COMPRESSIVE SENSING FOR WIFI-BASED PASSIVE BISTATIC RADAR

Patrick Maechler, Norbert Felber, and Hubert Kaeslin

Integrated Systems Laboratory, ETH Zurich
Gloriastr. 35, 8092 Zurich, Switzerland
{maechler,felber}@iis.ee.ethz.ch, kaeslin@ee.ethz.ch

ABSTRACT

Passive radars using WiFi base stations as illuminators are a promising option for local area surveillance. Unfortunately, the reflections of WiFi signals from a large object can easily mask any nearby objects making it difficult to detect all targets within range. Using the fact that most target scenes are only very sparsely populated makes it possible to apply compressive sensing (CS) for target detection. We analyze how well WiFi signals fit into the CS framework, propose corresponding detection schemes, and show how detection accuracy is increased and receiver complexity is reduced. It turns out that CS enables the detection of closely spaced targets with only a small number of recorded WiFi frames.

Index Terms— Localization, passive radar, compressive sensing, sparse approximation

1. INTRODUCTION

Passive radars offer a low-cost alternative to the commonly used active systems. The signals of existing radio transmitters (such as TV stations or mobile communication base stations) and their reflections on targets are analyzed by a passive radar receiver to determine the location of the targets. While passive radars are easy to setup and their presence is difficult to detect, they have to deal with non-cooperative transmitters, which puts higher demands on signal processing.

Since WiFi base stations (BS) are becoming ubiquitous nowadays, exploiting their signals might be interesting for local area surveillance. The first experiments to detect humans using WiFi signals were conducted in [1]. The regularly broadcasted beacons, which identify the BS to potential users, were utilized for localization. By correlating the received signal with the transmitted one, the detection of one person was possible in a wide open field without much clutter. Experiments in high clutter indoor environments with passive bistatic radars using OFDM-modulated WiFi signals were conducted in [2], where it was possible to detected one moving person even trough a wall. Two persons moving in opposite directions were, however, already difficult to detect. Experimental results of a WiFi-based passive radar system detecting range and speed of a target in a parking lot were presented in [3]. It was demonstrated that a moving vehicle can be detected, but a person standing next to it is severely masked by the strong reflection of the car. Only the joint application of disturbance removal techniques and ambiguity function control filters allowed for a detection of the moving vehicle and the person.

In a typical radar scenario, only a small number of interesting targets are usually present. This sparsity can be exploited in the detection process. CS [4] is a recently introduced framework based on sparsity and incoherent measurements, which offers the potential to reduce the number of required measurements and/or to increase accuracy. The goal is to overcome the limitations of WiFi-based passive radar when it comes to the detection of closely spaced targets. To this end, we first introduce CS and then apply it to WiFi-based localization problems. Multiple target detection schemes are proposed, depending on the number and modulation schemes of the available BS.

Notation: Lowercase and uppercase boldface letters stand for column vectors and matrices, respectively. $\mathbf{A}_{n,m}$ denotes the matrix element at the nth row and the mth column. The ℓ_2-norm of a vector $\mathbf{x}$ is denoted by $\|\mathbf{x}\|_2$ and $\|\mathbf{x}\|_1$ stands for its ℓ_1-norm.

2. COMPRESSIVE SENSING

Compressive sensing (CS), as introduced in [4], allows to sample a signal $\mathbf{x} \in \mathbb{C}^N$ with fewer measurements than the Nyquist rate suggests. Measurements $\mathbf{y} \in \mathbb{C}^M$ with $M < N$ are performed through non-adaptive, linear projections

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{n} \tag{1}$$

with a measurement matrix $\mathbf{\Phi} \in \mathbb{C}^{M\times N}$ and additive noise $\mathbf{n} \in \mathbb{C}^M$. Solving (1) for $\mathbf{x}$ is an underdetermined problem with no unique solution. If, however, $\mathbf{x}$ is sparse with only a few non-zero entries $K \ll N$, CS allows to reconstruct $\mathbf{x}$ provided the columns of $\mathbf{\Phi}$ (called atoms) are sufficiently *incoherent*, i.e. the inner product of any two columns of $\mathbf{\Phi}$ is small. The CS reconstruction problem can be formulated as a

This work was supported by armasuisse Wissenschaft & Technologie.

convex optimization problem

$$\hat{\mathbf{x}} = \min_{\tilde{\mathbf{x}}} \|\tilde{\mathbf{x}}\|_1 \quad \text{subject to } \|\mathbf{y} - \mathbf{\Phi}\tilde{\mathbf{x}}\|_2 \leq \varepsilon, \qquad (2)$$

where $\|\mathbf{n}\|_2 \leq \varepsilon$. Since exactly solving (2) implies complex computations, faster approximations are often preferred, especially in an application like radar, where real-time reconstruction is required.

Greedy algorithms are sparse recovery algorithms that iteratively approximate the sparse solution $\hat{\mathbf{x}}$. Matching pursuit (MP) [5] iteratively identifies the atom that is most correlated to the current signal estimate, followed by a simple update to compute an improved signal estimate. While each iteration of MP is computationally easy, the number of iterations may be large due to slow convergence. Orthogonal matching pursuit (OMP) [6], compressive sampling matching pursuit (CoSaMP) [7], and subspace pursuit (SP) [8] are more sophisticated greedy algorithms that incorporate a least squares (LS) step to compute the signal estimates. This LS step significantly reduces the number of required iterations, but demands high computational effort.

3. SPARSE LOCALIZATION

Ideas for exploiting sparsity and taking incoherent measurements have been around for a long time in the radar literature (some of them are reviewed in [9]), but only with the introduction of CS theory, those ideas gained more momentum. After a brief comment on sparsity, the most prominent applications of CS to radar are reviewed.

Sparsity Before CS can be applied, a discrete and finite-dimensional representation of the radar scene must be found, which results in a sparse vector $\mathbf{x}$. In many radar applications, this representation is established by a discrete grid defined in the range-Doppler plane. Assuming sparsity on the range-Doppler grid means that each object can be modeled as a point target located on a grid point and having a certain discrete velocity. This is a reasonable assumption for many small targets and fine enough grids. In order to obtain truly sparse representations, the grid must usually be chosen finer than what the bandwidth or ambiguity function suggest. A four times over-complete measurement matrix (i.e., four times more atoms are used than the number needed to span the whole solution space) has been shown to work well in many cases (e.g., [10]). Especially in indoor environments, many reflections will be received from walls, ceilings, etc., that cannot be sparsely represented. Therefore, all clutter must be removed, e.g. by background subtraction methods, before sparse recovery algorithms can be applied.

Incoherent pulses The first paper on CS radar imaging [11] proposed a system which transmits a pseudo noise (PN) or chirp sequence. Such sequences are incoherent to point targets on a range-Doppler grid. The proposed receiver can then be implemented as a low-rate analog-to-digital converter (ADC)[1] sampling below the Nyquist frequency. However, subsampling at the receiver leads to a loss in the receiver SNR. [12] proposed transmitting incoherent Alltop sequences and receiving reflections with a Nyquist-rate ADC. CS-based reconstruction then allows for higher detection performance.

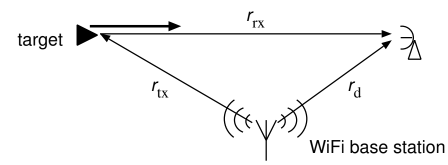

Fig. 1. Passive bistatic radar setup

Linear Projection CS can reduce the rate at which measurements are acquired by projecting the high-bandwidth signal at the receiver onto a PN sequence, integrating the result, and sampling it at a lower frequency. Using the example of a ground penetrating radar, [13] increased the resolution with this sampling scheme. While only very simple short pulses need to be transmitted, the receivers need additional elements for the linear projection, such as high-bandwidth mixers.

Pulse compression Step-frequency radars transmit pulses at discrete frequencies. CS-based schemes proposed, e.g., in [10] reduce the number of transmitted frequencies by selecting and transmitting only a random subset thereof. The different frequencies are either transmitted sequentially or at the same time. Thus, either measurement time or receiver complexity can be reduced. From a transmit energy point of view, this system is more efficient than the incoherent pulses or linear projection approaches, since no energy is lost.

A passive radar system for OFDM signals using CS was proposed in [14]. Digital video/audio broadcasting (DVB/DAB) stations are used as illuminators to identify flying targets. The number of symbols that must be known at the receiver from trainings or after decoding using outer codes was reduced while maintaining good detection accuracy.

4. HIGH ACCURACY WIFI RADAR

In this section, WiFi signals are analyzed for their suitability for any of the CS radar schemes introduced in Section 3 in order to increase target detection accuracy. IEEE 802.11 WiFi standards [15] use direct sequence spread spectrum (DSSS) modulation in the older 802.11b standard with 11 MHz bandwidth and orthogonal frequency division multiplexing

[1]While the sampling rate of the ADC can be low, the supported input bandwidth must still cover the full band. This leads to intentional aliasing.

(OFDM) with 20 MHz bandwidth in the newer a/g/n standards. The WiFi setup considered is shown in Fig. 1. We measure the differential bistatic delay $\tau = (r_{\text{tx}} + r_{\text{rx}} - r_{\text{d}})/c$ (the delay between the reference signal and the reflection), which translates into the target's position, and the Doppler frequency to obtain its velocity.

The *ambiguity function* [16] of a radar signal characterizes how well radar targets with a certain delay and Doppler difference can be distinguished. An ambiguity function analysis for WiFi signals was conducted in [16] and measured a range resolution of 25.4 m (27.3 m theoretical) for DSSS beacons and 18.8 m (18.1 m theoretical) for OFDM frames. The theoretical values are determined by the inverse of the actually occupied bandwidth $\Delta r = c_0/B$. The Doppler resolution is determined by the observation time; the observation of many subsequent frames is required to obtain a good velocity resolution. In both the range and Doppler dimensions, relatively large sidelobes were identified, which explains the masking of closely spaced targets as observed in [2, 3].

Since we aim at a high-resolution WiFi-radar, OFDM-based standards are analyzed first due to their higher initial bandwidth and, thus, better resolution.

4.1. OFDM-based WiFi

At the beginning of every 802.11 frame, known training symbols are transmitted that allow the receiver to obtain a channel estimate. In an OFDM training symbol, there are $N_{DT} = 52$ out of $N_S = 64$ OFDM subcarriers trained. Transforming the resulting estimate into the time domain will reveal all reflections, and CS reconstruction can then identify the ones corresponding to radar targets. With sparsity in a discrete time basis and measurements performed in a subset of the discrete Fourier basis, incoherence of the measurement matrix is given. Thus, measuring trained subcarriers meets CS preconditions for range estimations. The resulting system is similar to pulse compression in a step-frequency radar, but without much undersampling.

The maximum supported delay spread (and thus the maximum supported range) is defined by the length of the cyclic prefix. In the long training phase, a double length cyclic prefix of duration $T_t = 1.6\,\mu$s is used [15]. This results in a maximum supported range ($r_{\text{max}} = r_{\text{tx}} + r_{\text{rx}} - r_{\text{d}}$) of $r_{\text{max}} = T_t\, c_0 = 480$ m.

Also, previous WiFi-based passive radars [3] required a reference signal obtained from an antenna placed right next to the BS. This is, however, unnecessary in the described setup since all training tones are known to the receiver.

4.2. Formulation as CS problem

In order to obtain a CS formulation, range as well as Doppler frequencies must be described on a discrete grid. The sparse vector $\mathbf{x}$ is composed of all the delay profiles $\mathbf{x}_v \in \mathbb{R}^P$ (with P data points in the range dimension) at all considered Doppler frequencies $\omega_v,\ v \in [1, V]$ (with V data points in the Doppler dimension) stacked on top of each other (see (4)). This results in a vector $\mathbf{x}$ of size $N = PV$ with a sparsity K equal to the number of targets. The measurements vector $\mathbf{y}$ contains all the measured subcarriers from L subsequent frames, recorded at times $t_l,\ l \in [1, L]$. Thus, the measurement vector has a dimension of $M = LN_{DT}$.

Next, the measurement matrix is constructed, which includes multiple discrete Fourier transforms. The Doppler shift within one training sequence in assumed to be negligible (as in [14]).[2] We define a discrete Fourier transform (DFT) matrix $\mathbf{F}$, which contains only a subset of $\mathcal{S} = \{s_1\, s_2 \dots s_{N_{DT}}\}$ trained subcarriers and that limits the length of the delay profile to P.

$$\mathbf{F}_{n,m} = \frac{1}{\sqrt{N_{DT}}} \exp\left(-j2\pi \frac{s_n \cdot (m-1)}{N_S}\right) \tag{3}$$

with $n \in [1, N_{DT}]$, $m \in [1, P]$ for N_{DT} measured pilot tones and a delay profile length $P \leq N_S$. $\mathbf{F}$ is constructed from a $N_S \times N_S$ DFT matrix by selecting only the first P columns and the N_{DT} rows corresponding to the pilot tones. $\mathbf{\Phi}$ establishes a linear relation between the measurements $\mathbf{y}_l$ of multiple frames at times $t_l,\ l \in [1, L]$ (assuming $t_1 = 0$) and the range profile $\mathbf{x}_v$ at different Doppler shifts $\omega_v,\ v \in [1, V]$.

$$\underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_L \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathbf{F} & \dots & \mathbf{F} \\ e^{j\omega_1 t_2}\mathbf{F} & \dots & e^{j\omega_V t_2}\mathbf{F} \\ \vdots & \ddots & \vdots \\ e^{j\omega_1 t_L}\mathbf{F} & \dots & e^{j\omega_V t_L}\mathbf{F} \end{bmatrix}}_{\mathbf{\Phi}} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_V \end{bmatrix}}_{\mathbf{x}} \tag{4}$$

4.3. Simulations

A simulation environment for an OFDM WiFi passive bistatic radar was set up in order to test and evaluate CS-based target localization algorithms. The targets were distributed on arbitrary positions on the range-Doppler plane. Since they are not necessarily positioned on grid points, a fourfold overcomplete measurement matrix is applied (cf. Sec. 3). With a 20 MHz sampling rate, the delay profile is represented by 32 samples, which increases to $P = 128$ for fourfold overcompletion. A maximum velocity of 100 km/h and a resolution of 0.5 m/s in r_{rx} direction result in $V = 61$ grid points in the Doppler domain. Vector $\mathbf{x}$ thus has a dimension of $N = PV = 7807$. Assuming that 20 frames are recorded at random times within a 50 ms period, an underdetermined problem with only $M = LN_{DT} = 20 \cdot 52 = 1040$ measurements results. The sparse delay profile $\mathbf{x}$ is then recovered using (4) and MP [5]. Suppression of highly correlated

[2] Assuming a maximum velocity of 100 km/h, the maximal phase shift during the 8 μs training is $\Delta\omega = 2\pi v/c_0\, T f_c = 0.011$, which is negligible.

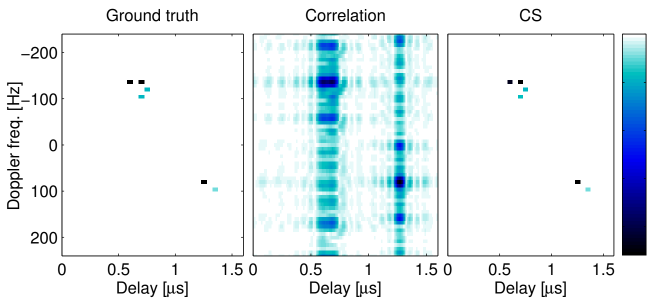

Fig. 2. OFDM passive radar simulation: correlation-based vs. CS-based detection ($L = 20$ frames, noise-free)

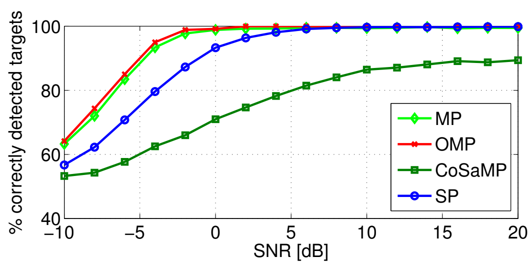

Fig. 3. OFDM passive radar simulation: SNR sweep with various greedy reconstruction algorithms

neighbors of selected targets is applied for better reconstruction results and FFTs are used for fast processing speed.

In Fig. 2, CS reconstruction of a sparse scene with six targets with different reflectivity (indicated by their color) is compared to the image produced by correlating the received and transmitted signals. One can clearly see that sidelobes in the ambiguity function mask closely spaced targets in the correlation image, while CS is still able to identify all six targets. The effect of noise together with the performance of different CS reconstruction algorithms is studied in Fig. 3 where the percentage of correctly detected targets (arranged as in Fig. 2) is evaluated. In this case, OMP shows the best performance.

5. MULTI-CHANNEL WIFI RADAR

CS-aided post processing can simplify the identification of targets as shown above but it cannot go much beyond the resolution limits set by the ambiguity function. To overcome the rather coarse resolution, increasing the measured bandwidth seems to be the only viable solution.

In urban environments multiple BS, transmitting in different channels, usually are within range. The trained subcarriers of all occupied channels can be measured while others are left unobserved, which corresponds to the principle of a step-frequency radar. CS reconstruction algorithms can 'fill in' the holes and reproduce a sparse scene with the accuracy offered by the entire available bandwidth instead of only one channel.

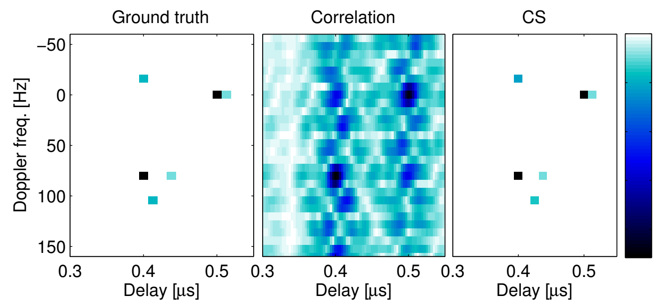

Fig. 4. High resolution OFDM passive radar simulation: correlation-based vs. CS-based detection with SNR 0 dB and $L = 20$ frames (zoomed)

We consider the 2.4 GHz industrial, scientific, and medical (ISM) band. The trained subcarriers of channels 1 through 13 (permitted in Europe) span a total bandwidth of 76.6 MHz. This leads to a theoretical range resolution of $\Delta r = c_0/76.6\,\text{MHz} = 3.92\,\text{m}$. The measurement matrix defined in (4) must now be extended to include measurements performed in different channels, which is achieved by only a few changes on how the subset of trained subcarriers is defined in (3). First, the sampling rate is increased from 20 MHz to 80 MHz (this is only the sampling rate at which $\mathbf{x}_v$ is represented; a physical sampling rate of 20 MHz is still possible if the center frequency is tunable). Those 80 MHz are subdivided into $N_S = 4 \cdot 64 = 256$ subcarriers. For each received frame, a subset $\mathcal{S} \subset [1, N_S]$ of $N_{DT} = 52$ subcarriers is measured and the corresponding DFT matrices must include only the defined subcarriers $\mathbf{F}_{\mathcal{S}}$.

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_L \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{\mathcal{S}_1} & \dots & \mathbf{F}_{\mathcal{S}_1} \\ e^{j\omega_1 t_2}\mathbf{F}_{\mathcal{S}_2} & \dots & e^{j\omega_V t_2}\mathbf{F}_{\mathcal{S}_2} \\ \vdots & \ddots & \vdots \\ e^{j\omega_1 t_L}\mathbf{F}_{\mathcal{S}_L} & \dots & e^{j\omega_V t_L}\mathbf{F}_{\mathcal{S}_L} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_V \end{bmatrix} \tag{5}$$

Simulations with fourfold increased resolution and therefore four times smaller possible distance between targets were performed. The results after measuring 20 frames randomly distributed on channels 1, 5, 9, and 13 with 0 dB SNR are presented in Fig. 4. Five of the six targets were exactly recovered in this scenario by CS-based detection, one target only approximately.

6. REDUCED SAMPLING RATE

When our goal is not to increase accuracy but to further reduce complexity, a different CS scheme needs to be applied. Under normal circumstances, achieving the necessary sampling rates for WiFi-signals is no problem. However, one could think of energy-starved sensor nodes, where reducing the number of acquired samples might be helpful to save

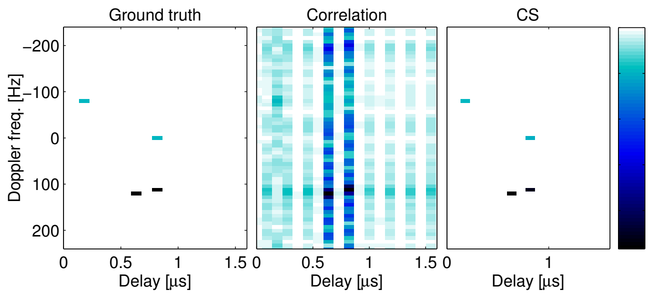

Fig. 5. DSSS passive radar simulation: correlation and CS-based detection with SNR 10 dB and $L = 10$ frames

power and memory. Transmitting DSSS-modulated frames corresponds to transmitting an incoherent PN sequence as proposed in [11]. The measurement matrix $\mathbf{\Phi}$ then contains shifted and subsampled versions of the transmitted Barker PN code.

The simulated system records the SYNC and AFD fields of an 802.11b frame [15]. It is assumed that 10 such frames are recorded within 50 ms. The examples shown in Fig. 5 contains 4 targets and receives signals with an SNR of 10 dB. Even with an undersampling factor of 128, CS reconstruction allows for a perfect detection of all targets.

7. CONCLUSION

It has been shown that compressive sensing reconstruction algorithms are suitable to detect targets with high accuracy in OFDM/WiFi-based passive bistatic radars. Not only can the proposed detection scheme distinguish closely spaced targets, which are difficult to separate from the correlation results, but it can also do so with only a few measurements at randomly distributed times. The accuracy is further increased when multiple base stations are in range, which can easily be included into the presented CS formulation. Also, a low-complexity scheme based on DSSS/WiFi was introduced. In all simulations, only training symbols were included for detection, which makes additional antennas to obtain a reference signal redundant.

8. REFERENCES

[1] H. Guo, K. Woodbridge, and C.J. Baker, "Evaluation of WiFi beacon transmissions for wireless based passive radar," in *Proc. IEEE Radar Conf.*, May 2008, pp. 1–6.

[2] K. Chetty, G. Smith, H. Guo, and K. Woodbridge, "Target detection in high clutter using passive bistatic WiFi radar," in *Proc. IEEE Radar Conf.*, May 2009, pp. 1–5.

[3] P. Falcone, F. Colone, C. Bongioanni, and P. Lombardo, "Experimental results for OFDM WiFi-based passive bistatic radar," in *Proc. IEEE Radar Conf.*, May 2010, pp. 516–521.

[4] D.L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, April 2006.

[5] S.G. Mallat and Z. Zhang, "Matching pursuits with time-frequency dictionaries," *IEEE Trans. Signal Process.*, vol. 41, no. 12, pp. 3397–3415, Dec. 1993.

[6] Y.C. Pati, R. Rezaiifar, and P.S. Krishnaprasad, "Orthogonal matching pursuit: recursive function approximation with applications to wavelet decomposition," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Nov. 1993, pp. 40–44.

[7] D. Needell and J.A. Tropp, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," *Applied and Computational Harmonic Analysis*, vol. 26, no. 3, pp. 301–321, May 2009.

[8] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2230–2249, May 2009.

[9] L.C. Potter, E. Ertin, J.T. Parker, and M. Cetin, "Sparsity and compressed sensing in radar imaging," *Proc. IEEE*, vol. 98, no. 6, pp. 1006–1020, June 2010.

[10] J.H.G. Ender, "On compressive sensing applied to radar," *Signal Processing*, vol. 90, no. 5, pp. 1402–1414, May 2010.

[11] R. Baraniuk and P. Steeghs, "Compressive radar imaging," in *IEEE Radar Conf.*, Apr. 2007, pp. 128–133.

[12] M.A. Herman and T. Strohmer, "High-resolution radar via compressed sensing," *IEEE Trans. Signal Process.*, vol. 57, no. 6, pp. 2275–2284, June 2009.

[13] A.C. Gurbuz, J.H. McClellan, and WR Scott, "Compressive sensing for GPR imaging," in *Proc. Asilomar Conf. Signals, Systems and Computers*. IEEE, Nov. 2007, pp. 2223–2227.

[14] C.R. Berger, S. Zhou, and P. Willett, "Signal extraction using compressed sensing for passive radar with OFDM signals," in *IEEE Int. Conf. Inf. Fusion*, July 2008, pp. 1–6.

[15] "IEEE standard for information technology, part 11: Wireless LAN medium access control (mac) and physical layer (phy) specifications," June 2007.

[16] F. Colone, K. Woodbridge, H. Guo, D. Mason, and C.J. Baker, "Ambiguity function analysis of wireless LAN transmissions for passive radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 47, no. 1, pp. 240–264, Jan. 2011.