2, 3, 4]. In [2] statistical distributions of distances from the mesh surface to the local principal data axis was used for watermarking relying on the local object symmetry. Noise like perturbations modify such distributions which otherwise would be quasi-uniform. The distance from the object center to vertices on its surface is considered as a statistical variable in [3] for embedding watermarks. Two watermarking statistical methods are used in [3] by changing either the mean or the variance in the statistics of such distances. Most of the existing 3-D mesh watermarking methods create bump like changes on surfaces of 3-D objects.

The geodesic distance, which takes into account the local surface variation, has been shown to be the most appropriate measure for calculating the distance between two different points on a mesh [5]. The Fast Marching Method (FMM) was proposed for the low-cost calculation of geodesic distances between two locations on the object mesh surface [6, 7]. FMM evaluates geodesic distances with respect to a reference point location.

In this paper we propose a 3-D watermarking method, based on the approach from [4], by embedding bits of information in distributions of geodesic distances calculated from the mesh surface. The watermark reference system is defined by the source location and by appropriate alignment. We split the object surface into strips of equal geodesic width, calculated by using FMM [6, 7]. The mean or the variance of distributions of geodesic distances corresponding to the vertices from each strip are changed when embedding each bit. The Vertex Placement Scheme (VPS) algorithm is proposed for displacing vertices along directions which are perpendicular to the geodesic front lines aiming to preserve the original object surface. Section 2 outlines the initialization procedure, while Section 3 describes how equal-sized object regions are generated on the mesh surface. Section 4 details the Vertex Placement Scheme (VPS) procedure for watermark embedding. Section 5 provides the experimental results, while Section 6 provides the conclusions of this study.

2. INITIALIZING THE GEODESIC FRONT PROPAGATION WATERMARKING METHOD

The proposed watermark embedding method has the following steps: defining the reference location, segmenting the object surface into strips, forming geodesic distance histograms and the vertex placement method for watermark embedding.

In order to define a robust source location we use the volume moment alignment. The principal axes of the object are obtained by eigen-decomposing the covariance matrix calculated from the coordinates of all 3-D object vertices. The resulting eigenvalues characterize the extension of the object along its principal axes whose directions are defined by the corresponding eigenvectors. In order to define a unique alignment, we propose two constraints to be used together. Firstly, the three principal axes must conform the right hand rule such that the direction of the third axis will be defined as the cross product of the first two. Furthermore, the valid alignment satisfies the condition that the third order moments of the object are positive. By following these constraints, the principal axis alignment is unique.

A random direction is cast from the object center according to a secret key. The starting point is defined as the intersection between the direction of this vector from the object center and the mesh surface. There are two extreme situations: when there is no intersection with the object surface and when there are multiple intersections. In the former case we proceed to generate additional random directions until an appropriate intersection with the object surface is found. In the latter case the intersection which is the furthest away from the object center is chosen as the source location.

3. ISO-GEODESIC MESH STRIP GENERATION

Let us define $T_{min} = \min(\{T(\mathbf{x}), \forall \mathbf{x} \in \mathcal{O}\})$ and $T_{max} = \max(\{T(\mathbf{x}), \forall \mathbf{x} \in \mathcal{O}\})$ as the minimum and maximum geodesic distances calculated for the object $\mathcal{O}$ from a source location $\mathbf{s}$. Generally, the number of vertices whose geodesic distance is close to T_{min} and T_{max} would be too small in order to be statistically relevant so they are not considered for watermark embedding. Therefore, we trim the range of acceptable geodesic distances to the range :

$$T(\mathbf{v}_j) \subset ((1-\varepsilon)T_{min} + \varepsilon T_{max}, \varepsilon T_{min} + (1-\varepsilon)T_{max}) \quad (1)$$

where $\varepsilon \in (0, 0.2)$ is used for the vertices which are close to extremes, according to their geodesic distance from the source location. Then, for a watermark code of M bits, the object mesh is segmented into M strips, each used for embedding a single bit. Consequently, the geodesic distance width for each strip is defined as:

$$T_b = \frac{(1-2\varepsilon)\,(T_{max} - T_{min})}{M}. \quad (2)$$

Let us consider $\mathcal{B}_i$ as the set of vertices which are located in a specific range of geodesic distances calculated from the source location $\mathbf{s}$ and characterizing a mesh strip on the object surface :

$$\mathcal{B}_i = \{\mathbf{v}_j \in \mathcal{O} \mid T_{min} + (i-1)T_b \le T(\mathbf{v}_j) < T_{min} + iT_b\} \quad (3)$$

for $i = 1, \ldots, M$. T_b should be large enough in order to define regions which contain a statistically consistent number of vertices available for watermark embedding.

After splitting the graphical object into strips of equal geodesic width, each strip is associated with a bit from the watermark code. In the following we consider statistical watermarking of distributions of geodesic distances for the vertices from inside each strip. For each bit of 0 or 1 we embed specific statistical changes, either in the value of the mean or in that of the variance of the local geodesic distances, by means of histogram mapping functions, as in [3, 4]. The shape of histograms is changed and the corresponding variables are mapped back into the displaced location of vertices onto the surface of objects [4].

4. CHANGING VERTEX GEODESIC DISTANCES BY VERTEX PLACEMENT SCHEME

In the following we describe how to displace vertices in order to conform with the distributions of the watermarked geodesic distance variables while not visibly perturbing the mesh surface. The proposed watermark embedding procedure for a particular triangle $\triangle ABC$ on the mesh surface, is called the vertex placement scheme (VPS). The study can be easily extended for all the vertices inside the strip $\mathcal{B}_i$ and to the entire object $\mathcal{O}$. Let us consider the vertices A and B as having their geodesic distances $T(A)$ and $T(B)$, $T(A) < T(B)$. When the angle $\angle C = \theta$ inside triangle $\triangle ABC$ is acute then the update scheme is monotone, *i.e.* $T(A) < T(B) < T(C)$. Let us assume that the lengths of the triangle sides are $a = \|BC\|$, $b = \|AC\|$ and $c = \|AB\|$ as shown in Fig. 1 and denote the geodesic distances between its vertices, calculated along the FMM front propagated with respect to the source location as:

$$t = T(C) - T(A) \quad (4)$$

$$u = T(B) - T(A) \quad (5)$$

$$h = T(C) - T(B) = t - u \quad (6)$$

Kimmel and Sethian have shown in [7] that the value of t can be calculated using FMM by assuming known $T(A)$, $T(B)$ and the geometry of $\triangle ABC$, according to the equation :

$$(a^2+b^2-2ab\cos\,\theta)t^2 + 2bu(a\cos\,\theta - b)t + b^2(u^2 - a^2\sin\,\theta) = 0 \quad (7)$$

The solution t must satisfy two conditions:

$$\begin{cases} u < t \\ a\cos\theta < \frac{b(t-u)}{t} < \frac{a}{\cos\theta} \end{cases} \quad (8)$$

$u < t$ means that $T(C) > T(B) > T(A)$ which conforms to the monotone property. The second condition of equation (8) means that $T(C)$ must be updated from within $\triangle ABC$. Thus, the updating procedure is given as:

$$T(C) = \begin{cases} min\{T(C), t + T(A)\}, & \text{if (8) are fulfilled} \\ min\{T(C), b + T(A), a + T(B)\}, & \text{otherwise} \end{cases} \quad (9)$$

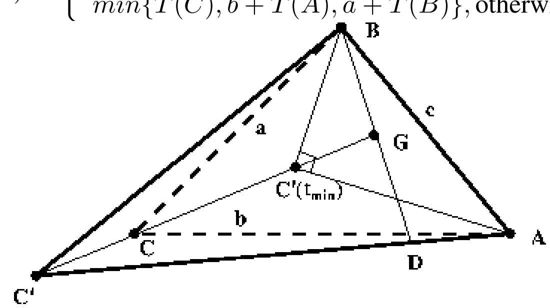


Fig. 1. Vertex perturbation from C to C'. Line BG is the approximation of the geodesic distance wave and CG and $C'G$ represent gradients to the wave front.

An extreme situation for t as solution from equation (7) is when $\theta = \pi/2$. In this case we have the minimum bound for t, constraining the vertex placement location, as :

$$t_{min} = \frac{b^2 u + ab\sqrt{a^2 + b^2 - u^2}}{a^2 + b^2} \qquad (10)$$

When θ is obtuse, triangles are unfolded and split into two acute angle triangles and then we can proceed with the updating scheme, [7].

In the following we show that perturbations of geodesic distances, according to the proposed VPS, can be embedded into the FMM method by ensuring a minimal change along the geodesic front. Let us consider that vertices from $\mathcal{F}$ define the current propagation front line for FMM while $\mathcal{D}$ corresponds to the set of all vertices from which the geodesic propagation front has passed. Let us assume that in the case of $\triangle ABC$ we have A and B fixed while C changes to C' following watermarking by VPS. Assuming that $\{A, B\} \in \mathcal{D}$, the watermark embedding is performed along the geodesic front vertices $C \in \mathcal{F}$. We associate the statistical variables g and $\hat{g}$, derived according to the histogram mapping functions, to the geodesic distance $T(C) = g$ and to that of a new location C', respectively, which would result after watermark embedding. The problem addressed in the following is about how to move the vertex C to a new location C', such that its new geodesic distance satisfies $T(C') = \hat{g}$, while ensuring that the graphical object suffers a minimal distortion.

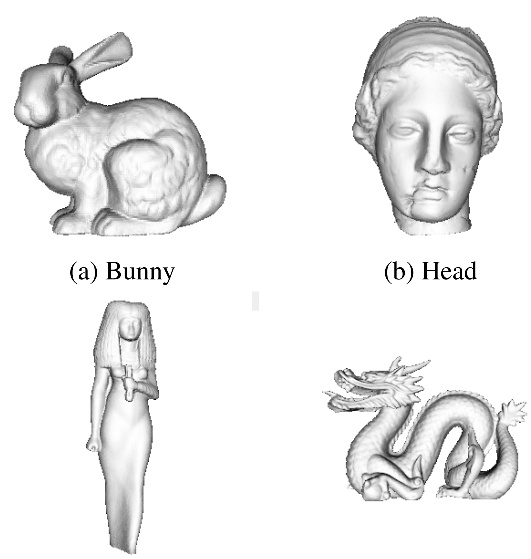
The proposed VPS consists of the following sequence of steps for vertices from a segmented strip $\mathcal{B}_i$:

1. Calculate $\hat{g}$ according to the histogram mapping.
2. Choose a vertex $C \in \mathcal{B}_i$ in the downwind direction of FMM and calculate $T(C)$.
3. While $T(C) \neq \hat{g}$ repeat Steps 4-6.
4. Locate $A, B \in \mathcal{N}(C)$ such that all three form a triangle which contributes to the minimum path calculation for $T(C)$.
5. Apply VPS in $\triangle ABC$ to move C to C' such that $T(C') = \hat{g}$.
6. Set $C \rightarrow C'$, update the geodesic distance $T(C)$ by using only vertices from the set $\mathcal{D}$ and go to Step 3.

Step 1 corresponds to the sampling of the watermarked distribution, while step 2 represents the current positioning on the object surface by using the Fast Marching Method. The VPS procedure is shown in Fig. 1. When the conditions from (8) are fulfilled, there is a point G inside $\triangle ABC$ such that $CG \perp BG$ and the Euclidean distance $\|CG\| = h$, with h defined in (6). When embedding the updated geodesic distances back into the surface and when replacing h with $\|CG\|$ in equation (6) results into $T(G) = T(B)$. BG is the approximation of the equal geodesic curve located at the distance $T(B)$ from the source location. Consequently, the VPS procedure transforms $\triangle ABC$ into $\triangle ABC'$ such that $T(C') = \hat{g}$, following watermarking, as illustrated in Fig. 1. The watermark extraction is blind and is performed by extracting histograms of geodesic distances corresponding to vertices from each mesh strip and applying a statistical test in order to determine the embedded bit [4].

5. EXPERIMENTAL RESULTS

In the following we provide the results when watermarking a set of four objects: Bunny, Head, Statue and Dragon. These objects are displayed in Fig. 2 and their mesh characteristics are provided in Table 1. We use the abbreviation ProMean or ProVar for the methods which embed watermarks by changing the mean or the variance, respectively, of the geodesic distance distribution. Before segmenting the objects into isogeodesic strips, they are trimmed by considering $\varepsilon = 0.1$ in (1). The proposed methodology is compared with the graphics watermarking methods proposed in [3] which are called ChoMean and ChoVar for changing the mean or variance of distributions of distances from the object center to its vertices.



(a) Bunny (b) Head (c) Statue (d) Dragon

Fig. 2. 3-D objects used in the experiments.

Object	No. Vertices	No. Faces
Bunny	34,835	69,666
Head	134,345	268,686
Statue	187,638	375,272
Dragon	422,335	844,886

Table 1. Characteristics of the graphical objects used in the study.

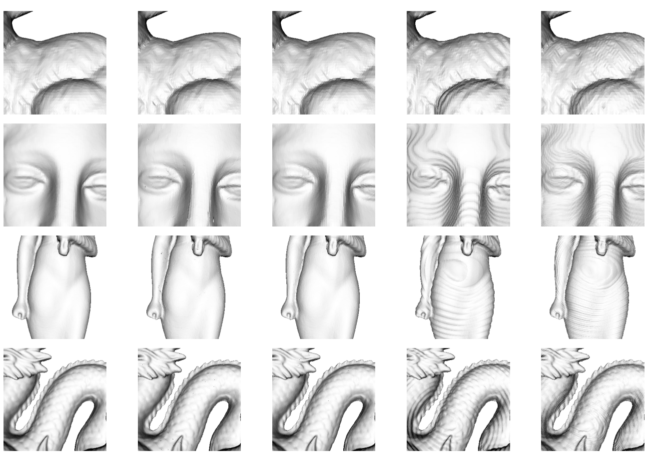
One of the aims for watermarking graphical objects is to achieve a minimal surface distortion. The visual quality of the watermarked objects is measured by a method called Metro, proposed in [8], which approximates the Hausdorff distance between two objects:

$$E(\mathcal{O}, \hat{\mathcal{O}}) = \max\{E_f(\mathcal{O}, \hat{\mathcal{O}}), E_b(\hat{\mathcal{O}}, \mathcal{O})\} \qquad (11)$$

where $E_f(\mathcal{O},\hat{\mathcal{O}})$ represents the root mean square error (RMS) of distances between the vertices from $\mathcal{O}$ and the closest points from the surface of $\hat{\mathcal{O}}$, while $E_b(\hat{\mathcal{O}},\mathcal{O})$ represents the RMS between the vertices of $\hat{\mathcal{O}}$ and the closest points from the surface of the object $\mathcal{O}$. All distances are calculated as fractions of the diagonal of the bounding box enclosing the mesh. Table 2, provides the distortion results, measured by $E(\mathcal{O},\hat{\mathcal{O}})$ from (11) for all four methods when watermarking the set of objects from Fig. 2. As can be observed from Table 2, the graphical object distortion introduced by the proposed watermarking methodology is much lower than that produced by ChoPro and ChoVar methods [3]. Fig. 3 displays the visual effects of the proposed watermarking methods using zoomed views of graphical object surfaces. The methods from [3] introduce visible staircase artifacts as it can be seen in these figures.

Object	ProMean	ProVar	ChoMean	ChoVar
Bunny	0.26	0.14	0.81	0.39
Head	0.11	0.06	0.37	0.19
Statue	0.23	0.12	0.94	0.40
Dragon	0.25	0.12	0.90	0.47

Table 2. Watermarked object distortion, where all results are multiplied with 10^{-4}.



(a) (b) (c) (d) (e)

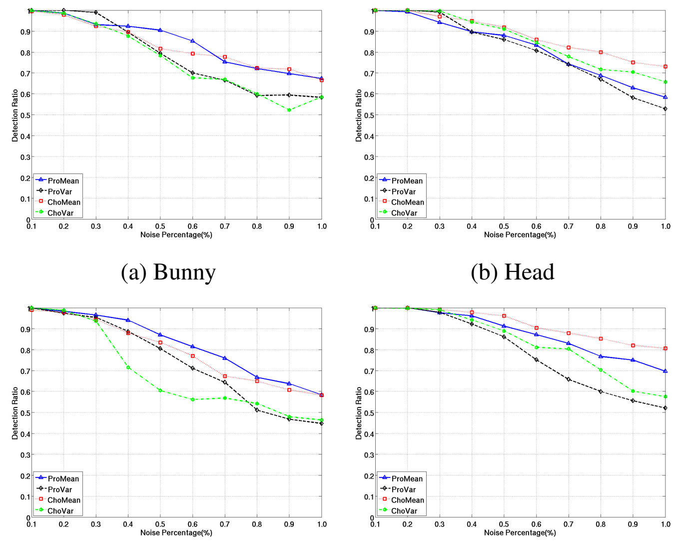
Fig. 3. Comparison of visual distortion by displaying zoomed details. (a) Original, and watermarked using (b) ProMean, (c) ProVar, (d) ChoMean, (e) ChoVar.

In the following we test the robustness of the 3-D watermarking methods to additive noise, smoothing, mesh simplification, and uniform re-meshing.

We consider additive random noise according to the following distortion equation:

$$\tilde{\mathbf{v}}_i = \hat{\mathbf{v}}_i + \epsilon \|\hat{\mathbf{v}}_{max}\| \overrightarrow{\mathbf{p}} \tag{12}$$

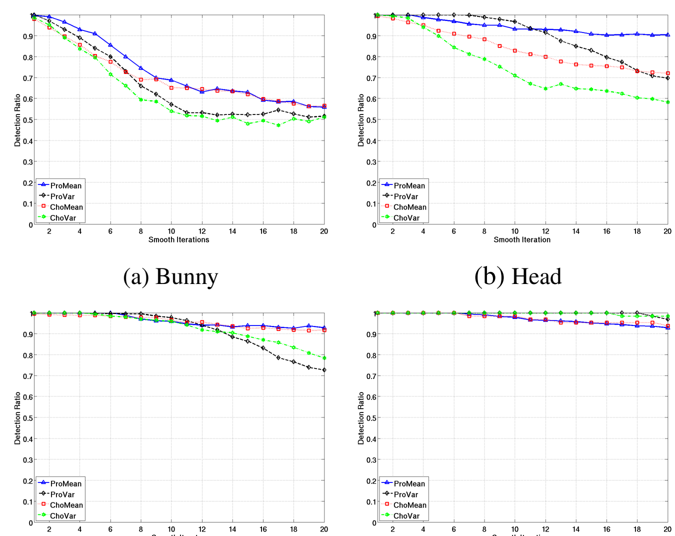
where $\tilde{\mathbf{v}}_i$ represents the distorted watermarked vertex $\hat{\mathbf{v}}_i$, $\epsilon \in [0,1]$ is the percentage of $\|\mathbf{v}_{max}\|$ which corresponds to the largest Euclidean distance measured from the object center to each vertex, $\overrightarrow{\mathbf{p}}$ is a unitary vector with random direction. The plots from Fig. 4 show the robustness against noise when varying ϵ for all four methods, ProMean, ProVar, ChoMean and ChoVar for all four graphical objects. From these plots it can be observed that ProMean and ChoMean methods provide better results than ProVar and ChoVar.



(a) Bunny (b) Head

(c) Statue (d) Dragon

Fig. 4. Watermark robustness to additive noise.



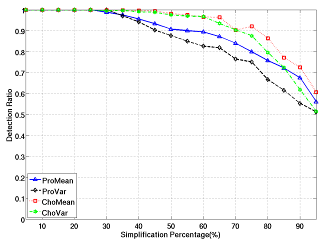
(a) Bunny (b) Head

(c) Statue (d) Dragon

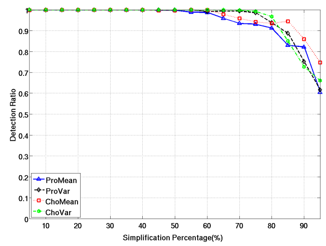
Fig. 5. Watermark robustness to surface smoothing.

We use Laplacian smoothing as in [9] for testing the robustness to surface smoothing. The plots displaying the smoothness robustness are provided in Fig. 5 and it can be observed that ProMean provides slightly better results for Bunny, Head and Statue graphical objects, while ProVar is better for Dragon. This is due to the fact that Dragon is an object characterized by a larger variance of its surface which benefits a watermarking method based on variance change.

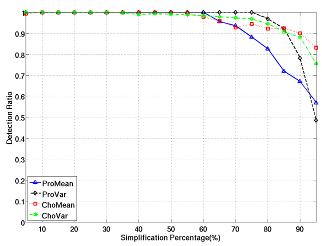
The quadratic metric simplification software, described in



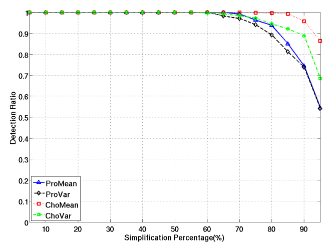
(a) Bunny



(b) Head

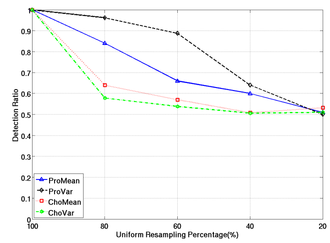


(c) Statue

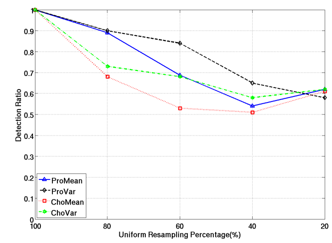


(d) Dragon

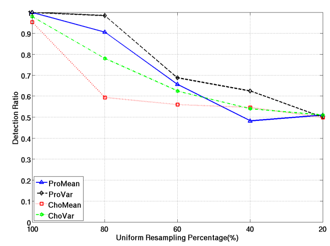
Fig. 6. Watermark robustness to mesh simplification.



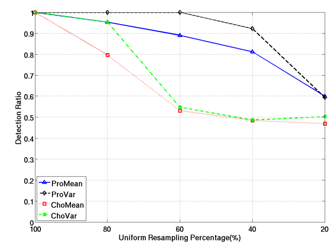
(a) Bunny



(b) Head



(c) Statue



(d) Dragon

Fig. 7. Watermark robustness to re-sampling.

[10], was used for testing the robustness at mesh simplification. Fig. 6 contains the plots showing the watermark resistance to mesh simplification attack for the objects when using all four methods. ProVar and ProMean are slightly less robust the Cho's methods at this attack. We compare the robustness of all four watermarking methods against the resampling attack by using the algorithm proposed in [11]. This attack consists of sampling vertices from the graphical object surface and connecting them in a way that is not related to the original mesh. The number of sampled vertices represent $\{100\%, 80\%, 60\%, 40\%, 20\%\}$ from the total number of vertices in the original object. The watermark detection results after this attack are shown in Fig. 7. The best results are provided by ProVar followed by ProMean for all four objects.

6. CONCLUSION

This paper proposes a 3-D watermarking methodology based on statistics of geodesic distances defined by using the Fast Marching Method (FMM). The graphical objects are segmented into strips and distributions of geodesic distances are defined according to the watermark code. Two different statistical methods are employed for watermark embedding, by modifying the mean or the variance of distributions of geodesic distances. The vertices are changed using the Vertex Placement Scheme such that the resulting object surface distortion is minimal. The proposed methodology has low computational demands and results in watermarks which are robust to various mesh attacks except for object cropping.

7. REFERENCES

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