CFO ESTIMATION ALGORITHM FOR OQAM-OFDM SYSTEMS BASED ON THE CONJUGATE SYMMETRY PROPERTY

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ABSTRACT

In this contribution, a carrier frequency offset (CFO) estimator for OQAM-OFDM systems based on the conjugate symmetry property of a properly designed synchronization sequence is proposed. It extends the concept of a previously described symbol-timing and phase offset recovery scheme to build a complete time-domain synchronization method. The outcome of numerical evaluations shows that the proposed CFO estimation algorithm is suitable as a coarse CFO correction for continuous stream transmission systems. It delivers satisfying results in case of AWGN channels as well as for channels with moderate delay spreads while exhibiting the drawback of a high sensitivity to symbol timing errors. Furthermore, the possibility to reduce the overhead of the synchronization sequence by allowing a certain amount of self-interference is investigated.

***Index Terms*—** CFO estimator, OQAM-OFDM, conjugate symmetry property

1. INTRODUCTION

The task of synchronization is essential to every digital communication system and therefore it is crucial for the systems overall performance. Speaking of multi-carrier based communication systems, synchronization algorithms have been extensively studied for the most prominent cyclic prefix (CP-) OFDM scheme. Besides the CP-OFDM based concepts, the Offset-QAM (OQAM-) OFDM scheme gained interest as a possible system design for future generations of wireless communication systems due to its potential benefits in spectral efficiency and the capability to adapt to double-dispersive channels [1]. For the OQAM-OFDM scheme, data-aided synchronization algorithms based on repetitive symbols in the time-domain have been studied in [2]. This class of algorithms makes it necessary to establish at least two identical parts within the time-domain synchronization period to estimate the frame start and carrier frequency offset (CFO). The resulting redundancy decreases the efficiency of the preamble structure within the OQAM-OFDM system. To minimize the overhead introduced by the demand for repetitive preamble parts, a symbol timing estimator based on the conjugate symmetry property (CSP) of a properly designed preamble has been introduced in [3] based on an idea proposed in [4]. The exploitation of the CSP in OQAM-OFDM systems has been further studied in [5] and [6] in case the synchronization sequence is located at the beginning of a transmission as it is the case, for example, in WiFi systems.

In this paper, we propose a CFO estimation algorithm based on a single synchronization sequence symbol as an extension to the symbol timing and phase offset estimator, described in [3]. The proposed method is suitable for a continuous stream transmission and reduces the preamble overhead in comparison to the algorithms presented in [2].

The remaining of this publication is structured as follows. In section 2 the system model is presented while the estimation algorithm together with the synchronization sequence is introduced in section 3. Section 4 shows the outcome of the numerical evaluation and the conclusion is drawn in section 5.

Notation: $\sqrt{-1} = j$, $\angle(\cdot)$ denotes the argument of a complex number in the range of $[-\pi, \pi)$. The symbol $*$ specifies the convolution operator and $\Re(\cdot)$ and $\Im(\cdot)$ defines the real and imaginary part of a complex value, respectively. Furthermore, $S = \mathcal{C}(\mathbb{S})$ is the cardinality of the set $\mathbb{S}$.

2. SYSTEM MODEL

We consider a critically-sampled OQAM-OFDM system, where the real and imaginary part of a complex-valued QAM symbol $c_{l,k}$ at the kth subchannel and the symbol index l are sent staggered in time by $T/2$. Thereby $T = KT_s$ is the time between two consecutive QAM symbols with sampling frequency $f_s = 1/T_s$. Introducing m as the new symbol index with symbol spacing $T/2$, the PAM symbols are defined as $d_{m,k} = \Re(c_{l,k})$ and $d_{m+1,k} = \Im(c_{l,k})$. Let $K = K_d + K_g$ be the total number of subchannels with $K_d = \mathcal{C}(\mathbb{K}_d)$ defining the number of data-bearing and $K_g = \mathcal{C}(\mathbb{K}_g)$ the number of guard subchannels. The term T_s and normalization factors will be discarded for simplicity in the following.

The time-discrete baseband signal $s[n]$ can be written as

$$s[n] = \sum_{m=-\infty}^{\infty} \sum_{k \in \mathbb{K}_d} j^{mod(m+k,2)} d_{m,k} p_k[n - mT/2] \quad (1)$$

where p_k defines the filter function according to [1] as

$$p_k[n] = p[n] e^{j\frac{2\pi}{K}kn}. \quad (2)$$

The prototype filter functions p considered here fulfill the symmetric property such that

$$p[n] = p[\alpha K - n] \quad (3)$$

holds for $n \in \{1, 2, \cdots, \frac{\alpha}{2}K\}$, where α is the overlapping factor of the prototype filter used in the filter bank.
The signal at the receiver is defined as

$$r[n] = (s[n] * h[n])\, e^{j\frac{2\pi}{K} f_\Delta n + \varphi} + \nu[n]. \quad (4)$$

Thereby, $h[n]$ denotes the normalized time-discrete channel impulse response, $\nu[n]$ describes zero mean, circular complex white Gaussian noise and φ stands for a random phase offset. The carrier frequency offset f_Δ is normalized with respect to the subchannel spacing.

3. PROPOSED CFO ESTIMATOR

Let m_0 be the position of the synchronization sequence symbol within a frame. The number of guard symbols left unused around the position m_0 is given by N_g, as depicted in Fig. 1. An OQAM-OFDM symbol that exhibits the conjugate symmetry property at the output s of the modulation filter bank can be constructed if the following condition on the symbols $d_{m_0,k}$ is fulfilled.

$$d_{m_0,k} = \begin{cases} \sqrt{2} b_k & \text{k even} \\ 0 & \text{k odd} \end{cases} \quad (5)$$

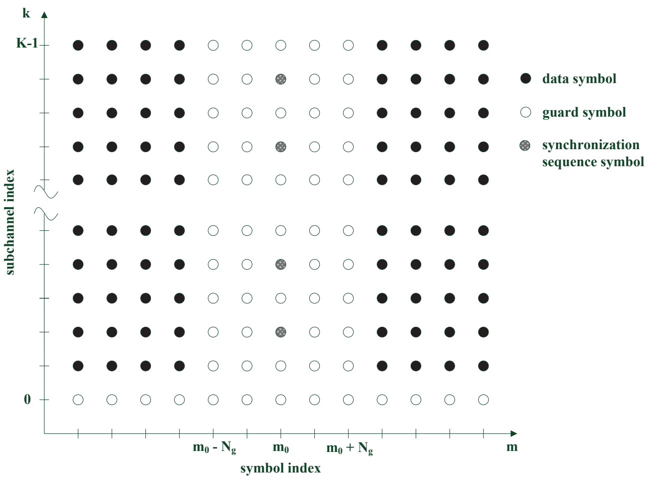

Fig. 1. Frame section containing real-valued data symbols, unused symbols and the synchronization sequence symbols.

with $b_k \in \{-1, 1\}$ and $k \in \mathbb{K}_d$. The sum of the energy of the synchronization sequence is normalized to the sum of the energy of the payload symbols. Based on s the symbol-timing can be recovered as proposed in [3]. The estimation of the CFO requires the conjugate symmetry property to become a real symmetry property within the signal s such that for $N_g = \alpha$ and $n \in \{1, 2, \cdots, K/2 - 1\}$ the equality

$$s[(m_0 + \alpha)\frac{K}{2} + n] = s[(m_0 + \alpha)\frac{K}{2} - n] \quad (6)$$

holds. This is achieved if b_k fulfills the additional symmetry condition

$$b_k = b_{K-k} \quad (7)$$

for $k \in \{1, 2, \cdots, K/2 - 1\}$. For the considered prototype filter function, which is specified in section 4, the evaluation range of $\pm(K/2 - 1)$ around the center of the synchronization pulse contains more than 90% of the energy of the synchronization signal. Therefore, this is considered an appropriate range for CFO estimation with respect to the objective of minimization of the overhead for synchronization within a frame. The proposed CFO estimation scheme assumes that the utilized prototype filter itself is real and symmetric as it is one prerequisite of the considered OQAM-OFDM transmission scheme. Let the received signal be already synchronized in time with respect to the reference signal's position m_0. The signal r can then be split in

$$x_+[n] = r[(m_0 + \alpha)\frac{K}{2} + n] \quad (8)$$
$$\text{and } x_-[n] = r[(m_0 + \alpha)\frac{K}{2} - n].$$

The CFO estimation algorithm for $\hat{f}_\Delta$ can be directly obtained through calculation of the phase differences between these signal parts according to

$$\hat{f}_\Delta = \sum_{n=1}^{K/2-1} w_n \hat{\zeta}[n] \quad (9)$$

with the individual CFO estimate per sample

$$\hat{\zeta}[n] = \frac{K}{4\pi n} \angle \left(\frac{x_+[n]}{x_-[n]} \right). \quad (10)$$

Thereby, the multiplication of each CFO estimate $\hat{\zeta}[n]$ with the weight

$$w_n = \frac{n}{\sum_{i=1}^{K/2-1} i} \quad (11)$$

in combination with taking the sum of these products define a linear weighted average calculation. It is to note here that the distance between samples for each individual phase estimate is varying according to the position of the samples with respect to the symmetric center. The weighted average calculation accounts for the circumstance that estimation values for the phase offset obtained from nearby reference values are more prone to the influence from noise than values further spaced apart. The acquisition range of the proposed estimator is within half a subchannel spacing.

4. NUMERICAL RESULTS

The numerical results are obtained using the assumptions listed below. Thereby, the root mean square (RMS) of the error $\varepsilon = \hat{f}_\Delta - f_\Delta$ is taken as the metric for the following evaluation for each of the 10^4 runs of the Monte-Carlo simulation.

- The number of subchannels K is 512, except for the evaluation presented in Fig. 3. $K_d = 300$ out of K subchannels are used for data transmission and the rest is left unused. The ratio between K_d and K is similar for all values of K in Fig. 3. The sampling frequency is $f_s = 7.68$MHz. The prototype filter is derived with the frequency sampling technique as described in [7] using $\alpha = 4$ and a filter design parameter equal to 0.97195983. The number of guard symbols on each side of the reference symbol is $N_g = 4$, except for the results shown in Fig. 4 where N_g is varying.
- Besides the AWGN channel, the proposed estimator is evaluated for three additional channel types (A,B,C) which are derived from an exponential decaying power delay profile (PDP) according to $h[n] = e^{-\beta n}$. β and the RMS delay spread are provided in Table 1.
- The symbol timing is assumed to be known for the results shown here but can be recovered through the symbol timing estimation proposed in [3]. For each run the CFO f_Δ is drawn out of a uniform distribution in the range of $\{-0.25, 0.25\}$ of the subchannel spacing.

Table 1. Attenuation factor β and the resulting RMS delay spread (δ_{RMS}) for each channel type.

Channel A	$\beta = 3$	$\delta_{RMS} = 31ns$
Channel B	$\beta = 2$	$\delta_{RMS} = 55ns$
Channel C	$\beta = 1$	$\delta_{RMS} = 125ns$

The numerical evaluation of the proposed algorithm in terms of the RMS error as a function of the signal-to-noise ratio (SNR) is provided in Fig. 2 for the self-interference free case with $N_g = 4$. The degradation of the algorithm's performance due to multi-path propagation results in an error floor which is present even for a channel with a short delay spread around $\delta_{RMS} = 31ns$ and rises for channels that

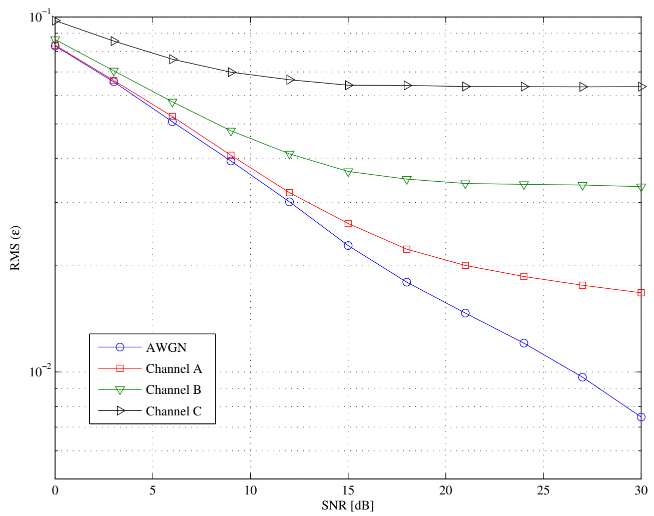

Fig. 2. RMS of CFO estimation error for the AWGN channel and channel type A, B and C depending on the SNR.

incorporate longer delay spreads. From the outcome of the investigation in Fig. 2 it can be concluded that the proposed algorithm is mainly suitable for environments where multi-path propagation is rare. Thereby, the effect of the channel impulse response distorts the sample pairs differently and leads to an increase of the estimation errors. The robustness against the effects of noise and multi-path components rises only marginally as the number of subchannels increase, as depicted in Fig. 3 and the influence on the proposed algorithm can be neglected. In a practical application, the vulnerability to the influences of multi-path propagation dominates the per-

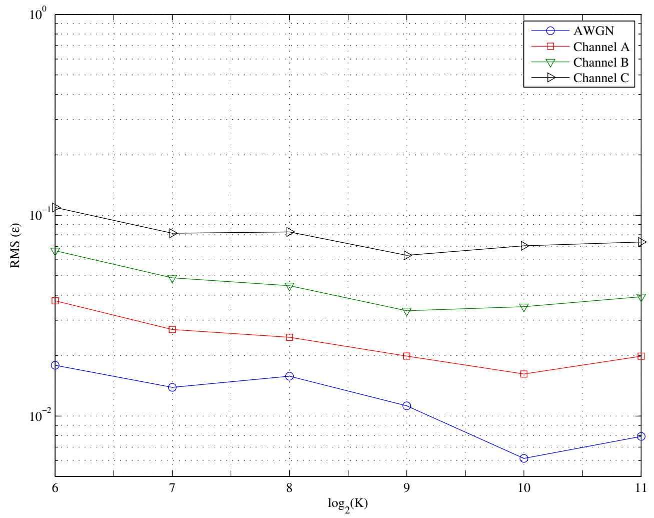

Fig. 3. RMS of CFO estimation error for the AWGN channel and channel type A, B and C depending on the number of subchannels K. The SNR is fixed to 25dB.

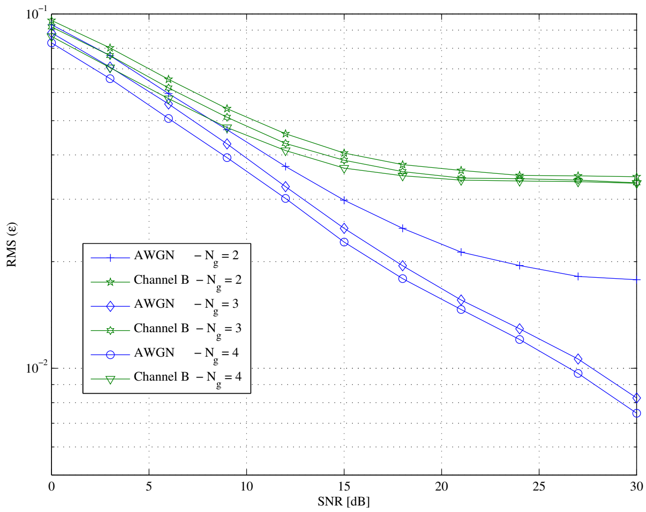

Fig. 4. RMS of CFO estimation error for the AWGN channel and channel type B depending on the SNR and the self-interference rate.

formance. Therefore, the influence due to self-interference introduced by the removal of guard symbols can be nearly neglected in a frequency-selective channel, which is illustrated in Fig. 4 for various numbers of guard symbols. The robustness against a reduction of the number of guard symbols is due to the aforementioned property of the prototype filter function that more than 90% of the energy is concentrated within the K samples around the center of the pulse. In Fig. 5, the high susceptibility of the algorithm to a timing offset can be observed for different timing offsets at an SNR of 25dB.

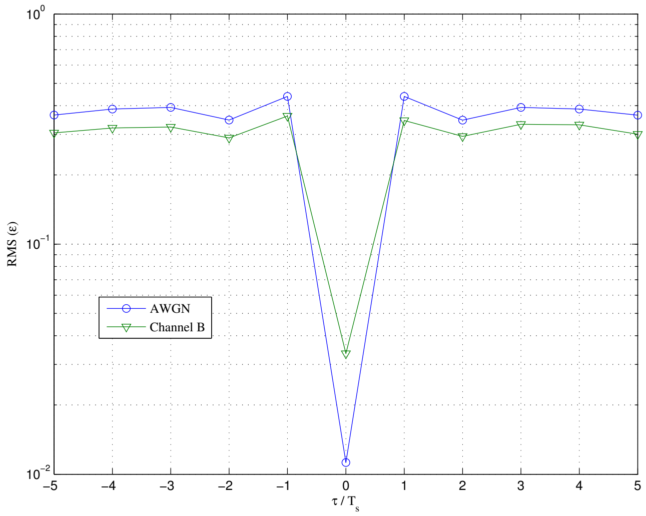

Fig. 5. RMS of CFO estimation error for the AWGN channel and channel type B depending on the timing offset.

5. CONCLUSION

We proposed a CFO estimation algorithm for OQAM-OFDM systems based on the conjugate symmetry property of a well-designed synchronization sequence. The CFO estimator shows a sufficient performance for an initial coarse frequency offset correction in the time-domain in case of low delay-spread multi-path propagation channels. In case of a more severe multi-path environment or symbol timing errors, the estimation performance decreases significantly due to the structure of the reference sequence. Additionally, the overhead within a frame imposed by the synchronization sequence can be reduced by allowing a certain amount of self-interference without harming the CFO estimation significantly.

6. REFERENCES

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