A NETWORK SHADOW FADING MODEL FOR AUTONOMOUS INFRASTRUCTURE WIRELESS NETWORKS

Bijan Golkar and Elvino Sousa

Deparment of Electrical and Computer Engineering, University of Toronto
10 King's College Road, Toronto, Ontario, Canada, M5S 3G4
Emails: {bijan,sousa}@comm.utoronto.ca

ABSTRACT

In traditional cellular networks the base stations (BSs) are more or less regularly deployed. In the channel model for such networks, correlated channel gains are only considered between a BS and multiple terminals (or one terminal and multiple BSs). Although the literature has introduced efficient techniques to produce correlated channel gains, to our knowledge it has not reported on the correlation between pairs of links with no common end. This may be a reasonable model for large cell sizes (or equivalently the large inter-site-distances) in the traditional networks. In future networks, however, it is expected that a large number of BSs will be deployed in random positions. With the dense deployment of BSs, appropriate correlation channel models should be developed. This paper proposes a novel shadow fading model which enables the generation of correlated shadow fading gains between all links in the network.

Index Terms— Correlated shadow fading, Autonomous cellular network, Self-organizing network (SON), Channel modeling

1. INTRODUCTION

The propagation of electromagnetic waves in a wireless communication system is studied from two main standpoints. The first deals with the signal power attenuation on a macroscopic level and is often referred to as large-scale (or shadow) fading. The second class considers the fine structure of the multipath propagation and is known as small-scale (or multipath) fading. Multipath fading studies the time and frequency variations of the channel impulse response due to the differences in delays, phases and amplitudes of the multiple reflections of the transmitted signal arriving from different directions at the moving/static receiver. In this paper, we exclusively focus on the former class.

Large scale fading predicts the average received signal power level at a given distance from the transmitter. This power is not only a function of the distance but also highly dependent on the unique topographical properties of the environmnet. The physical man made/natural obstructions in the environment should be incorporated into the signal attenuation model. However, due to the lack of accurate detailed propagation measurements and landscape information — or to develop a generic model — the effect of the environment on signal propagation is modeled by introducing a degree of randomness in the average received power level. Based on empirical results, the randomness has been modeled by a lognormal random variable in the signal attenuation model and the effect is referred to as shadow (lognormal) fading.

Due to the inherent spatial correlation in the topography of the environment, a proper spatial correlation model should also be considered. In the special case of a link between a fixed base station and a moving terminal, the channel gain experienced by the terminal cannot change abruptly after having traveled a relatively small distance. The first such model was introduced in [1]. It was further developed to capture the correlation between one mobile terminal and multiple base stations in a cellular network [2, 3]. More recently, two-dimensional correlation patterns were proposed to facilitate the generation of correlated channel gains [4, 5].

Although the literature has introduced efficient techniques to produce correlated channel gains, to our knowledge it has not reported on the correlation between pairs of links with no common end. The existing models consider the correlation between links either between a terminal and multiple BSs or between a BS and multiple terminals. This type of correlation, however, is particularly important in autonomous networks where the base stations are no longer deployed according to a regular pattern. The irregularity of the deployment of base stations can result in a dense deployment within the same locality (with similar/correlated topographical properties). In this case, the correlation between a pair of links between two terminals and two base stations becomes crucial for a realistic evaluation of the system performance.

This paper presents a network correlation model for shadow fading. Section 2 provides a review of the shadow fading principles together with the correlation models available in the literature. In section 3, we introduce the concept of potential field based on which the network correlation model is derived. Different properties of the proposed correlation model are also examined in this section. Finally section 4 concludes the paper with a summary of the model and future work.

2. BACKGROUND MATERIAL

Consider the link between a transmitter and a receiver. In free space with unobstructed line of sight between the transmit-receive pair, the Friis formula gives the received signal power level, P_r, as a function of the transmit power level P_t, the transmitter antenna gain G_t and the receiver antenna gain G_r as follows:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$$

where λ is the wavelength of the transmitted signal and d is the distance between the transmitter and the receiver.

In terrestrial communication, the received signal power level is a complex function of the topography. To begin with, the rate at which the received power decays with distance is a function of the communication environment (e.g. urban, rural or indoor). On average this rate is mathematically modeled as the pathloss exponent γ. Let us take two links of the same length but sufficiently far from each other in the same environment. Although the distance between the transmitter and the receiver is the same for both, the unique topographical properties cause the received signal powers to be different. Due to the lack of detailed propagation measurements, this effect is statistically studied by introducing a random variable into the equation. Hence the received signal power is given by,

$$P_r = K \cdot \frac{1}{d^\gamma} \cdot L$$

where the constant K is a function of the antenna gains, transmission frequency, transmission power and other factors. L is the introduced random variable and is referred to as the shadow fading gain. Based on empirical results, L is modeled as a lognormal random variable, i.e. $\log L \sim \mathcal{N}(0, \sigma_0^2)$, where $\mathcal{N}(0, \sigma_0^2)$ represents the Gaussian distribution with a mean of zero and a standard deviation of σ_0.

Due to the fact that the topographical properties of the terrain do not change abruptly — the L values for different links are spatially correlated — an appropriate correlation model should be adopted. In 1991, Gudmundson proposed a correlation model [1] as follows: Let us consider the link from point A to point B and the link from point A to point C in Figure 1. The distance between points B and C is denoted by d. The correlation between the links A-B and A-C is a function of d as follows:

$$R(d) = \sigma_0^2 e^{-d/d_c} \quad (1)$$

where d_c is the correlation distance.

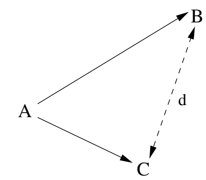

Fig. 1. Two links from a common point to two points apart by a distance of d

In [4] a two-dimensional spatial correlation model is proposed for a cellular network. The model generates the correlated shadow fading gains between a BS and multiple terminals. In order to do so, for each BS, a two-dimensional Gaussian random field with appropriate marginal distribution and spatial correlation is generated[1]. The shadow fading gain between the BS and any terminal in the field is the value of the field at the terminal.

In this paper, we propose a general network shadow fading model to generate correlated shadow fading gains between any two links in the network. In particular, consider the following two links A-B and C-D (see Figure 2). The two transmitters A and C are located close to each other. Similarly receivers B and D are also located close to one another. Clearly the shadow fading gains of the two links should be correlated. However, since the two links do not share a common end, the Gudmundson correlation model would not introduce correlation between the shadow fading gains for the above two links. Our model, on the other hand, introduces an appropriate correlation between such links.

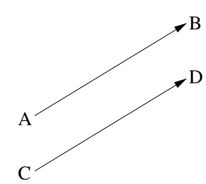

Fig. 2. A pair of links with the same topographical properties

3. NETWORK CORRELATION MODEL

We introduce a Gaussian field in the network coverage area, which unlike previous work is not dependent on a particular BS. The Gaussian field is a random process where each realization is a function from the plane to the real numbers. If we fix a point in the plane, the outcome is a Gaussian random variable. Random variables corresponding to 2 different points are jointly Gaussian with a standard deviation of $\frac{\sigma_0}{\sqrt{2}}$

[1]The underlying spatial correlations are generated based on the Gudmundson model.

and the correlation model in (1). Figure 3 illustrates a realization of this field which is referred to as the *shadow fading potential field*. The value of the field at point A defines the *potential level* of point A and is denoted by $X_{\rm A}$. As two points A and B come closer to each other, the correlation between their corresponding potential levels increases, i.e. the random variable formed by the difference should have a variance that approaches zero.

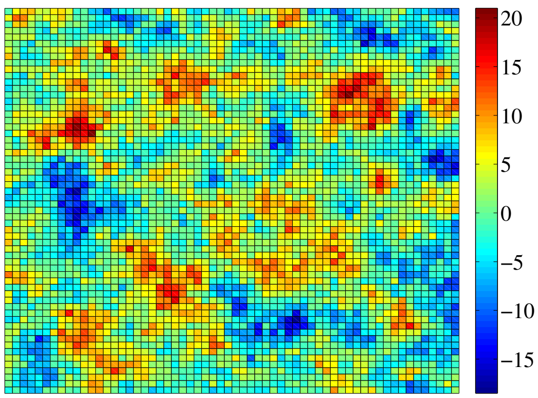

Fig. 3. A realization of the network potential field

The shadow fading gain of a link between point A and point B is defined as the exponential of the function $\mathfrak{f}(X_{\rm A}, X_{\rm B})$. In order to comply with the properties of shadow fading, the governing function $\mathfrak{f}(.,.)$ should have the following three properties:

- Symmetry: The channel gain is intrinsically symmetric. Regardless of which end the transmitter (or the receiver) lies, the channel gain should be the same.
- Continuity: As the two ends of a link come closer to each other, the randomness due to the topography of the environment diminishes. This should translate into a smaller variance for the shadow fading gain.
- Compatibility: Based on empirical results, the shadow fading gain in the logarithmic scale (dB) is modeled as a zero-mean Gaussian random variable with a standard deviation of σ_0dB.

With this set of requirements, the following function is proposed for the shadow fading gain (in dB) between point A and point B:

$$\mathfrak{f}(X_{\rm A}, X_{\rm B}) = \mathrm{sgn}(X_{\rm A} + X_{\rm B}) \cdot |X_{\rm A} - X_{\rm B}|$$

where, $\mathrm{sgn}(t)$ is the signum function.

The proposed function satisfies the above required conditions. It clearly satisfies the symmetry and continuity conditions. The compatibility condition should however be verified.

LEMMA: If S and T are two independent Gaussian random variables with zero mean and standard deviations of $\sigma_{\rm S}$ and $\sigma_{\rm T}$ respectively, the random variable $Z = \mathrm{sgn}(S) \cdot |T|$ has a Gaussian distribution with zero mean and a standard deviation of $\sigma_{\rm T}$.

PROOF: Since X is a zero-mean Gaussian random variable, $t = \mathrm{sgn}(X)$ is a discrete random variable with $P(t = 1) = P(t = -1) = 0.5$. The cumulative distribution function of Z is

$$F_Z(z) = P(Z \le z) = 0.5P(|T| \le z) + 0.5P(-|T| \le z)$$

or,

$$F_Z(z) = \begin{cases} \frac{1}{2}P(|T| \le z) + \frac{1}{2} & ,z \ge 0 \\ \frac{1}{2}P(-|T| \le z) & ,z < 0 \end{cases}$$

It is well-known that the random variable $L = |T|$ has a half-normal distribution with probability distribution function

$$f_L(l) = \sqrt{\frac{2}{\pi\sigma_{\rm T}^2}} \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right)$$

Hence when $z \ge 0$,

$$\begin{aligned} F_Z(z) &= \frac{1}{2}P(L \le z) + \frac{1}{2} \\ &= \frac{1}{2}\left(1 + \sqrt{\frac{2}{\pi\sigma_{\rm T}^2}} \int_0^z \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right) dl\right) \\ &= \frac{1}{2}\left(2 - \sqrt{\frac{2}{\pi\sigma_{\rm T}^2}} \int_z^{+\infty} \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right) dl\right) \\ &= 1 - \frac{1}{\sqrt{2\pi\sigma_{\rm T}^2}} \int_z^{+\infty} \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right) dl \\ &= \frac{1}{\sqrt{2\pi\sigma_{\rm T}^2}} \int_{-\infty}^{z} \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right) dl \end{aligned}$$

and for $z < 0$,

$$\begin{aligned} F_Z(z) &= \frac{1}{2}P(-L \le z) \\ &= \frac{1}{2}P(L \ge -z) \\ &= \frac{1}{\sqrt{2\pi\sigma_{\rm T}^2}} \int_{-z}^{+\infty} \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right) dl \\ &= \frac{1}{\sqrt{2\pi\sigma_{\rm T}^2}} \int_{-\infty}^{z} \exp\left(-\frac{l^2}{2\sigma_{\rm T}^2}\right) dl \end{aligned}$$

Hence, $Z \sim \mathcal{N}(0, \sigma_{\rm T}^2)$.

■

Let S and T take the values of $X_{\rm A} + X_{\rm B}$ and $X_{\rm A} - X_{\rm B}$ respectively. Hence,

$$\mathrm{E}\{ST\} = \mathrm{E}\{(X_\mathrm{A} + X_\mathrm{B})(X_\mathrm{A} - X_\mathrm{B})\} = 0$$

And since S and T are both zero mean Gaussian random variables, it is concluded that S and T are independent. Applying the lemma to the shadow fading gain function the variance of the shadow fading gain is derived as:

$$\sigma^2 = \mathrm{E}\left\{(X_\mathrm{A} - X_\mathrm{B})^2\right\} = \sigma_0^2 - 2\mathrm{E}\{X_\mathrm{A}X_\mathrm{B}\} \\ = \sigma_0^2\left(1 - e^{-d_\mathrm{AB}/d_c}\right)$$

where d_AB is the distance between the points A and B. Figure 4 depicts the normalized variance (σ/σ_0) as a function of the normalized distance (d/d_c). It is observed that when the distance between the transmitter and receiver increases to more than 5 times the correlation distance, the variance matches that of the Gudmundson model, i.e. σ_0^2.

It is important to note that the variance of the Gudmundson model is not a function of distance. This violates the continuity property at d equal to zero. By introducing the underlying potential field, we are able to maintain this important property in our shadow fading model.

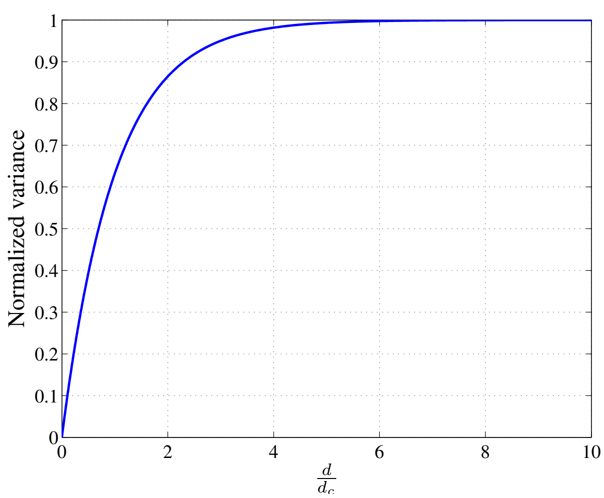

Fig. 4. Normalized variance of the shadow fading gain

3.1. Outage probability

The outage probability for a link in a wireless network is defined as follows:

$$P_\text{out}(d) = \text{Prob}\left(P_r \leq P_\text{min}\right) = \text{Prob}\left(\frac{K}{d^\gamma} \cdot l \leq P_\text{min}\right)$$

where, K is directly proportional to the transmit power of the BS and P_min is the minimum required received power. Let us define the *outage radius* R as the distance (from the BS) where the outage probability is equal to 5%. By setting the outage probability to 0.05 and solving for R, the outage radius becomes,

$$R = \left(\frac{K}{P_\text{min}} 10^{-\frac{\sigma\sqrt{2}}{10}\text{erfc}^{-1}(0.1)}\right)^{\frac{1}{\gamma}}$$

where erfc(.) is the complementary error function. By increasing the transmit power K, the outage radius will move farther away from the BS and hence a larger cell is formed. In a cell with an outage radius R, the outage probability can be written as:

$$P_\text{out}(d) = \text{Prob}\left(\frac{K}{d^\gamma} \cdot L \leq P_\text{min}\right) \\ = \frac{1}{2}\text{erfc}\left(-\frac{10\log_{10}\left(\frac{d}{R}\right)^\gamma - \sigma\sqrt{2}\text{erfc}^{-1}(0.1)}{\sigma\sqrt{2}}\right) \tag{2}$$

The outage probability as a function of the normalized distance for the Gudmundson and the proposed models are provided in Figure 5 and Figure 6 for an outage radius of $100d_c$ (large cell) and d_c (small cell) respectively. It is evident that the two models predict the same outage probability for large cells. In small cells, however, the proposed model achieves lower outage probabilities for distances lower than the outage radius. This was expected due to the smaller variance of the model for short distances.

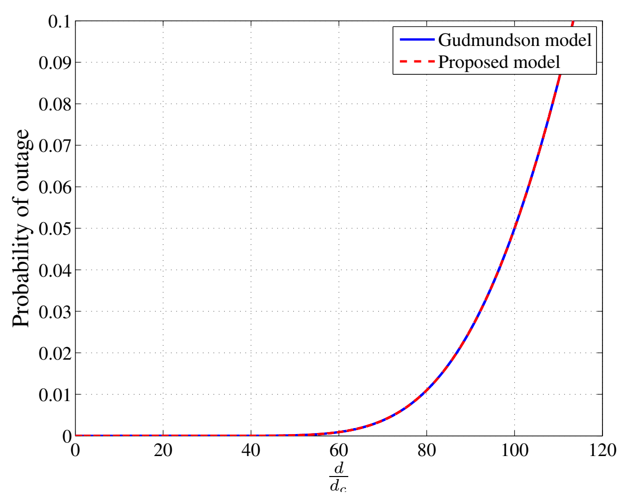

Fig. 5. Outage probability for large cells ($R = 100d_c$)

3.2. Correlation between two links with a common end

The proposed model provides a unified approach to generate correlated channel gains between any pairs of links in the net-

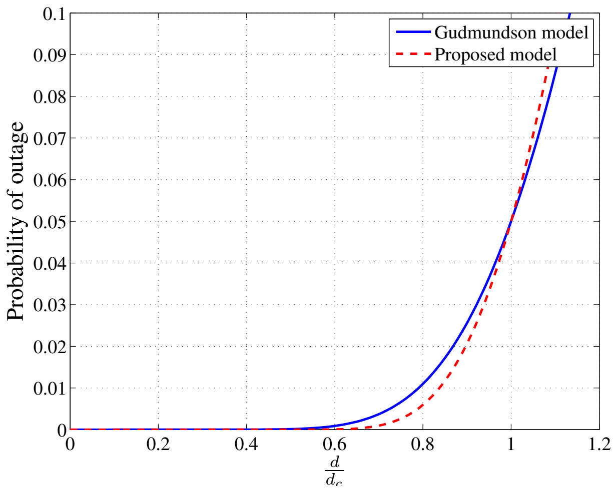

Fig. 6. Outage probability for small cells ($R = d_c$)

work. The Gudmundson model, however, produces correlation between the channel gains only when the two links share a common end. In order to compare the correlation properties of the two models, we consider this special case. Let the two terminals k_1 and k_2 be located at a distance $D \gg d_c$ from the BS as illustrated in Figure 7. We study the correlation between the shadow fading channel gains g_1 and g_2 between the BS (the common end) and the two terminals k_1 and k_2 respectively. By varying the angle θ, the correlation at different distances d are observed while keeping the distance between the terminals and the BS fixed.

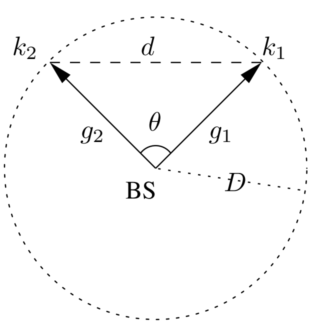

Fig. 7. Common end simulation model

Figure 8 depicts the normalized correlation functions of the two models. In the proposed model, although the correlation decreases for a larger d, it does not approach zero due to the common end shared by both links.

4. SUMMARY

In this paper we have proposed a new model for the generation of correlated shadow fading gains. The network correlation

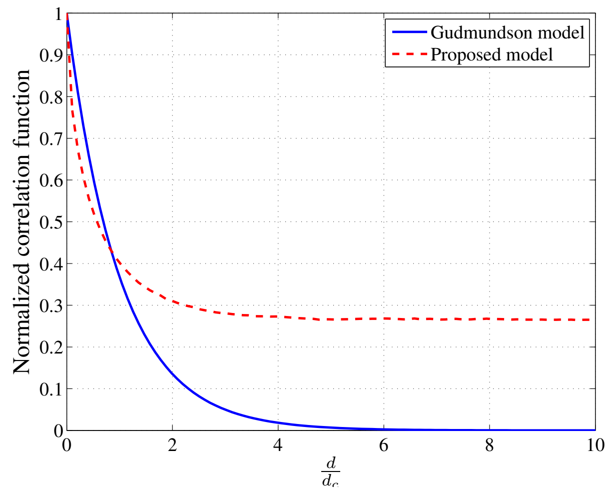

Fig. 8. Normalized correlation function comparison

model has been constructed based on an underlying potential field and a shadow fading gain function which satisfies the principles of shadow fading. The proposed model enables the generation of correlated gains between any pair of links in the network. With the advent of autonomous networks and dense irregular deployment of base stations, this model is crucial for a realistic performance evaluation of the network algorithms. Some relevant statistics of the model have also been compared against the Gudmundson model.

5. REFERENCES

[1] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electronics Letters*, vol. 27, no. 23, pp. 2145–2146, Nov. 1991.

[2] F. Graziosi, M. Pratesi, M. Ruggieri, and F. Santucci, "A multicell model of handover initiation in mobile cellular networks," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 3, pp. 802–814, May 1999.

[3] F. Graziosi and F. Santucci, "A general correlation model for shadow fading in mobile radio systems," *IEEE Communications Letters*, vol. 6, no. 3, pp. 102–104, 2002.

[4] Krishnan Kumaran, Steven E. Golowich, and Sem Borst, "Correlated shadow-fading in wireless networks and its effect on call dropping," *Wireless networks*, vol. 8, no. 1, pp. 61–71, 2002.

[5] I. Forkel, M. Schinnenburg, and M. Ang, "Generation of two-dimensional correlated shadowing for mobile radio network simulation," *In Proceedings of the symposium on Wireless Personal Multimedia Communications, WPMC 2004 (September 2004)*, 2004.