BLOCK-BASED DECISION-FEEDBACK EQUALIZERS WITH REDUCED REDUNDANCY

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ABSTRACT

In recent years many works related to the design of block-based transceivers have been published. The main target of this research activity is to optimize the use of the spectral resources in broadband transmissions. A possible way to address this problem is to reduce the amount of redundancy required by block transmissions to avoid interblock interference. An efficient solution is to employ zero-padding zero-jamming (ZP-ZJ) transceivers, which allow the transmission with reduced redundancy. ZP-ZJ systems have been successfully employed in the context of linear transceivers. This paper shows how the ZP-ZJ concept can be applied in decision-feedback equalization. Some performance analyses based on the resulting mean-square error and error probability of symbols are included to show the possible degrading effects of the reduction in the amount of redundancy. Nevertheless, simulation results illustrate that data throughput and average mutual information between transmitted and estimated symbols can be enhanced significantly without affecting the system performance, for a certain level of signal-to-noise ratio at the receiver.

Index Terms— Block DFE, reduced redundancy, throughput.

1. INTRODUCTION

Equalization plays an important role in any modern digital transmission scheme. Linear equalizers are still the preferred choice in practical systems due to their computational simplicity. However, the constant performance improvements of digital processors have enabled the use of nonlinear equalizers as well. The nonlinearities induce certain degrees of freedom which are not exploited in linear equalization. Among the nonlinear receivers, *decision-feedback equalizers (DFE)* [1, 2, 3, 4] are the most popular since they feature good tradeoff between performance improvements and computational complexity. Indeed, the computational burden related to DFE systems does not increase too much since the nonlinearity is introduced through a simple hard-decision detection which takes place before feeding back the symbols in the equalization process.

In modern communications, it is common practice the segmentation of the overall data string into smaller blocks that are transmitted separately in the so-called *block-based transmission*. Such separation in blocks is rather useful in block-based DFEs, since any symbol error within a given data block is not propagated across different blocks, thus limiting the harmful effects commonly caused by feedback-based equalization. Nonetheless, the undesired superposition of signals inherent to broadband communications generates *interblock interference (IBI)* between adjacent transmitted data blocks. IBI is a degrading effect present in block transmissions that can be eliminated by transmitting redundant signals, such as zero-padded or cyclic-prefixed signals [4, 5].

Zero-padding (ZP) is a very efficient way of adding guard intervals between data blocks in order to mitigate IBI [4, 6]. For multipath channels modeled as finite impulse-response (FIR) filters with order L, traditional approaches include at least L zeros as prefix or suffix of each transmitted data block (see, e.g., [4] and references therein). The inclusion of such redundant signals obviously reduces the spectral efficiency associated with the related transceivers. Lin and Phoong [7] have proposed a method to diminish this waste of bandwidth, namely, *zero-padding zero-jamming (ZP-ZJ)* transceivers [7, 8]. Such systems allow one to transmit a reduced amount of redundant zeros, ranging from the minimum, $\lceil L/2 \rceil$, to the most commonly used value L [7]. However, just few works have taken into account this feature and *all of them consider only linear equalizers* (see, e.g., [8, 9, 10, 11]).

This work shows that ZP-ZJ techniques can also be successfully applied in the context of DFE systems. The paper describes how to apply *known* minimum mean-square error (MMSE) solutions with zero-forcing (ZF) constraints to design block-based DFEs within the framework of reduced-redundancy systems. The paper also includes some mathematical results describing the monotone behavior of several figures of merit related to the proposed *ZP-ZJ DFE* systems (such as MSE of symbols and error probability of symbols) The proposed analyses indicate that the reduction in the amount of redundancy leads to loss in performance of these figures of merit, not including throughput. In fact, *throughput may increase by reducing the amount of redundant signals*, as will be clearer in the simulation results.

1.1. Organization

This paper is organized as follows: Section 2 contains the description of the proposed block-based DFE with reduced redundancy (ZP-ZJ DFE). In Section 3 we state some mathematical results which describe formally the monotone behavior of several figures of merit associated with the proposed DFE. Simulation results are in Section 4, whereas the concluding remarks are in Section 5.

1.2. Notation

Given a real number x, $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. The notations $\mathsf{E}[\cdot]$ and $[\cdot]^H$ stand for expected value and Hermitian transpose operations on $[\cdot]$, respectively. The set $\mathbb{C}^{M \times N}$ denotes all $M \times N$ matrices comprised of complex-valued entries, whereas $\mathbb{C}^{M \times N}[x]$ denotes all polynomials in the variable x with $M \times N$ complex-valued matrices as coefficients.

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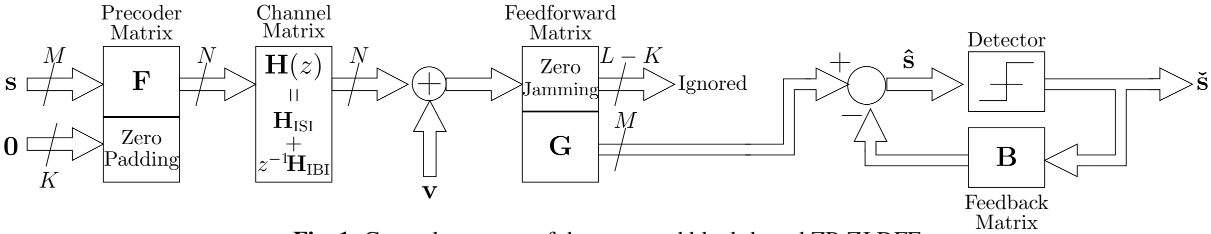

Fig. 1. General structure of the proposed block-based ZP-ZJ DFE.

2. DFE WITH REDUCED REDUNDANCY

Assume that we want to transmit a vector $\mathbf{s} \in \mathcal{C}^{M\times 1} \subset \mathbb{C}^{M\times 1}$, with $M \in \mathbb{N}$ symbols drawn from a given constellation $\mathcal{C}$, through an FIR channel whose transfer function is

$$H(z) \triangleq h(0) + h(1)z^{-1} + \cdots + h(L)z^{-L}, \tag{1}$$

with $h(l) \in \mathbb{C}$, for each $l \in \{0, 1, \cdots, L\} \subset \mathbb{N}$. It is possible to show that the matrix representation of such block-transmission scheme is given as [4, 7]

$$\mathbf{H}(z) \triangleq \mathbf{H}_{\text{ISI}} + z^{-1}\mathbf{H}_{\text{IBI}} \in \mathbb{C}^{N\times N}[z^{-1}], \tag{2}$$

in which $\mathbb{N} \ni N \geq \max\{M, L\}$ is the number of transmitted elements in a block, while $\mathbf{H}_{\text{ISI}}$ and $\mathbf{H}_{\text{IBI}}$ are Toeplitz matrices.

The first row of $\mathbf{H}_{\text{ISI}}$ is $[h(0)\ \mathbf{0}^T_{(N-1)\times 1}]$, whereas the first column is $[h(0)\ h(1)\ \cdots\ h(L)\ \mathbf{0}^T_{(N-L-1)\times 1}]^T$. In matrix $\mathbf{H}_{\text{IBI}}$, the first row is $[\mathbf{0}^T_{(N-L)\times 1}\ h(L)\ h(L-1)\ \cdots\ h(1)]$, whilst the first column is $\mathbf{0}_{N\times 1}$.

In order to eliminate the IBI effect modeled by matrix $\mathbf{H}_{\text{IBI}}$, one can append $K \triangleq N - M$ zeros to the transformed vector $\mathbf{Fs}$ at the transmitter end, in which $\mathbf{F} \in \mathbb{C}^{M\times M}$ is a *precoder matrix*. The received vector of size N will still suffer from IBI effects in its first $L - K$ elements. The receiver thus ignores these first $L - K$ signals, working only with the remaining $N - (L - K) = (M + K) - (L - K) = M + 2K - L$ elements. These elements are first transformed into M signals by the *feedforward matrix* $\mathbf{G} \in \mathbb{C}^{M\times(M+2K-L)}$, as depicted in Fig. 1. The process of adding zeros at the transmitter end, and discarding elements at the receiver end is denominated zero-padding zero-jamming (ZP-ZJ) [8].

The authors in [7] show that, if one assumes that matrices $\mathbf{F}$ and $\mathbf{G}$ are full-rank, the zero-forcing solution to ZP-ZJ transceivers can be achieved, i.e.,

$$[\mathbf{0}_{M\times(L-K)}\ \mathbf{G}]\mathbf{H}(z)[\mathbf{F}^T\ \mathbf{0}_{M\times K}]^T = \mathbf{I}_M, \tag{3}$$

as long as the number of redundant elements K satisfies the inequality $2K \geq L$. From now on, we shall assume that $K \in \{\lceil L/2\rceil, \lceil L/2\rceil + 1, \cdots, L\} \subset \mathbb{N}$.

As illustrated in Fig. 1, after the multiplication by the feedforward matrix, the received vector passes through a usual decision-feedback processing [1, 2, 3, 4]. In this figure, $\check{\mathbf{s}} \in \mathcal{C}^{M\times 1}$ denotes the vector containing the detected symbols and $\mathbf{B} \in \mathbb{C}^{M\times M}$ is the *feedback matrix*. This matrix is chosen *strictly upper triangular*, so that the symbol estimation within a data block is sequentially performed, guaranteeing the causality of the process [4].

The ZP-ZJ structure of the DFE proposed in Fig. 1 can be simplified if one incorporates the ZP-ZJ processing into the channel model, yielding an *effective channel matrix* $\mathbf{H}$,[1] which is Toeplitz and has dimension $(M + 2K - L) \times M$. In this case, the first row of $\mathbf{H}$ is $[h(L-K)\ h(L-K-1)\ \cdots\ h(0)\ \mathbf{0}^T_{(M+K-L-1)\times 1}]$, whereas the first column is $[h(L-K)\ h(L-K+1)\ \cdots\ h(L)\ \mathbf{0}^T_{(M+K-L-1)\times 1}]^T$. The equivalent transceiver structure is depicted in Fig. 2.

Under the common simplifying assumption of *perfect decisions* [4], one has $\check{\mathbf{s}} = \mathbf{s}$, yielding $\hat{\mathbf{s}} = (\mathbf{GHF} - \mathbf{B})\mathbf{s} + \mathbf{G}\bar{\mathbf{v}}$ (see Fig. 2). Hence, the overall MSE of symbols, $\mathcal{E}$, is given as [4]

$$\mathcal{E} \triangleq \mathsf{E}\{\|\hat{\boldsymbol{s}} - \boldsymbol{s}\|_2^2\} = \sigma_s^2\|\mathbf{GHF} - \mathbf{B} - \mathbf{I}_M\|_{\text{F}}^2 + \sigma_v^2\|\mathbf{G}\|_{\text{F}}^2, \tag{4}$$

where $\|\cdot\|_2$ stands for the standard norm-2 of a vector, whereas $\|\cdot\|_{\text{F}}$ denotes the standard Frobenius norm of a matrix. In addition, we have assumed that the transmitted vector $\mathbf{s}$ and the channel-noise vector $\bar{\mathbf{v}}$ are respectively drawn from zero-mean jointly wide-sense stationary (WSS) random sequences $\boldsymbol{s}$ and $\bar{\boldsymbol{v}}$.[2] Moreover, we have assumed that $\boldsymbol{s}$ and $\bar{\boldsymbol{v}}$ are uncorrelated, i.e., $\mathsf{E}\{\boldsymbol{s}\bar{\boldsymbol{v}}^H\} = \mathsf{E}\{\boldsymbol{s}\}\mathsf{E}\{\bar{\boldsymbol{v}}\}^H = \mathbf{0}_{M\times(M+2K-L)}$, and that $\sigma_s^2, \sigma_v^2 \in \mathbb{R}_+$.

Now, the design of matrices $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{B}$ can be formulated as an MSE-based optimization problem, as follows [4]:

$$\min_{\mathbf{F},\mathbf{G},\mathbf{B}} \left\{\sigma_s^2\|\mathbf{GHF} - \mathbf{B} - \mathbf{I}_M\|_{\text{F}}^2 + \sigma_v^2\|\mathbf{G}\|_{\text{F}}^2\right\}, \tag{5}$$

subject to:

$$(\mathbf{GHF} - \mathbf{B} - \mathbf{I}_M) = 0, \tag{6}$$

$$\|\mathbf{F}\|_{\text{F}}^2 = M, \tag{7}$$

$$[\mathbf{B}]_{mn} = 0,\ \forall m \geq n, \tag{8}$$

where, in order to simplify the forthcoming mathematical descriptions, we focus only on MMSE solutions that meet the ZF constraint in (6). In addition, the constraint in (7) implies that the average transmitted power is not modified by the precoder matrix $\mathbf{F}$, whereas the constraint in (8) means that the feedback matrix is strictly upper triangular.

The equivalent structure of the proposed block-based ZP-ZJ DFE illustrated in Fig. 2 matches the general block-based DFE model described, for instance, in [4]. Therefore, the solutions to the above optimization problem are already known and can be described as [4] (p. 816):

$$\mathbf{F} = \mathbf{V}_{\mathbf{H}}\mathbf{S}, \tag{9}$$

$$\mathbf{G} = \mathbf{R}\mathbf{S}^H\boldsymbol{\Sigma}_{\mathbf{H}}^{-1}[\mathbf{I}_M\ \ \mathbf{0}_{M\times(2K-L)}]\mathbf{U}_{\mathbf{H}}^H, \tag{10}$$

$$\mathbf{B} = \mathbf{R} - \mathbf{I}_M, \tag{11}$$

in which the above matrices come from the SVD decomposition of $\mathbf{H}$ and the QRS decomposition [4] (pp. 646–652) of $\boldsymbol{\Sigma}_{\mathbf{H}}$, as follows:

$$\mathbf{H} = \underbrace{\mathbf{U}_{\mathbf{H}}}_{(M+2K-L)\times(M+2K-L)} \underbrace{\begin{bmatrix}\boldsymbol{\Sigma}_{\mathbf{H}} \\ \mathbf{0}_{(2K-L)\times M}\end{bmatrix}}_{(M+2K-L)\times M} \underbrace{\mathbf{V}_{\mathbf{H}}^H}_{M\times M}, \tag{12}$$

$$\boldsymbol{\Sigma}_{\mathbf{H}} = \underbrace{\sqrt[M]{\prod_{m=0}^{M-1}\sigma_m}}_{\triangleq\alpha}\,\mathbf{QRS}^H = \alpha\mathbf{QRS}^H, \tag{13}$$

[1] Sometimes, we shall denote $\mathbf{H}$ as $\mathbf{H}(K)$ in order to emphasize that the related effective channel matrix is built considering the transmission of K redundant zeros.

[2] The time index was omitted for the sake of simplicity.

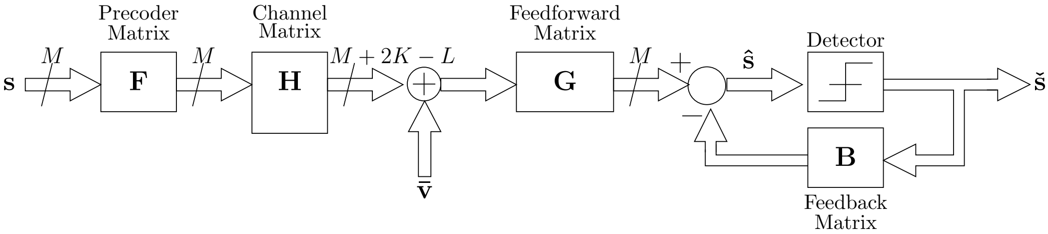

Fig. 2. Equivalent structure of the proposed block-based ZP-ZJ DFE.

where $\mathbf{\Sigma_H} = \mathbf{\Sigma_H}^H > O$ is an $M \times M$ diagonal matrix containing the M nonzero singular values of $\mathbf{H}$. The mth diagonal element of $\mathbf{\Sigma_H}$ is denoted as σ_m. In addition, $\mathbf{Q}$ and $\mathbf{S}$ are $M \times M$ unitary matrices, whereas $\mathbf{R}$ is an $M \times M$ upper triangular matrix containing only 1s in its main diagonal. Note that $\mathbf{Q}^H \mathbf{\Sigma_H} \mathbf{S} = \alpha \mathbf{R}$, which means that, in the special case of a diagonal matrix $\mathbf{\Sigma_H} > O$, the QRS decomposition is closely connected with the SVD decomposition of a upper triangular matrix $\alpha \mathbf{R}$ whose diagonal elements are constant and equal to α. See [4, 12] and references therein for further detailed information on QRS decompositions.

It is worth mentioning that other optimal solutions[3] can be derived for ZP-ZJ DFE systems whose equivalent building-block description is given in Fig. 2.

3. PERFORMANCE ANALYSIS

Several physical-layer figures of merit related to the proposed ZP-ZJ DFE have close connections with the singular values of the effective Toeplitz channel matrix $\mathbf{H}$. Lemma 1, which is borrowed from [13], characterizes the monotone behavior of the singular values of $\mathbf{H}$ with respect to the number of transmitted redundant elements, K.

Lemma 1. *Given two fixed natural numbers L and M, let us assume that each effective channel matrix $\mathbf{H}(K) \in \mathbb{C}^{(M+2K-L)\times M}$ is constructed from the same Lth-order channel-impulse response, with $K \in \{\lceil L/2 \rceil, \lceil L/2 \rceil + 1, \cdots, L\}$. Then*

$$\sigma_m(K+1) \geq \sigma_m(K), \tag{14}$$

where each $\sigma_m(K) \in \mathbb{R}_+$ is a singular value of $\mathbf{H}(K)$.

By using Lemma 1, we can derive a very general result (Theorem 1) which encompasses as particular cases the majority of the popular figures of merit of practical interest (e.g., MSE of symbols, mutual information, and error probability of symbols).

Theorem 1. *Let us assume that, for each $m \in \{0, 1, \cdots, M-1\}$, there exists a function $f_m : \mathbb{R}_+ \to \mathbb{R}$ such that a performance quantifier $\mathcal{J} : \{\lceil L/2 \rceil, \lceil L/2 \rceil + 1, \cdots, L\} \to \mathbb{R}$ associated with the proposed ZP-ZJ DFE transceiver can be defined as*

$$\mathcal{J}(K) \triangleq \frac{1}{M} \sum_{m=0}^{M-1} f_m(\sigma_m(K)) \ \text{ or } \ \mathcal{J}(K) \triangleq \sqrt[M]{\prod_{m=0}^{M-1} f_m(\sigma_m(K))}. \tag{15}$$

If f_m is monotone increasing for all m, then $\mathcal{J}(K+1) \geq \mathcal{J}(K)$, for all K. Likewise, if f_m is monotone decreasing for all m, then $\mathcal{J}(K+1) \leq \mathcal{J}(K)$, for all K.

Proof. The result follows from the application of Lemma 1 along with the hypotheses of the theorem. □

[3]For instance, MMSE-based solutions with channel-independent unitary precoder or Pure MMSE-based solutions [4].

Since the resulting MSE of symbols, $\mathcal{E}(K)$ (see (4)), the total mutual information between transmitted and estimated symbols in a block, $\mathcal{I}(K)$, and the *total* error probability of symbols in a block, $\mathcal{P}(K)$, are respectively given by [4]:

$$\mathcal{E}(K) = M\sigma_v^2 \sqrt[M]{\prod_{m=0}^{M-1} \frac{1}{\sigma_m^2(K)}}, \tag{16}$$

$$\mathcal{I}(K) = M \ln \left(1 + \frac{\sigma_s^2}{\sigma_v^2} \sqrt[M]{\prod_{m=0}^{M-1} \sigma_m^2(K)} \right), \tag{17}$$

$$\mathcal{P}(K) = cM\mathcal{Q} \left(\frac{A}{\sigma_v} \sqrt[M]{\prod_{m=0}^{M-1} \sigma_m(K)} \right), \tag{18}$$

in which c and A are positive real scalars that depend on the particular digital constellation $\mathcal{C}$, whereas $\mathcal{Q}(\cdot)$ is a decreasing function of its argument, being defined as

$$\mathbb{R} \ni x \mapsto \mathcal{Q}(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \mathrm{e}^{-w^2/2} \mathrm{d}w, \tag{19}$$

then, the following corollary from Theorem 1 holds.

Corollary 1. *Given the definitions in Lemma 1, we have*

$$\mathcal{E}(K+1) \leq \mathcal{E}(K), \ \ \mathcal{I}(K+1) \geq \mathcal{I}(K), \ \ \mathcal{P}(K+1) \leq \mathcal{P}(K), \tag{20}$$

with $K \in \{\lceil L/2 \rceil, \lceil L/2 \rceil + 1, \cdots, L-1\}$.

Proof. The inequalities come from the application of Theorem 1, along with the fact that $\mathcal{E}(K)$ is monotone decreasing, $\mathcal{I}(K)$ is monotone increasing, and $\mathcal{P}(K)$ is monotone decreasing with respect to each singular value $\sigma_m(K)$. □

Even though the results from Theorem 1 and Corollary 1 seem rather reasonable, no previous work has given formal proofs for their validity.

As a by product of Corollary 1, we have the following result: let $\overline{\mathcal{E}}(K) \triangleq \frac{\mathcal{E}(K)}{M+K}$ be the average MSE of symbols and let $\overline{\mathcal{P}}(K) \triangleq \frac{\mathcal{P}(K)}{M+K}$ be the average error probability of symbols, where the averaging process is taken with respect to the number of transmitted signals within a block, $M+K$. Thus, it follows from Corollary 1 that $\overline{\mathcal{E}}(K)$ and $\overline{\mathcal{P}}(K)$ feature the same monotone behavior of $\mathcal{E}(K)$ and $\mathcal{P}(K)$, i.e., $\overline{\mathcal{E}}(K+1) \leq \overline{\mathcal{E}}(K)$ and $\overline{\mathcal{P}}(K+1) \leq \overline{\mathcal{P}}(K)$.

The previous results may lead us to a *wrong* conclusion that it is not worth reducing the number of transmitted redundant elements. Nevertheless, if we define the average mutual information between transmitted and estimated symbols within a block as $\overline{\mathcal{I}}(K) \triangleq \frac{\mathcal{I}(K)}{M+K}$, then we *cannot* say that $\overline{\mathcal{I}}(K)$ is a monotone function of K. This

Table 1. Average mutual information (in nats).

Channel	A		B		C	
SNR [dB]	10	20	10	20	10	20
$K = L$	1.52	2.99	$\mathbf{1.34}$	2.69	$\mathbf{1.20}$	$\mathbf{2.53}$
$K = L - 1$	$\mathbf{1.64}$	$\mathbf{3.24}$	1.32	2.77	1.03	2.42
$K = L - 2$	1.08	2.70	1.29	$\mathbf{2.85}$	0.83	2.26

means that the mutual statistical dependence between transmitted and estimated symbols may increase, decrease, or even keep the same value in the average. Indeed, some numerical examples (see Table 1) in Section 4 show that, for some channels and signal-to-noise ratios (SNRs), it is worth reducing the amount of redundant elements from an average mutual-information viewpoint. Note that this discussion only makes sense when K is not much smaller than M, otherwise $M + K \approx M$ and $\overline{\mathcal{I}}(K)$ would be almost constant. But this is not a drawback since the proposed ZP-ZJ systems are specially suitable for channels whose order L is large, as compared to M, so that the cost of sending redundant data is not negligible, thus calling for reduced-redundancy solutions.

If on one hand we need to use as much redundancy as possible in order to achieve lower probability of error or MSE of symbols (as described in Corollary 1), on the other hand we must reduce the transmitted redundancy to save bandwidth, which is paramount in high data-rate systems, and *maybe* to increase the average mutual information between transmitted and estimated symbols. In fact, in order to take both effects (performance and bandwidth usage) into account, one should consider *throughput* as figure of merit.

We shall assume that

$$\text{Throughput} \triangleq b r_c \frac{M}{M+K}(1 - \text{BLER}) f_s \text{ bps}, \qquad (21)$$

in which b denotes the number of bits required to represent one constellation symbol, r_c denotes the code rate assuming the protection of channel coding, K denotes the amount of redundancy, f_s denotes the sampling frequency, and BLER stands for block-error rate [14].

As one can see, throughput is also a function of the bit-error protection that is implemented at higher layers of a given communication protocol, entailing a sort of cross-layer design. Section 4 shows some setups where the proposed reduced-redundancy DFE outperforms the traditional *ZP DFE*[4] with respect to the throughput performance.

4. SIMULATION RESULTS

The aim of this section is to assess the performance of the proposed DFE with reduced redundancy through numerical examples. We consider the transmission of $50,000$ data blocks containing $M = 8$ 16-QAM symbols, which means that $b = 4$ (see (21)). In order to generate each data block, we produce 16 random bits that, after passing through a convolutional channel-coding process with code rate $r_c = 1/2$, are transformed into 32 bits, which are mapped into 8 16-QAM symbols. The channel coding has constraint length 7 and octal generators $\mathbf{g_0} \triangleq [\mathbf{133}]$ and $\mathbf{g_1} \triangleq [\mathbf{165}]$ [9, 10]. We assume that the sampling frequency is $f_s = 400$ MHz. In order to compute the BLER, we assume that a data block is discarded when at least one of the original bits is incorrectly decoded at the receiver end.

We consider the following channel models [13]: *Channel A*, whose transfer function is $0.1659 + 0.3045z^{-1} - 0.1159z^{-2} - 0.0733z^{-3} - 0.0015z^{-4}$; *Channel B*, whose zeros are 0.999, -0.999, 0.7j, -0.7j, and -0.4j; and *Channel C*, whose zeros are 0.8, -0.8, 0.5j, -0.5j, and -0.8j. In the case of Channel A, the number of redundant elements is such that $K \in \{2, 3, 4\}$, whereas for Channels B and C we have $K \in \{3, 4, 5\}$.

Table 1 contains the average mutual information between transmitted and estimated symbols, $\overline{\mathcal{I}}(K)$, for Channels A, B, and C, and for two distinct SNR values, namely 10 dB and 20 dB. The boldface numbers illustrate the best values, thus showing that $\overline{\mathcal{I}}(K)$ is not a monotone function of K. Indeed, the best values of $\overline{\mathcal{I}}(K)$ depend on both the channel model and noise level.

Fig. 3 depicts the obtained results. Figs. 3(a), (b), and (c) contain the *uncoded* bit-error rate (BER) results, i.e., the BER computed *before* the channel-decoding process. In addition, Figs. 3(d), (e), and (f) contain the throughput results. There are four curves in these figures which describe the performance of the following systems: (i) ZP-ZJ DFE with $K = L - 2$ (minimum-redundancy), (ii) ZP-ZJ DFE with $K = L - 1$ (reduced-redundancy), (iii) ZP DFE with $K = L$ (full-redundancy), and (iv) ZP DFE with $K = L$ (full-redundancy) with no error propagation, in which *the exact symbols are fed back*. This last system will be used as a benchmark for our comparisons.

By observing Fig. 3(a), one can verify that the uncoded BER of both reduced- and full-redundancy systems are quite close to the benchmark transceiver for transmissions through Channel A. Only the minimum-redundancy system does not perform well in this particular case. Such uncoded-BER performances are reflected in the good throughput results obtained by the reduced-redundancy transceiver in Fig. 3(d). Indeed, the ZP-ZJ DFE with $K = L - 1 = 3$ can outperform the benchmark transceiver in up to 50 Mbps, whereas for SNRs larger than 22 dB, the ZP-ZJ DFE with $K = L - 2 = 2$ can outperform the benchmark transceiver in up to 100 Mbps.

For Channel B, Fig. 3(b) shows that minimum- and reduced-redundancy transceivers have closer uncoded-BER performances, but both of them are not as close to the benchmark transceiver as in the previous case of Channel A. By observing Fig. 3(e), we can verify that the throughput performance of the traditional ZP DFE is better than the proposed ZP-ZJ DFE with minimum and reduced redundancies for SNRs smaller than 15 dB. Nevertheless, for typical SNR values around 20 dB, the gain from using the proposed transceivers is remarkable, outperforming the benchmark system in almost 100 Mbps (minimum-redundancy transceiver).

For Channel C, the error propagation is critical since the already known ZP DFE without error propagation achieves much higher throughputs than the other transceivers for SNRs smaller than 16 dB, as depicted in Figs 3(c) and (f). In this low SNR range, the proposed DFEs do not perform as good as the traditional full-redundancy DFE ($K = 5$). On the other hand, for SNRs larger than 20 dB (commonly found in practical systems), the proposed reduced-redundancy DFE ($K = 4$) can outperform the benchmark transceiver in up to 40 Mbps, whereas the proposed minimum-redundancy DFE ($K = 3$) can outperform the benchmark transceiver in up to 85 Mbps.

5. CONCLUDING REMARKS

In this work we proposed block-based ZP-ZJ transceivers with decision-feedback equalization. These transceivers allowed the tradeoff between transmission-error performance and data throughput, enabling the optimization of the spectral resources in broadband transmissions. This was possible by choosing the amount of redundancy ranging from the minimum to the channel order, which is usually employed. Some tools to analyze the transceivers were proposed based on the resulting MSE of symbols, mutual information between transmitted and estimated symbols, and error probability of symbols.

[4] In this work, the traditional ZP DFE can be seen as a particular type of the proposed ZP-ZJ DFE for $K = L$ (full-redundancy). Hence, the proposed ZP-ZJ DFE extends the standard ZP-based DFE systems [4].

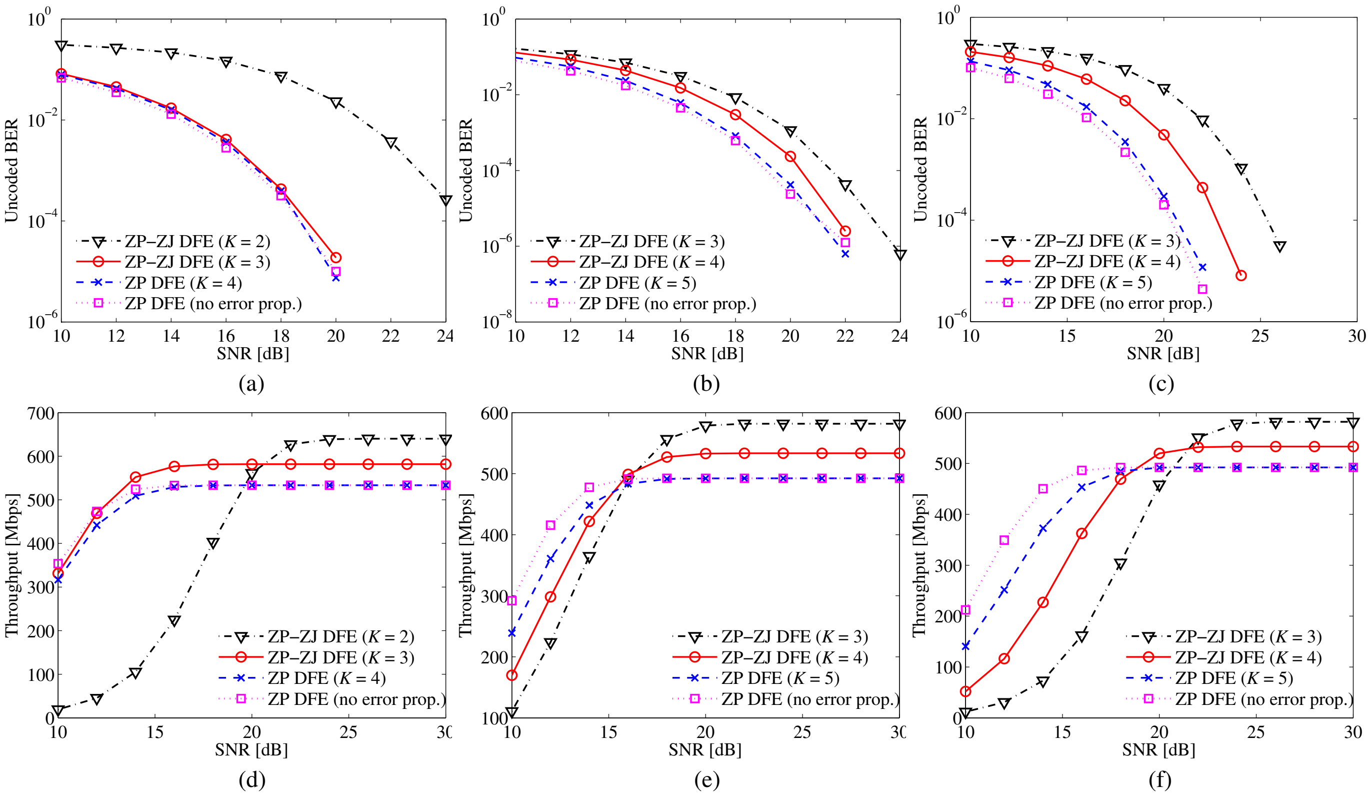

Fig. 3. Uncoded BER × SNR [dB] for (a), (b), and (c); Throughput [Mbps] × SNR [dB] for (d), (e), and (f). Figures (a) and (d) are associated with Channel A, while (b) and (e) are associated with Channel B, and (c) and (f) are associated with Channel C.

The main conclusion from this work is that, for ZP-ZJ-based DFE transceivers, it is possible to increase the data throughput for a certain level of SNR at the receiver, without affecting the system performance, as confirmed by the simulation results. These are preliminary results from investigations that are in progress. An interesting future research direction is the development of efficient algorithms to implement the proposed optimal solutions.

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