LOW-COMPLEXITY BLIND BEAMFORMING BASED ON CYCLOSTATIONARITY

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ABSTRACT

A blind adaptive beamformer with real-valued weights for cyclostationary signals is proposed by introducing a preprocessing stage. It is proved that the optimum weight vector of the proposed method is real-valued. Thus we can discard the imaginary part of the weight vector during the update process, which reduces the computational complexity of the algorithm. Moreover, since the optimum solution is real-valued, a result closer to the optimum one is reached, leading to a much increased convergence rate.

Keywords: Blind beamforming, cyclostationarity, real-valued, self-coherent restoral (SCORE), linear array.

1. INTRODUCTION

Adaptive beamforming is a technique for receiving the signal of interest (SOI) from specific directions while suppressing interference signals from other directions [1, 2]. The minimum variance distortionless response (MVDR) beamformer and reference signal based (RSB) beamformer are two commonly used adaptive beamformers. For the MVDR beamformer, the direction of the SOI is known or can be estimated in advance and the beamformer weights are obtained by minimizing the output power subject to some linear constraints; for the RSB beamformer, a reference signal is available to the system and the beamformer weights can be obtained by minimizing the mean-square error (MSE) between the reference signal and the beamformer output [3]. For both cases, some prior information, either the direction of arrival (DOA) of the SOI or a reference signal, is needed.

When this prior information is not available, we can rely on some statistical properties of the SOI and employ the so-called blind methods for beamforming, such as the constant modulus based methods [4] and the cyclostationarity based methods [5, 6, 7, 8, 9], leading to the class of blind beamformers [10, 11, 12, 13, 14, 15, 16, 17]. In this paper, we focus our study on the cyclostationarity based blind adaptive beamformer. A cyclostationary signal has the statistical property of correlating with either a frequency-shifted version of itself or its complex conjugate [18], which can be used to extract the SOI and suppress the interference without knowing the DOA of the desired signal. A class of spectral self-coherent restoral (SCORE) algorithms was developed for this class of signals [5], and a representative example is the least-square SCORE (LS-SCORE) algorithm. To improve the convergence speed of the traditional LS-SCORE algorithm, the Cross-SCORE algorithm was developed [5]. However, the Cross-SCORE algorithm needs to solve the generalized eigenvalue problem, which has a very high computational complexity and is difficult for an online implementation. Recently, a subspace projection approach was proposed [15], with a similar performance as the Cross-SCORE algorithm. This approach requires an eigendecomposition operation to the data covariance matrix and an estimate of the number of signals, leading to a high computational complexity too. Moreover, if the number of signals is not estimated correctly, the performance of the eigendecomposition-based method will degrade severely.

Most recently, with a preprocessing stage, an RSB beamformer with real-valued coefficients was proposed based on the uniform linear array (ULA) structure [19]. Its advantage is twofold: 1) with real-valued coefficients, the computational complexity of the overall system is reduced significantly; 2) a faster convergence speed is achieved. In this paper, we apply the idea to the cyclostationary based blind beamformer and propose a novel structure using the LS-SCORE algorithm. It can be proved that with this new structure, the optimum solution for the LS-SCORE algorithm will be real-valued. Therefore, in the update of the algorithm, the imaginary part of the weight vector can be discarded, leading to a solution closer to the optimum one. As a result, the convergence speed of the algorithm is increased and its computational complexity reduced.

This paper is structured as follows. In Section II, a review of the maximizing output SINR (signal-to-interference-plus-noise ratio) beamformer and its real-valued optimum solution based on a preprocessing transformation is provided. Our proposed LS-SCORE algorithm with real-valued weight vector is given in Section III. Simulation results are presented in Section IV and conclusions drawn in Section V.

2. MAXIMIZING OUTPUT SINR BEAMFORMER WITH REAL-VALUED COEFFICIENTS

Consider an M-antenna ULA with K far-field narrowband signals impinging from the directions $\theta_1, \theta_2, \cdots, \theta_K$, respectively. The received data vector at the nth snapshot can be expressed as

$$\mathbf{x}[n] = [x_1[n], \cdots, x_M[n]]^T = \sum_{i=1}^{K} \mathbf{a}(\theta_i) s_i[n] + \mathbf{n}[n] \quad (1)$$

where $x_i[n]$ is the signal received by the ith antenna of the array, $s_i[n]$ is the ith source signal, $\mathbf{n}[n]$ is the noise with a power σ^2 and

$$\mathbf{a}(\theta_i) = [1, e^{-j2\pi d \sin(\theta_i)/\lambda}, \cdots, e^{-j2\pi(M-1)d\sin(\theta_i)/\lambda}]^T \quad (2)$$

is the $M \times 1$ steering vector of the ith source, with d being the adjacent antenna spacing and λ denoting the signal wavelength.

Assume that all impinging signals and noise are uncorrelated with each other. Then the data covariance matrix can be expressed as

$$\mathbf{R}_{xx} = E[\mathbf{x}[n]\mathbf{x}^H[n]] = \sum_{i=1}^{K} \sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i) + \sigma^2 \mathbf{I} \quad (3)$$

where $E[\cdot]$ denotes the expectation operation, $[\cdot]^H$ represents the Hermitian transpose, σ_i^2 is the ith source power and $\mathbf{I}$ is the identity matrix.

Without loss of generality, we assume that the first signal is the SOI. Then the optimum weight vector for maximizing the output SINR can be expressed as [1]

$$\mathbf{w}_{SINR} = \eta \mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_1), \quad (4)$$

where η is a scalar.

Based on the specific structure of ULA and the resultant generalized conjugate symmetric property of the data covariance matrix, a preprocessing transformation matrix can be employed to transform the complex-valued optimum weight vector to a real-valued one [19].

Suppose the response of the beamformer to the SOI s_1 is g^*, which is complex-valued, i.e.,

$$\mathbf{a}^H(\theta_1)\mathbf{w}_{SINR} = [\mathbf{w}_{SINR}^H \mathbf{a}(\theta_1)]^H = (g^*)^* = |g|e^{j\psi} \quad (5)$$

where ψ is the angle of g.

For a ULA, its steering vector has the following structure [3]:

$$\mathbf{a}(\theta) = e^{j\phi}\mathbf{J}\mathbf{a}^*(\theta) \quad (6)$$

where $*$ denotes the conjugate operation, $\mathbf{J}$ is the exchange matrix defined as

$$\mathbf{J} = \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{bmatrix} \quad (7)$$

and $\phi = -(M-1)2\pi d \sin(\theta)/\lambda$. Because the covariance matrix $\mathbf{R}_{xx}$ is Hermitian Toeplitz, we have $\mathbf{R}_{xx} = \mathbf{J}\mathbf{R}_{xx}^*\mathbf{J}$. As $\mathbf{J} = \mathbf{J}^{-1}$, we then have [3, 19]

$$\mathbf{R}_{xx}^{-1} = (\mathbf{J}\mathbf{R}_{xx}^*\mathbf{J})^{-1} = \mathbf{J}(\mathbf{R}_{xx}^{-1})^*\mathbf{J}. \quad (8)$$

Using equations (6) and (8), it can be shown that the optimum weight vector $\mathbf{w}_{SINR}$ for maximizing the output SINR has the following generalized conjugate symmetric structure (see [3] for details):

$$\mathbf{w}_{SINR} = e^{j\phi_s}\mathbf{J}\mathbf{w}_{SINR}^* \quad (9)$$

where $\phi_s = 2\psi - \phi$.

We can then construct an $M \times M$ transformation matrix $\mathbf{T} = [\mathbf{t}_0, \mathbf{t}_1, \cdots, \mathbf{t}_{M-1}]^T$ where the vector $\mathbf{t}_i$ satisfies the following generalized conjugate symmetric property $\mathbf{t}_i = e^{-j\phi_s}\mathbf{J}\mathbf{t}_i^*, (i = 0, \cdots, M-1)$.

Define

$$\bar{\mathbf{w}}_{SINR} = \mathbf{T}\mathbf{w}_{SINR}. \quad (10)$$

Substituting (9) into (10), and using the property that $\mathbf{T}\mathbf{J} = e^{-j\phi_s}\mathbf{T}^*$, we have [19]:

$$\bar{\mathbf{w}}_{SINR} = \mathbf{T}\mathbf{w}_{SINR} = \mathbf{T}e^{j\phi_s}\mathbf{J}\mathbf{w}_{SINR}^* = \bar{\mathbf{w}}_{SINR}^*, \quad (11)$$

i.e.,

$$\bar{\mathbf{w}}_{SINR} \in \Re^M \quad (12)$$

Based on (11), a class of adaptive beamfomers with real-valued weights is proposed [19], which has a low computational complexity and a fast convergence rate.

The transformation matrix $\mathbf{T}$ is not unique, and in this paper it is adopted as [19, 20]

$$\mathbf{T} = \begin{cases} \frac{e^{-j\phi_s/2}}{\sqrt{2}} \begin{bmatrix} \mathbf{I} & \mathbf{J} \\ j\mathbf{J} & -j\mathbf{I} \end{bmatrix} & \text{if } M \text{ is even} \\ \frac{e^{-j\phi_s/2}}{\sqrt{2}} \begin{bmatrix} \mathbf{I} & 0 & \mathbf{J} \\ 0 & \sqrt{2} & 0 \\ j\mathbf{J} & 0 & -j\mathbf{I} \end{bmatrix} & \text{if } M \text{ is odd.} \end{cases} \quad (13)$$

Note that $\mathbf{T}$ defined in this way is unitary, i.e., $\mathbf{T}^{-1} = \mathbf{T}^H$.

3. PROPOSED METHOD BASED ON CYCLOSTATIONARITY

In this section, we will first review the concept of cyclostationarity and the LS-SCORE algorithm and then propose our new structure with a real-valued weight vector.

3.1. Cyclostationarity

When a signal is of second-order periodicity with a cycle frequency α, its cyclic conjugate autocorrelation function

$$r_{ss^*} = \langle s(t)\{s^*(t-\tau)\}^* e^{-j2\pi\alpha t}\rangle_\infty \qquad (14)$$

does not equal zero at frequency α for a delay τ, where $\langle\cdot\rangle$ denotes the average over the time interval [0 ∞] [15, 18].

Now we define the cyclic conjugate autocorrelation matrix as

$$\mathbf{R}_{xx^*}(\alpha,\tau) = \langle \mathbf{x}(t)\{\mathbf{x}^*(t-\tau)\}^H e^{-j2\pi\alpha t}\rangle_\infty. \qquad (15)$$

Assume that the SOI has a cyclic frequency α_1, and the cyclic frequencies of the interferences and noise are different from α_1. Then the cyclic conjugate autocorrelation matrix of the received array signal can be expressed as

$$\mathbf{R}_{xx^*}(\alpha_1,\tau) = r_{ss^*}(\alpha_1,\tau)\mathbf{a}(\theta_1)\mathbf{a}^H(\theta_1). \qquad (16)$$

Define a reference signal $z(t)$ as

$$z(t) = \mathbf{c}^H\mathbf{x}^*(t-\tau)e^{j2\pi\alpha_1 t} = \mathbf{c}^H\mathbf{u}(t) \qquad (17)$$

where $\mathbf{u}(t) = \mathbf{x}^*(t-\tau)e^{j2\pi\alpha_1 t}$ and $\mathbf{c}$ is a control vector.

The LS-SCORE algorithm is obtained by minimizing the difference between the reference signal $z(t)$ and the beamformer output $y(t) = \mathbf{w}^H x(t)$, i.e.,

$$f(\mathbf{w},\alpha_1) = \min_{\mathbf{w}}\langle\|z(t) - \mathbf{w}^H\mathbf{x}(t)\|^2\rangle_\infty. \qquad (18)$$

The solution to the problem in (18) is given by [5]

$$\mathbf{w}_{ls} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{xz}(\alpha_1) \qquad (19)$$

where

$$\mathbf{r}_{xz}(\alpha_1) = \mathbf{R}_{xx^*}\mathbf{c} = r_{ss^*}(\mathbf{a}^H(\theta_1)\mathbf{c})\mathbf{a}(\theta_1). \qquad (20)$$

The optimum weight vector of the LS-SCORE algorithm can also be expressed as [6, 15]

$$\mathbf{w}_{ls} = \mu\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_1) \qquad (21)$$

where $\mu = r_{ss^*}\mathbf{a}^H(\theta_1)\mathbf{c}$.

Using the stochastic gradient method with discrete time n instead of continuous time t in (18), we can obtain the following update equation for the weight vector $\mathbf{w}[n]$ with a stepsize u_1 [8]:

$$\mathbf{w}[n+1] = \mathbf{w}[n] - u_1\mathbf{P}[n] \qquad (22)$$

where $\mathbf{P}[n] = -(z[n] - \mathbf{w}^H\mathbf{x}[n])^*\mathbf{x}[n]$ is the gradient of $f(\mathbf{w},\alpha_1)$ with respect to $\mathbf{w}^*$.

3.2. The Proposed LS-SCORE Algorithm

We can see that $\mathbf{w}_{ls}$ in (21) has the same form as $\mathbf{w}_{SINR}$ in (4). Therefore, the LS-SCORE beamformer also achieves the maximum output SINR as the optimum beamformer given in (4).

Similarly, we can prove that $\mathbf{w}_{ls}$ has the same generalized conjugate symmetric structure as given in (9)

$$\mathbf{w}_{ls} = e^{j\varphi}\mathbf{J}\mathbf{w}_{ls}^*. \qquad (23)$$

$Proof$: Substituting (6) and (8) into (21), and noticing that $\mathbf{JJ} = \mathbf{I}$, we have

$$\begin{aligned}\mathbf{w}_{ls} &= \mu\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_1)\\ &= \mu\mathbf{J}(\mathbf{R}_{xx}^{-1})^*\mathbf{J}e^{-j\phi}\mathbf{J}\mathbf{a}^*(\theta_1)\\ &= \frac{\mu}{\mu^*}e^{-j\phi}\mathbf{J}\mu^*(\mathbf{R}_{xx}^{-1})^*\mathbf{JJ}\mathbf{a}^*(\theta_1)\\ &= \frac{\mu}{\mu^*}e^{-j\phi}\mathbf{J}\mathbf{w}_{ls}^*. \qquad (24)\end{aligned}$$

By defining

$$e^{-j\varphi} = \frac{\mu}{\mu^*}e^{-j\phi}, \qquad (25)$$

and substituting (25) into (24), we then obtain the result in (23).

Then $\mathbf{w}_{ls}$ can be transformed into a real-valued form using the transformation matrix $\mathbf{T}$ defined in (13) with ϕ_s replaced by φ, i.e.,

$$\bar{\mathbf{w}}_{ls} = \mathbf{T}\mathbf{w}_{ls} = \bar{\mathbf{w}}_{ls}^*. \qquad (26)$$

Since we do not know the phase of φ in $\mathbf{T}$, we can not use the transformation matrix $\mathbf{T}$ directly to form a blind adaptive beamformer with real-valued weights as in the constant modulus based blind beamformer case [16]. Following the RSB structure proposed in [16], noting that here we can consider $z(t)$ as the reference signal in the RSB beamformer, we can decompose $\mathbf{T}$ into two parts as follows $\mathbf{T} = \beta\mathbf{T}_1$ where $\beta = e^{-j\varphi/2}$ is an unknown parameter with a unit magnitude and $\mathbf{T}_1 = e^{j\varphi/2}\mathbf{T}$. Now $\mathbf{T}_1$ is independent of φ.

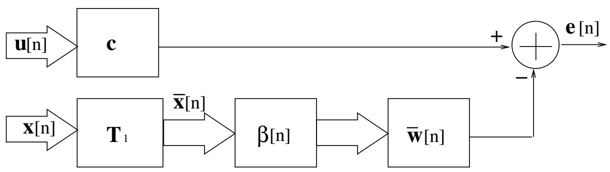

Fig. 1: The proposed LS-SCORE blind beamformer

Define $\bar{\mathbf{w}} = \mathbf{T}_1\mathbf{w}$ and $\bar{\mathbf{x}}[n] = \mathbf{T}_1\mathbf{x}[n]$. The new structure for the proposed LS-SCORE algorithm is shown in Fig. 1.

Now we can formulate the proposed LS-SCORE algorithm as follows

$$J(\bar{\mathbf{w}},\beta) = \min_{\bar{\mathbf{w}}}||(z[n] - \bar{\mathbf{w}}^H\beta\bar{\mathbf{x}}[n])||^2. \qquad (27)$$

By taking the gradient of (27) with respect to $\bar{\mathbf{w}}^*$ and β, respectively, we have

$$\frac{dJ(\bar{\mathbf{w}},\beta)}{d\bar{\mathbf{w}}^*} = (\bar{\mathbf{x}}^H[n]\bar{\mathbf{w}} - \beta z^*[n])\bar{\mathbf{x}}[n], \quad (28)$$
$$\frac{dJ(\bar{\mathbf{w}},\beta)}{d\beta} = -\bar{\mathbf{w}}^H\bar{\mathbf{x}}[n]z^*[n]. \quad (29)$$

(28) is a function of β and (29) is a function of $\bar{\mathbf{w}}$. Now we introduce an iterative method to solve this problem. We can see that if β is known, then we can update $\bar{\mathbf{w}}$ in the negative direction of the gradient of (28) using the stochastic gradient method; once $\bar{\mathbf{w}}$ is updated, we then use the same approach to update β. Then we obtain a set of update equations for both β and $\bar{\mathbf{w}}$ as follows:

$$\begin{cases} \hat{\mathbf{w}}[n+1] = \bar{\mathbf{w}}[n] - u_1(\bar{\mathbf{x}}^H[n]\bar{\mathbf{w}}[n] - \beta[n]z^*[n])\bar{\mathbf{x}}[n] \\ \hat{\beta}[n+1] = \beta[n] + u_2\bar{\mathbf{w}}^H[n]\bar{\mathbf{x}}[n]z^*[n]. \end{cases} \quad (30)$$

Since the optimum solution $\mathbf{w}_{ls}$ after this transformation is real-valued, we can discard the imaginary part of $\mathbf{w}$ at each update, which is closer to the optimum one and generally gives a better output SINR result. Moreover, since β is located on the unit circle, we also need to normalise β after each update. As a result, we arrive at the following set of update equations:

$$\begin{cases} \hat{\mathbf{w}}[n+1] = \bar{\mathbf{w}}[n] - u_1(\bar{\mathbf{x}}^H[n]\bar{\mathbf{w}}[n] - \beta[n]z^*[n])\bar{\mathbf{x}}[n] \\ \bar{\mathbf{w}}[n+1] = \frac{1}{2}(\hat{\mathbf{w}}[n+1] + \hat{\mathbf{w}}^*[n+1]) \\ \hat{\beta}[n+1] = \beta[n] + u_2\bar{\mathbf{w}}^H[n]x\bar{\mathbf{x}}[n]z^*[n] \\ \beta[n+1] = \hat{\beta}[n+1]/|\hat{\beta}[n+1]|. \end{cases} \quad (31)$$

3.3. Computational Complexity

The number of real multiplications of the traditional LS-SCORE algorithm for each update is $8M+2$ for an even M. However, the number of real multiplications of the proposed LS-SCORE algorithm is only $4M + 19$. So the computational complexity of the proposed LS-SCORE is only about half of the traditional LS-SCORE algorithm for a large M. For an odd M, one of elements of $\mathbf{T}$ is $\sqrt{2}$ and additional 2 real multiplications are needed for the proposed LS-SCORE algorithm.

4. SIMULATIONS

Simulations are performed based on a 4-antenna ULA. The SOI and two interferences impinge on the array from $0°$, $-40°$ and $50°$, respectively. Both the SOI and interferences are binary phase-shift-keying (BPSK) signals with a raised-cosine pulse shape and the roll-off factor is 1. The baud rate for the SOI and interferences is 50MBd, 30MBd, and 20MBd, respectively. The SOI and interferences have the same carrier frequency.The received signals are converted to baseband and then sampled at a frequency of 250MHz. The cycle frequency for the SOI is set to 50MHz. The antenna spacing between adjacent antennas is half wavelength at the carrier frequency. We assume that all signals have the same power with a signal-to-noise ratio (SNR) of 10dB. The control vector $\mathbf{c}$ is given by $\mathbf{c} = [1, 0, \cdots, 0]^T$ and the time lag $\tau = 0$ is chosen.

Two different values of the stepsize u_1 are chosen, which are 0.0001 and 0.00001, respectively, and the stepsize u_2 for β is set to 0.1. Fig. 2 shows the output SINR result with respect to the snapshot number. We can see that an improved performance has been achieved in terms of both convergence rate and output SINR for the proposed LS-SCORE algorithm.

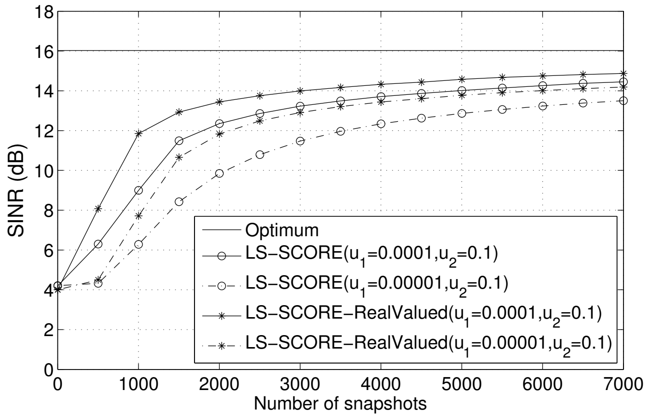

Fig. 2: Output SINR versus the number of snapshots.

Now consider the evolution of the normalized weight vector defined as $\|\mathbf{w} - \mathbf{w}_{opt}\|^2/\|\mathbf{w}_{opt}\|^2$. The result corresponding to Fig. 2 is shown in Fig. 3, where the convergence rate of the weight vector of the proposed LS-SCORE algorithm is much faster than that of the traditional one.

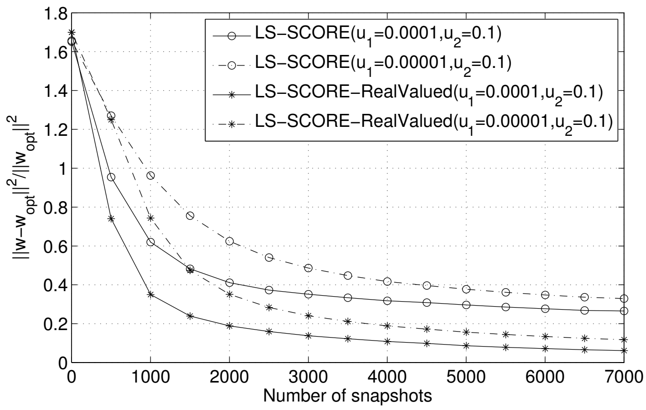

Fig. 3: Normalized weight vector error versus the number of snapshots.

5. CONCLUSION

A blind adaptive beamformer with real-valued weight vector for cyclostationary signals has been proposed by preprocessing the received data. After this preprocessing stage, the complex-valued optimum weight vector of the traditional LS-SCORE algorithm is transformed into a real-valued one. Therefore, we can ignore the imaginary part of the weight vector in the update of the algorithm. As a result, the computational complexity of the proposed LS-SCORE algorithm is much less than that of the traditional one. Simulation results have shown that the proposed method outperforms the traditional LS-SCORE algorithm in terms of both convergence rate and output SINR.

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