EXPLOITING STRUCTURE OF SPATIO-TEMPORAL CORRELATION FOR DETECTION IN WIRELESS SENSOR NETWORKS

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ABSTRACT

In dense Wireless Sensor Networks (WSN) consecutive measurements obtained by sensors are spatio-temporally correlated in applications that involve the observation of the variation of a physical phenomenon. To exploit this spatio-temporal structure for event detection, the the traditional GLRT test degenerates in the case where dimensionality of data is equal to the sample size or larger. It is because the spatio-temporal sample covariance matrix becomes ill-conditioned or near singular. To circumvent this problem, we modify the traditional GLRT detector by splitting the large spatio-temporal covariance matrix into spatial and temporal covariance matrices. In addition, several detectors are proposed that are robust in the case of high dimensionality and small sample size. Numerical results are drawn, which show that the proposed detection schemes indeed out perform the traditional approaches when the dimension of data is larger than the sample size.

***Index Terms*—** Spatio-Temporal Correlation, Kronecker Structure, GLRT, Wireless Sensor Network.

1. INTRODUCTION

In wireless senor networks (WSNs), dense deployment of sensor nodes makes the sensor observations highly correlated in the space domain. In other words, the existence of spatial correlation implies that the readings from sensor nodes which are geographically close to each other are expected to be largely correlated. In environment monitoring applications, sensor nodes periodically sample and communicate the data to the fusion center. The nature of the energy-radiating physical phenomenon yields temporal correlation between each consecutive observation of a sensor node [1]. Existence of temporal correlation implies that the readings observed at one time instant are related to the readings observed at the previous time instants. To sum it up this means that the physical phenomena often exhibit both spatial and temporal correlation. By noticing the fact that sensor readings are both spatially and temporally correlated, the detection of an event can be performed by capturing the spatio-temporal correlation in the sensor readings.

While considering large scale WSN, we assume that a subset of sensors (i.e. those located close to the event) receive the signal emitted by the event and send their measurements to the fusion center. Intuitively, spatial correlation present in observations of these sensors indicates that measurements are received from same neighborhood and it is most likely that some real event has happend. Similarly, if there is strong correlation between the consecutive time measurements then it further confirms the actual presence of an event. There have been some attempts to consider the correlated measurements into formulation of the signal detection. However, many of these studies consider the presence of correlation as a deleterious effect [2]. On the contrary, there are some contributions that focus on the discrimination between only spatially correlated and spatially independent observations by exploiting the structure of covariance matrix [3, Ch. 9-10][4]. However, these works consider observations received as temporally independent and do not capture spatio-temporal characteristics of the physical phenomenon.

The GLRT approach in [5] detects spatial correlation in time series based on the decision whether the spatio-temporal sample covariance matrix is block diagonal or not. This approach typically ends up with a simple quotient between the determinant of the spatio-temporal sample covariance matrix and the determinant of its diagonal version. As the GLRT involves estimation of unknown parameters (i.e. covariance matrix), therefore, it depends on the sample size and the dimensionality. In practice, GLRT is used based on the assumption that the sample size is large while the sample dimension is small. In the case of large WSN when the sample support available for estimating the covariance matrix is limited, the GLRT degenerates due to singular and ill-conditioned covariance matrix [6]. To cope with this problem we propose novel detectors that are robust against the high dimensionality of spatio-temporal data. Consequently, the proposed detectors are based on splitting the spatio-temporal covariance matrix

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into spatial and temporal covariance matrices. By doing so the demand for large sample size reduces as the dimensions of the resultant spatial and temporal covariance matrices become much smaller than the dimension of full spatio-temporal covariance matrix.

The remaining paper is organized as follows. In section 2, problem statement and details of signal models are presented. In section 3 we present the traditional detectors and section 4 introduces a modification of the traditional GLRT and two robust ad hoc tests. In section 5 we present simulation results and section 6 concludes the work.

2. PROBLEM FORMULATION

2.1. Problem statement

We consider an infrastructure based sensor network where a fusion center receives observations from sensor nodes and it makes the final decision about the presence of an event in the field. Furthermore, we assume that the signal emitted by the presence of the event can be modeled as an electromagnetic field that can be measured by a radio-frequency receiver. The region where the physical phenomenon occurs is called event region. While considering large WSN and limited sensing range, we assume that the event will be detected by only L sensors, which are present in the event region (i.e. the region where signal power is present) and we call them *active sensors*. To save power and communication costs, we consider local censoring technique such that only L active sensors communicate their observations to the fusion center, which is also to say that only these sensors have locally decided that the event is present in the field. Herein, we assume that every sensor is equipped with a local energy detector to decide whether to communicate their observations to the fusion center or not. After receiving the measurements, the fusion center draws the final decision based on the exploitation of both spatial and temporal correlations present in the received measurements.

2.2. Signal Model

Each active node measures its total received power (in dB), normalizes the measurements by subtracting the mean noise power (in dB) and sends the normalized measurement x_i (in dB), i = {1, 2, . . .,L } to the fusion center. Under the null hypothesis $\mathcal{H}_0$, the power at the output of the energy detectors will be simply the sum of noise power and the power of any interfering signals that may be present. Given that the noise level can vary among the nodes, and that the exact values are unknown, the noise power at the nodes are modeled as independent lognormally distributed random variables [7]. Consequently, the measurements x_i (in dB) are assumed to be independently distributed Gaussian random variables with zero mean (after normalization) and variance $\sigma^2_{\eta_i}$. Under the alternative hypothesis $\mathcal{H}_1$, the total received power measured at each node is the sum of the noise power and the signal received from the event. For shadow fading propagation environments, the event's signal at each node can be approximated by a set of correlated lognormally distributed random variables, in which the mean is a function of the distance dependent path-loss and the variance σ_s^2 [7]. More specifically, the received signal power at a node located d_i meters from the event is defined as (in dB).

$$P_i(dB) = P_0(dB) - 10\beta\log\left(d_i\right) + s_i \quad \text{i=1,2,}\cdots\cdots\text{,L} \tag{1}$$

where β is is the signal decay exponent and $s_i = \mathcal{N}\left(0, \sigma_s^2\right)$ quantifies the uncertainty due to shadowing. Given this definition for the received signal power, the total received power under the alternative hypothesis is equal to the sum of two independent lognormally distributed random variables. Various studies have shown that the sum of two independent lognormal random variables can be approximated as a lognormal random variable [8]. Using this approximation, the measurements x_i, given $\mathcal{H}_1$, are assumed to be Gaussian distributed with means [7].

$$\mu_i = \mathrm{E}\left[10\mathrm{log}_{10}\left(1 + SNR_i\right)\right] \tag{2}$$

and the covariances

$$\rho_{i,j} = \sigma_i\sigma_j e^{-ad_{i,j}} \quad i,j = 1,2,\cdots,L \tag{3}$$

where σ_i (σ_j) is the variance of x_i under $\mathcal{H}_1$, $d_{i,j}$ is a unknown separation between i^{th} and j^{th} sensor, and a is the correlation coefficient. At time instant n, the observation vector received at the fusion center can be represented as $\mathbf{x}(n) \doteq \left[\begin{array}{cccc} x_1(n) & x_2(n) & \cdots & x_L(n) \end{array}\right]^T$. Under $\mathcal{H}_0$, $\mathbf{x}(n) \sim \mathcal{N}\left(0, \mathbf{\Sigma}_\eta\right)$ and under $\mathcal{H}_1$, $\mathbf{x}(n) \sim \mathcal{N}\left(\boldsymbol{\mu}, \mathbf{\Sigma}_s\right)$, where the $i-$th element of vector $\boldsymbol{\mu}$ is given by (2), the i-jth element of $\mathbf{\Sigma}_s$ is given in (3) and $\mathbf{\Sigma}_\eta = \text{diag}\left\{\begin{array}{cccc} \sigma^2_{\eta_1} & \sigma^2_{\eta_2} & \cdots & \sigma^2_{\eta_K} \end{array}\right\}$. At the fusion center measurements collected from L active sensors, where each sensor observe the field N times are stored in the $L \times N$ matrix $\mathbf{X} \doteq \left[\begin{array}{cccc} \mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(N) \end{array}\right]$. Taking columns of the matrix $\mathbf{X}$, stack in $LN \times 1$ vector $\mathbf{z}$ as: $\mathbf{z} \doteq \text{vec}\left(\mathbf{X}\right)$, the signal model can be written as :

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_0) \\ \mathcal{H}_1 &: \mathbf{z} \sim \mathcal{N}(\mathbf{u}, \mathbf{\Sigma}_1) \end{aligned} \tag{4}$$

where $\mathbf{u} \doteq \left[\begin{array}{cccc} \boldsymbol{\mu}^T(1) & \boldsymbol{\mu}^T(2) & \cdots & \boldsymbol{\mu}^T(N) \end{array}\right]^T \in \mathbb{R}^{KN \times 1}$. Under $\mathcal{H}_0$, the covariance matrix $\mathbf{\Sigma}_0 = \mathrm{E}\left[\mathbf{z}\mathbf{z}^H\right] \in \mathbb{R}^{LN \times LN}$ has block diagonal elements $\mathbf{\Sigma}_\eta^{(n_i,n_i)} \in \mathbb{R}^{L \times L}$, $1 \leq n_i \leq N$ and off-diagonal blocks are zero matrices. Under $\mathcal{H}_1$, the spatio-temporal covariance matrix $\mathbf{\Sigma}_1 \doteq \mathrm{E}\left[\left(\mathbf{z} - \mathbf{u}\right)\left(\mathbf{z} - \mathbf{u}\right)^H\right] \in \mathbb{R}^{LN \times LN}$ has block elements $\mathbf{\Sigma}_s^{(n_i,n_j)} \doteq E\left[\left(\mathbf{x}(n_i) - \boldsymbol{\mu}(n_i)\right)\left(\mathbf{x}(n_j) - \boldsymbol{\mu}(n_j)\right)^H\right] \in \mathbb{R}^{L \times L}$ with $1 \leq n_i, n_j \leq N$ captures the spatial correlation between sensors.

3. TRADITIONAL TEST STATISTICS

In this section we derive decision schemes for the detection problem introduced in (4), by adopting two traditional multivariate approaches. The performance of these schemes will be compared with the novel approaches that will be proposed in section 4. To solve the hypothesis testing problem (4) following the the traditional GLRT approach, the test statistic can be formulated as:

$$L_{G_1}(x) = \frac{\max\limits_{\boldsymbol{\Sigma}_0} p\left(\mathbf{z}(1), \mathbf{z}(2), \cdots, \mathbf{z}(M), \boldsymbol{\Sigma}_0\right)}{\max\limits_{\boldsymbol{\Sigma}_1, \mathbf{u}} p\left(\mathbf{z}(1), \mathbf{z}(2), \cdots, \mathbf{z}(M); \mathbf{u}, \Sigma_1\right)} \gtrless_{\mathcal{H}_1}^{\mathcal{H}_0} \gamma \tag{5}$$

where we have assumed that M independent realizations of the data matrix $\mathbf{X}$, or equivalently $\mathbf{z}$ are available, and $p\left(\mathbf{z}(1), \mathbf{z}(2), \cdots, \mathbf{z}(M), \boldsymbol{\Sigma}_0\right)$ and $p\left(\mathbf{z}(1), \mathbf{z}(2), \cdots, \mathbf{z}(M); \mathbf{u}, \Sigma_1\right)$ are likelihood functions under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Solving (5), we can get the final expression as:

$$L_{G_1}(\mathbf{x}) = \frac{\det\hat{\boldsymbol{\Sigma}}_1}{\det\hat{\boldsymbol{\Sigma}}_0} \gtrless_{\mathcal{H}_1}^{\mathcal{H}_0} \gamma. \tag{6}$$

By using [3, Lema 3.2.1], we can write $\hat{\boldsymbol{\Sigma}}_1 = \hat{\boldsymbol{\Sigma}} - \hat{\mathbf{u}}\hat{\mathbf{u}}^T$ with $\hat{\boldsymbol{\Sigma}} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{z}(m)\mathbf{z}^H(m)$ and $\hat{\mathbf{u}} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{z}(m)$. Similarly, at a very low SNR we can approximate $\hat{\boldsymbol{\Sigma}}_0 \approx \text{diag}\hat{\boldsymbol{\Sigma}}_1$ as $\mu_i = \mathrm{E}\left[10log_{10}\left(1 + SNR_i\right)\right] \approx 0$. When $M \ggg KN$, the GLRT in (5) is an optimal detector but in the case of large-scale WSN, when the dimensionality of data (i.e. LN) is larger than the sample size M, the singularity of the sample covariance matrix makes the GLRT degenerate [6]. One way to mitigate this problem, is to design a detector that avoids matrix operations that are vulnerable to the singularity of the sample covariance matrix. Taking into account this fact the CAV detector proposed in [4] can be used to avoid problems due to ill-conditioned sample covariance matrix. The CAV detector is a ratio between the sum of elements of the spatio-temporal sample covariance matrix and the sum of diagonal elements of that matrix as:

$$L_{G_2}(x) = \frac{\sum_{\tau}^{NL}\sum_{\upsilon}^{NL}\left|\rho_1^{(\tau,\upsilon)}\right|}{\sum_{\tau}^{NL}\left|\rho_0^{(\tau,\tau)}\right|} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma \tag{7}$$

where $|\cdot|$ represents absolute value, $\rho_1^{(\tau,\upsilon)}$ is (τ,υ)-th element of the spatio-temporal sample covariance matrix $\hat{\boldsymbol{\Sigma}}_1$ and $\rho_0^{(\tau,\tau)}$ (τ,τ)-th diagonal element of $\hat{\boldsymbol{\Sigma}}_0$. The detector in (7) does not need any prior information of the signal, the channel nor the noise power.

4. PROPOSED TEST STATISTICS

In section 3, we have presented the traditional GLRT approach for detection problem in (4) and argued that it degenerates due to singularity issues that arises in detection problems with small samples sizes and large number of sensors. Consequently, to circumvent the singularity issue we adopted CAV detector for our detection. In this section, to cope with this problem of practical importance, we will move one step further by factoring $\boldsymbol{\Sigma}_1$ into a purely spatial and a purely temporal components based on the Kronecker product [9] as

$$\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_{1,\mathrm{T}} \otimes \boldsymbol{\Sigma}_{1,\mathrm{S}} \tag{8}$$

where $\boldsymbol{\Sigma}_{1,\mathrm{S}}$ captures the spatial correlation between the observations of sensors and $\boldsymbol{\Sigma}_{1,\mathrm{T}}$ captures the time correlation between repeated measurements. Herein, we remark that the covariance structure (8) makes an implicit assumption that the temporal correlation structure remains the same at all spatial locations and the spatial correlation structure is same during observation time (i.e. $1 : N$) . The structure of separable covariances dramatically reduces the number of parameters in the covariance matrix and therefore demands small M to estimate the spatio-temporal covariance matrix [10]. Based on the factored $\boldsymbol{\Sigma}_1$, in sub-section 4.1, we derive the proposed novel GLRT, and in sub-sections 4.2 and 4.3, we propose two novel ad hoc tests.

4.1. Kronecker Structure based GLRT

The GLRT based on Kronecker structure can be written as:

$$L_{G_3}(z) = \frac{\max\limits_{\boldsymbol{\Sigma}_{0,\mathrm{T}},\boldsymbol{\Sigma}_{0,\mathrm{S}}} p\left(\mathbf{z}; \boldsymbol{\Sigma}_{0,\mathrm{T}} \otimes \boldsymbol{\Sigma}_{0,S}\right)}{\max\limits_{\mathbf{u},\boldsymbol{\Sigma}_{1,\mathrm{T}},\boldsymbol{\Sigma}_{1,\mathrm{S}}} p\left(\mathbf{z}; \mathbf{u}, \boldsymbol{\Sigma}_{1,\mathrm{T}} \otimes \boldsymbol{\Sigma}_{1,S}\right)} \gtrless_{\mathcal{H}_1}^{\mathcal{H}_0} \gamma. \tag{9}$$

Let $\mathbf{z}(1), ..., \mathbf{z}(M)$ be a random realizations of $\mathbf{z} \sim \mathcal{N}\left(\mathbf{u}, \boldsymbol{\Sigma}_{k,\mathrm{S}} \otimes \boldsymbol{\Sigma}_{k,\mathrm{T}}\right)$, where $\boldsymbol{\Sigma}_{k,\mathrm{S}}$ and $\boldsymbol{\Sigma}_{k,\mathrm{T}}$ are spatial and temporal covariance matrices, respectively. Whereas, $k = 0, 1$ represent hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. The estimates of unknown parameters, required in the GLRT (9) are typically found by using maximum likelihood estimation (MLE), because it is asymptotically an unbiased and efficient estimator for these unknowns. Under $\mathcal{H}_1$, the likelihood function of $\mathbf{z}$ can be written as [9]:

$$p\left(\mathbf{z}; \mathbf{U}, \boldsymbol{\Psi}_1\right) = c\left(\det\boldsymbol{\Sigma}_{1,\mathrm{T}}\right)^{-\frac{NM}{2}}\left(\det\boldsymbol{\Sigma}_{1,\mathrm{S}}\right)^{\frac{-LM}{2}} \exp\left[-\frac{1}{2}\sum_{m=1}^{M}\mathrm{tr}\left\{\boldsymbol{\Sigma}_{1,\mathrm{S}}^{-1}\mathbf{X}_M\left(m\right)\boldsymbol{\Sigma}_{1,\mathrm{T}}^{-1}\mathbf{X}_M^H\left(m\right)\right\}\right] \tag{10}$$

where $\boldsymbol{\Psi}_1 = \boldsymbol{\Sigma}_{1,\mathrm{T}} \otimes \boldsymbol{\Sigma}_{1,\mathrm{S}}$, $\mathbf{X}_M\left(m\right) = \mathbf{X}\left(m\right) - \mathbf{U}$ with $\mathbf{u} = \mathrm{vec}\left(\mathbf{U}\right)$ and $c = (2\pi)^{-\frac{1}{2}LNM}$. The MLE of unknowns $\mathbf{U}$, $\boldsymbol{\Sigma}_{1,\mathrm{T}}$ and $\boldsymbol{\Sigma}_{1,\mathrm{S}}$ is found by taking the derivative of $\log p\left(\mathbf{z}; \mathbf{U}, \boldsymbol{\Psi}_1\right)$ with respect to the unknown $\mathbf{U}$, $\boldsymbol{\Sigma}_{1,\mathrm{T}}$ and $\boldsymbol{\Sigma}_{1,\mathrm{S}}$ and then equating to 0. By doing so, the MLE under the hypothesis $\mathcal{H}_1$ can be written as [9]:

$$\hat{\boldsymbol{\Sigma}}_{1,\mathrm{S}} = \frac{1}{NM}\sum_{m=1}^{M}\left(\mathbf{X}(m) - \bar{\mathbf{X}}\right)\hat{\boldsymbol{\Sigma}}_{1,\mathrm{T}}^{-1}\left(\mathbf{X}(m) - \bar{\mathbf{X}}\right)^H \tag{11}$$

$$\hat{\boldsymbol{\Sigma}}_{1,\mathrm{T}} = \frac{1}{LM}\sum_{m=1}^{M}\left(\mathbf{X}(m) - \bar{\mathbf{X}}\right)^H\hat{\boldsymbol{\Sigma}}_{1,\mathrm{S}}^{-1}\left(\mathbf{X}(m) - \bar{\mathbf{X}}\right) \tag{12}$$

where $\bar{\mathbf{X}} = \hat{\mathbf{U}} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{X}(m)$. Expression (11) and (12) suggest that $\hat{\boldsymbol{\Sigma}}_{1,\mathrm{S}}$ and $\hat{\boldsymbol{\Sigma}}_{1,\mathrm{T}}$ can be estimated using an iterative method such as the Flip-Flop algorithm. The Flip-Flop

Algorithm 1 Flip-Flop

1. Choose a starting value for $\hat{\Sigma}_{1,\mathrm{S}}$ as in (13)
2. Find the following
   - Estimate $\hat{\mathbf{\Sigma}}_{1,\mathrm{T}}^{r}$ from (12) with $\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}^{1} = \hat{\mathbf{\Sigma}}_{1,\mathrm{S}}^{0}$.
   - Estimate $\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}^{1}$ from (11) with $\hat{\mathbf{\Sigma}}_{1,\mathrm{T}}^{1}$.

algorithm is obtained by alternately minimizing (10) w.r.t. $\mathbf{\Sigma}_{1,\mathrm{T}}$ keeping the last available estimate of $\mathbf{\Sigma}_{1,\mathrm{S}}$ fixed and vice versa. In [10], numerical experiments have been reported which indicate that the Flip-Flop algorithm performs very well and is much faster than a more general purpose optimization algorithm such as Newton–Raphson [10]. In [11], it has been discussed that for the case of large M, there is no need to iterate the algorithm. Taking into account this fact, we adopt non-iterative Flip-Flop approach and only perform the the steps given in Algorithm 1. To begin the process of non-iterative Flip-Flop, we use an initial value of $\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}$ as:

$$\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}^{0} = \frac{1}{NM}\sum_{m=1}^{M}\left(\mathbf{X}(m) - \bar{\mathbf{X}}\right)\left(\mathbf{X}(m) - \bar{\mathbf{X}}\right)^{H}. \quad (13)$$

Under $\mathcal{H}_0$ and considering low SNR regime we have $\mathbf{\Sigma}_{0,\mathrm{T}} \approx \mathrm{diag}\mathbf{\Sigma}_{1,\mathrm{T}}$ and $\mathbf{\Sigma}_{0,\mathrm{S}} \approx \mathrm{diag}\mathbf{\Sigma}_{1,\mathrm{S}}$, and the Likelihood function is:

$$p\left(\mathbf{z};\mathbf{\Psi}_0\right) = c\left(\det\mathbf{\Sigma}_{0,\mathrm{T}}\right)^{-NM/2}\left(\det\mathbf{\Sigma}_{0,\mathrm{S}}\right)^{-LM/2} \exp\left[-\frac{1}{2}\sum_{m=1}^{M}\mathrm{tr}\left\{\mathbf{\Sigma}_{0,\mathrm{S}}^{-1}\mathbf{X}_M\left(m\right)\mathbf{\Sigma}_{0,\mathrm{T}}^{-1}\mathbf{X}_M^{H}\left(m\right)\right\}\right]. \quad (14)$$

Taking derivative of $\log p\left(\mathbf{z};\mathbf{\Sigma}_{0,\mathrm{T}}\otimes\mathbf{\Sigma}_{0,\mathrm{S}}\right)$ with respect to unknown $\mathbf{\Sigma}_{0,\mathrm{T}}$ and $\mathbf{\Sigma}_{0,\mathrm{S}}$ and then equating to 0, the MLE of these parameters can be written as: $\hat{\mathbf{\Sigma}}_{0,\mathrm{S}} \approx \mathrm{diag}\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}$ and $\hat{\mathbf{\Sigma}}_{0,\mathrm{T}} \approx \mathrm{diag}\hat{\mathbf{\Sigma}}_{1,\mathrm{T}}$. With all these estimated values of unknowns, (9) can be written as:

$$L_{G_3}(z) = \frac{\left(\det\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}\right)^{N}\left(\det\hat{\mathbf{\Sigma}}_{1,\mathrm{T}}\right)^{L}}{\left(\det\hat{\mathbf{\Sigma}}_{0,\mathrm{S}}\right)^{N}\left(\det\hat{\mathbf{\Sigma}}_{0,\mathrm{T}}\right)^{L}} \gtrless_{\mathcal{H}_1}^{\mathcal{H}_0} \gamma. \quad (15)$$

This structured GLRT has several advantages over the traditional. Under $\mathcal{H}_1$ instead of $\frac{1}{2}NL(NL+1)$ parameters, it has only $\frac{1}{2}N(N+1)+\frac{1}{2}L(L+1)$ parameters to estimate. The dimensions of these two covariance matrices $\mathbf{\Sigma}_{1,\mathrm{S}}$ and $\mathbf{\Sigma}_{1,\mathrm{T}}$ are much smaller than the dimension of full spatio-temporal covariance matrix $\mathbf{\Sigma}_1$, that is why the computations are much less demanding.

4.2. Sum-Sum test

Inspired by CAV detector (7), based on spatial and temporal covariance matrices we propose,

$$L_{G_4}(z) = \frac{\left(\sum_{\tau}^{L}\sum_{\upsilon}^{L}\left|\rho_{1,\mathrm{S}}^{(\tau,\upsilon)}\right|\right)\left(\sum_{\tau}^{N}\sum_{\upsilon}^{N}\left|\rho_{1,\mathrm{T}}^{(\tau,\upsilon)}\right|\right)}{\left(\sum_{\tau}^{L}\left|\rho_{0,\mathrm{S}}^{(\tau,\tau)}\right|\right)\left(\sum_{\tau}^{L}\left|\rho_{0,\mathrm{T}}^{(\tau,\tau)}\right|\right)} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma \quad (16)$$

where $\rho_{1,S}^{(\tau,\upsilon)}$ and $\rho_{1,\mathrm{T}}^{(\tau,\upsilon)}$ are (τ,υ)-th elements of spatial covariance matrix $\hat{\mathbf{\Sigma}}_{1,\mathrm{S}}$ and temporal covariance matrix $\hat{\mathbf{\Sigma}}_{1,\mathrm{T}}$, respectively. Similarly, $\rho_{0,S}^{(\tau,\tau)}$ and $\rho_{0,\mathrm{T}}^{(\tau,\tau)}$ are (τ,τ)-th diagonal elements of $\hat{\mathbf{\Sigma}}_{0,\mathrm{S}}$ and $\hat{\mathbf{\Sigma}}_{0,\mathrm{T}}$, respectively. Contrary to GLRT based detector (15), detector (16) does not involve determinants so it will have robustness against the high dimensionality. Similar to (7), it does not need any prior information of the signal distribution, the channel and noise power.

4.3. Trace-Trace Test

Motivated by the John's U-statistic [6], here we propose an ad hoc test statistic that uses the two separate spatial and temporal sample covariance matrices achieved through Algorithm 1. The expression for this Ad hoc test is

$$L_{G_5}(z) = \frac{1}{LN}\mathrm{tr}\left(\hat{\Psi}_{\mathrm{S}} - \mathbf{I}_K\right)^2 \mathrm{tr}\left(\hat{\Psi}_{\mathrm{T}} - \mathbf{I}_N\right)^2 \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma \quad (17)$$

where $\hat{\Psi}_{\mathrm{S}} = \frac{\hat{\mathbf{\Sigma}}_{1,S}}{\frac{1}{K}\mathrm{tr}\left(\hat{\mathbf{\Sigma}}_{1,S}\right)}$ and $\hat{\Psi}_{\mathrm{T}} = \frac{\hat{\mathbf{\Sigma}}_{1,T}}{\frac{1}{N}\mathrm{tr}\left(\hat{\mathbf{\Sigma}}_{1,T}\right)}$. The detector (17) has robustness against the high dimensionality and it does not assume any prior information about the signal and noise distribution.

5. SIMULATIONS

For simulations we consider a wireless sensor network, where sensors are distributed following a uniform probability distribution and we assume that the event appears at an unknown position. We analyze the performance of the detection approaches for different values of L, the number of active sensors and N, the number of samples based on the analysis of area under the ROC curve (AUC), which varies between 0.5 (poor performance) and 1 (good performance).

In Fig.1 we analyze the the detection schemes for $L = 20$, $N = 5$. The results show that by increasing the value of M, of course the detection performances of all detectors increase, but the proposed schemes perform better than the traditional schemes. For example for a specific value of AUC, lets say AUC = 0.9, the proposed detectors $L_{G_3}(z)$, $L_{G_4}(z)$ and $L_{G_5}(z)$ need $M \leq 50$, while the traditional GLRT $L_{G_1}(z)$ and $L_{G_2}(z)$ need $M \geq 100$.

In Fig.2, we simulate the detectors for scenario where we have $L = 30$ and $N = 10$. From the results it can be seen that to achieve a good detection performance, the required sample size M for the traditional approaches is much higher than the proposed approaches. We can clearly see that for AUC = 0.9, the proposed detectors need sample size $M \leq 100$ while for the same level of AUC, $L_{G_2}(z)$ needs $M \geq 300$. We can also see that the traditional GLRT suffers severely due

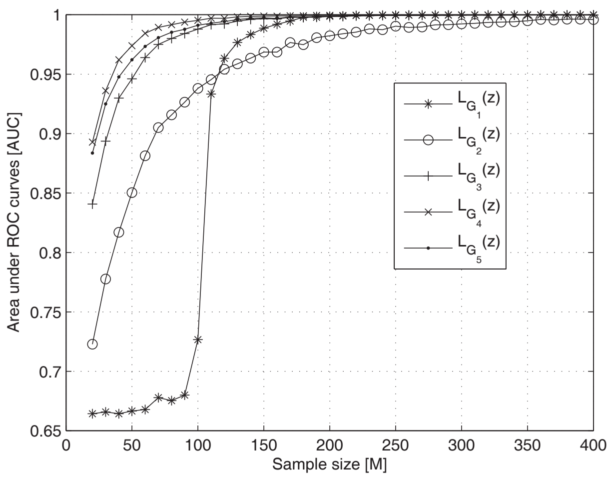

Fig. 1. AUC curves: L = 20, N = 5, $\sigma_s^2 = 2$dB, $\sigma_n^2 = 2$dB $P_0 = -20$dB

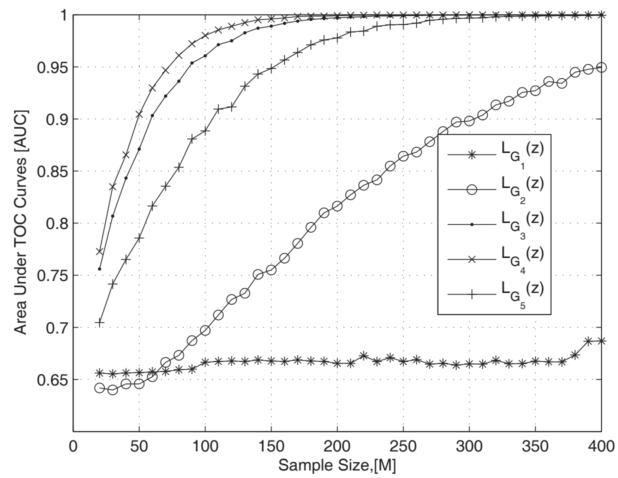

Fig. 2. AUC curves: L = 30, N = 10, $\sigma_s^2 = 2$dB, $\sigma_n^2 = 4$dB $P_0 = -20$dB

to ill-conditioned sample covariance matrix. From these results we can conclude that the proposed detection schemes are performing better than the traditional schemes. Amongest the five detectors discused in this paper, the novel detectors $L_{G_4}(z)$ and $L_{G_5}(z)$ show more robustness. In all of the cases they have consistently better detection performance as the AUC curves reach to the reasonable high value at a very small M .

6. CONCLUSION

In this paper novel signal detection schemes have been proposed with the aim to detect an event by exploiting spatio-temporal correlation present in the observations received from the sensors. The traditional detection approaches suffer due to singularity and ill-conditioned spatio-temporal sample covariance matrix. To address this problem, we have proposed detection schemes by splitting the large spatio-temporal covariance matrix into spatial and temporal covariance matrices. Simulation results obtained, have shown that the proposed detection schemes consistently have better detection performance in the case when the sample size is much smaller than the dimensionality of data.

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