CONSTRAINED EIGENFILTER ALLPASS DESIGN FOR PHOTONIC SYSTEMS

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ABSTRACT

We present a least squares based allpass filter design with a prescribed maximum pole magnitude constraint suitable for photonic systems. The constraint comes from the physical limitations of realizing digital filters using photonic components, and must be addressed in the designing phase. We develop our design algorithm following the eigenfilter approach for allpass filters. The constraint is formulated using the Argument Principle and modified to match the eigenfilter objective function. An iterative approach is then employed to obtain the filter coefficients that best satisfy both the objective function and the constraint. Our algorithm will be ideal for yielding allpass filters suitable for narrowband designs that are common to the optical domain. Examples relevant to filtering in the optical frequency range are subsequently demonstrated. Physical origins of the pole magnitude constraints are also briefly explained.

***Index Terms*—** allpass filter, photonic filter design, constrained filter design, eigenfilter, narrowband filter

1. INTRODUCTION

Designing filters for photonic systems has become an attractive topic in recent years due to a renewed interest in signal processing for optical communications. The research area of photonic integrated circuits has been receiving a high level of attention as modern electrical communications systems are strained by heavy bandwidth demands [1, 2]. Optical signal processors not only demonstrate superiority in bandwidth, but also exhibit advantages in terms of resilience to electromagnetic interference, and to transmission loss [3].

The development in photonic circuits subsequently opens up new areas for signal processing research. In this paper, we focus on Discrete Time Coherent Optical Proccesing (DTCOP) [3]. In the DTCOP scheme, a radio frequency signal (RF) in the GHz range is imposed on an optical carrier with an operational frequency near 100THz. In a photonic system that employs the DTOP scheme, filtering operations must be carried out in the optical domain. Such basic operation requirements in a photonic system translate into a stringent requirement for signal processing designs. A center frequency in the THz frequency range along with the typical MHz bandwidth allocations in optical communication setups imply that the filtering operations are narrowband.

A narrowband filter is difficult to design because it will not only require a high filter order, but also poles extremely close to the unit circle–a characteristic highly undesirable in traditional digital signal processing research. A pole with magnitude close to unity implies that the system may demonstrate stability issues under the finite word length effect, which has been the subject of many research papers [4]. A photonic system has similar constraints because the pole magnitudes directly relate to the feasibility of realizing the filter using optical components. Photonic systems are constituted of basic building elements realized using nanoscale dielectric waveguides and resonators. The maximum component values for these devices are limited by current fabrication capabilities. Therefore, a constrained approach to designing narrowband filters must be devised.

Allpass filters are critical in creating realizable photonic systems. While a narrowband lowpass filter can be designed directly, we instead consider a lowpass filter of the form [5]

$$H(z) = \frac{1}{2}(A_1(z) + A_2(z)) \tag{1}$$

where $A_1(z)$ and $A_2(z)$ are two allpass filters. The design works by creating different phase profiles for the two allpass filters in desired frequency ranges. Being able to realize frequency selective filters using purely allpass substructures is highly beneficial in photonic applications. While a frequency selective filter can be directly mapped to various nanoscale photonic elements, a design based on allpass filters can keep power consumption and power dissipation at minimum level. A photonic allpass does not intrinsically attenuate the optical signal, and therefore does not require additional amplification. Realizable photonic allpass filters such as the Bragg mirror topology [6] have been proven to be an excellent building block for various optical communication schemes [7].

Although constrained frequency selective filters algorithms exist [8, 9], allpass designs with constraints are scarce. While constrained allpass design algorithm has been presented in the minimax sense [10], a well defined least squares (LS) approach still does not exist. We demonstrate in this paper a LS algorithm with pole magnitude constraints that is able to generate allpass designs suitable for generalized photonic filtering needs. We derive the constraint based on the Argument Principle, following the works shown in [8, 9], and reformulate the objective function for an allpass design.

The rest of the paper is organized as follows: Section 2 discusses the chracteristics of an allpass filter, and its realization using the Bragg mirror topology. Section 3 reviews the eigenfilter design approach for allpass filters first presented in [11]. The details for the formulation of the pole magnitude constraint are shown in Section 4. Section 5 presents the modified eigenfilter allpass design problem with maximum pole magnitude limitations. Section 6 demonstrates example designs suitable for optical communications, and Section 7 concludes the paper.

2. ALLPASS FILTERS

An allpass filter describes a system that exhibits no attenuation in the magnitude for all frequencies, but alters the phase of the input signal according to a prescribed profile. The transfer function for an N-th order allpass filter with real coefficients can be written as

$$\begin{aligned} A(z) &= \frac{a_N + a_{N-1}z^{-1} + \cdots + a_1 z^{-(N-1)} + a_0 z^{-N}}{a_0 + a_1 z^{-1} + \cdots a_{N-1} z^{-(N-1)} + a_N z^{-N}} \\ &= z^{-N}\frac{D(z^{-1})}{D(z)} \end{aligned} \tag{2}$$

The allpass filter is highly suitable for physical realization using optical components because it can be easily modularized through a cascade or lattice of first order sections. A first order allpass filter can be realized using a variety of nanoscale photonic elements, one of which is the Bragg Mirror topology shown in Figure 1.

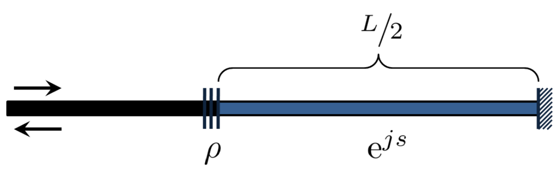


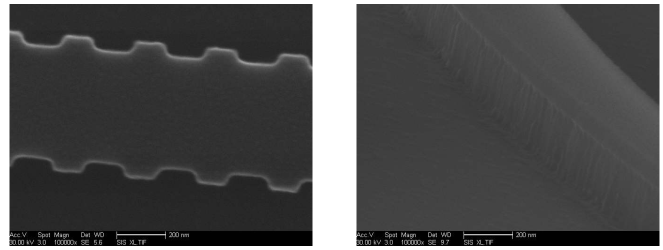
Fig. 1: First order allpass realization using a Bragg mirror topology. The optical path length is represented by L, s is the phase delay of the unit, and ρ is the reflection coefficient.

The unit cell consists of two Bragg mirrors shown in Figure 2(a) as its primary elements, one with amplitude reflection coefficient ρ, and one with perfect reflection. The waveguide section between the two reflectors creates a phase shift of e^{js} on the input. The traversal time of the waveguide loop serves as the discrete delay element z^{-1}, and is directly related to the optical path length L.

Under ideal fabrication conditions, the transfer function of the Bragg mirror topology is

$$A_B(z) = \mathrm{e}^{js}\frac{\rho \mathrm{e}^{-js} - z^{-1}}{1 - \rho \mathrm{e}^{js} z^{-1}} \tag{3}$$

which directly matches the form of a first order allpass filter. Individual sections of this unit cell can then be stitched together to form higher order allpass systems. In real world fabrication, however, the performance of the filter is degenerated by the waveguide loss α, which arises from factors such as roughness of the sidewall, as shown in Figure 2(b). The actual transfer function of the Bragg mirrors topology is



(a) Top-down image of a Bragg mirror waveguide created by introducing periodic perturbation.

(b) Perspective image showing the roughness of the waveguide side wall–one of the sources of loss.

Fig. 2: Waveguide SEM images.

$$A_B(z) = \mathrm{e}^{js}\frac{\rho \mathrm{e}^{-js} - \alpha z^{-1}}{1 - \alpha\rho \mathrm{e}^{js} z^{-1}} \tag{4}$$

It is immediately evident that the maximum realizable pole magnitude is determined by the quantity $\alpha\rho$. Both parameters are governed by the fabrication capabilities, and it is expected that $\alpha\rho$ as high as 0.97 should be realizable with current technology [12]. It should be also noted that with the introduction of α, the transfer function $A_B(z)$ deviates from the profile of an ideal allpass filter. However, with existing low loss waveguides, the effect is minimal and is neglected in this paper.

3. EIGENFILTER APPROACH

The eigenfilter approach, originally introduced in [11], is a powerful design technique for allpass filters in the least squares (LS) sense. The design starts by formulating the phase response of the allpass filter from (2)

$$\begin{aligned} \Theta_A(\omega) &= -N\omega + 2\arctan\frac{\sum_{k=0}^{N} a_k \sin k\omega}{\sum_{k=0}^{N} a_k \cos k\omega} \\ &= -N\omega + 2\arctan\frac{\mathbf{a}^\mathsf{T}\mathbf{s}(\omega)}{\mathbf{a}^\mathsf{T}\mathbf{c}(\omega)} \end{aligned} \tag{5}$$

where $\mathbf{a}$ is the column vector containing the allpass filter coefficients, and

$$\mathbf{s}(\omega) = \begin{bmatrix} 0 & \sin\omega & \sin 2\omega & \cdots & \sin N\omega \end{bmatrix}^\mathsf{T} \tag{6}$$

$$\mathbf{c}(\omega) = \begin{bmatrix} 0 & \cos\omega & \cos 2\omega & \cdots & \cos N\omega \end{bmatrix}^\mathsf{T} \tag{7}$$

Given a desired phase response $\Theta_d(\omega)$ The objective function in the eigenfilter design is then

$$E_{\mathrm{EF}} = \int_R W(\omega)|\Theta_d(\omega) - \Theta_A(\omega)|^2 d\omega \tag{8}$$

where R is the frequency ranges of interest, and $W(\omega)$ is a weighing function. Through trigonometric manipulation, the objective function can be rewritten as

$$E_{\mathrm{EF}} = \int_R W(\omega) \left| 2 \arctan \frac{\mathbf{a}^\mathsf{T} \mathbf{s}_\beta(\omega)}{\mathbf{a}^\mathsf{T} \mathbf{c}_\beta(\omega)} \right|^2 d\omega \tag{9}$$

where

$$\mathbf{s}_\beta(\omega) = \begin{bmatrix} \sin\beta(\omega) & \cdots & \sin(\beta(\omega) - N\omega) \end{bmatrix}^\mathsf{T} \tag{10}$$

$$\mathbf{c}_\beta(\omega) = \begin{bmatrix} \cos\beta(\omega) & \cdots & \cos(\beta(\omega) - N\omega) \end{bmatrix}^\mathsf{T} \tag{11}$$

with $\beta(\omega) = \frac{1}{2}(\Theta_d(\omega) + N\omega)$. Using Taylor series expansion on $\arctan$, and keeping only the first term, we arrive at

$$E_{\mathrm{EF}} \approx 4 \int_R W(\omega) \frac{\mathbf{a}^\mathsf{T} \mathbf{S}_\beta(\omega) \mathbf{a}}{\mathbf{a}^\mathsf{T} \mathbf{C}_\beta(\omega) \mathbf{a}} d\omega \tag{12}$$

where $\mathbf{S}_\beta(\omega) = \mathbf{s}_\beta(\omega)\mathbf{s}_\beta(\omega)^\mathsf{T}$, $\mathbf{C}_\beta(\omega) = \mathbf{c}_\beta(\omega)\mathbf{c}_\beta(\omega)^\mathsf{T}$. To obtain a closed form solution, an iterative approach is used by modifying E_{EF} to

$$\begin{aligned} E_{\mathrm{EF}}^{(i)} &\approx \mathbf{a}^{(i)\mathsf{T}} \left(4 \int_R W(\omega) \frac{\mathbf{S}_\beta(\omega)}{\mathbf{a}^{(i-1)\mathsf{T}} \mathbf{C}_\beta(\omega) \mathbf{a}^{(i-1)}} d\omega \right) \mathbf{a}^{(i)} \\ &= \mathbf{a}^{(i)\mathsf{T}} \mathbf{P}^{(i-1)} \mathbf{a}^{(i)} \end{aligned} \tag{13}$$

The approach here is to use $\mathbf{a}^\mathsf{T} \mathbf{C}_\beta(\omega) \mathbf{a}$ from the previous iteration as a weight in the current step. To solve for $\mathbf{a}^{(i)}$, we note that the setup in each iteration is an optimization problem of the form

$$\begin{aligned} \underset{\mathbf{a}^{(i)}}{\text{minimize}} \quad & \mathbf{a}^{(i)\mathsf{T}} \mathbf{P}^{(i-1)} \mathbf{a}^{(i)} \\ \text{subject to} \quad & \mathbf{a}^{(i)\mathsf{T}} \mathbf{a}^{(i)} = 1 \end{aligned} \tag{14}$$

where the constraint $\mathbf{a}^{(i)\mathsf{T}} \mathbf{a}^{(i)} = 1$ is enforced to avoid the null solution. The objective matrix $\mathbf{P}^{(i-1)}$ is real valued, symmetric and positive definite. According to the Rayleigh principle [13], the vector $\mathbf{a}^{(i)}$ that minimizes the objective function can be readily found by solving for the eigenvector corresponding to the minimum eigenvalue of $\mathbf{P}^{(i-1)}$. Note that while an optimal allpass filter coefficient vector $\mathbf{a}_{\mathrm{opt}}$ can then be found through iteration, the eigenfilter approach contains no restriction on the magnitude of the poles. We must therefore modify the approach to include the constraint for designs that are realizable using photonic components.

4. CONSTRAINT SETUP

To properly formulate the constraint on the maximum pole magnitudes, we use the setup based on the Argument Principle [14] following the approaches shown in [8, 9]. The Argument Principle states that for a function $\hat{D}(z)$ that is differentiable inside a contour $\mathcal{C}$ except at a number of singularities, the following condition is satisfied

$$N_z - N_p = \frac{1}{2\pi j} \oint_{\mathcal{C}} \frac{\hat{D}'(z)}{\hat{D}(z)} dz \tag{15}$$

where N_z is the number of zeros of the function inside region $\mathcal{C}$, and N_p is the number of poles. The contour $\mathcal{C}$ in our setup is a circle defined by the maximum allowable pole radius $r_{\max}$. The maximum radius is directly related to the quantity $\alpha\rho$ from the manufacturable component values of a photonic allpass filter. Notice that as long as the roots of $D(z) = \sum_{k=0}^{N} a_k z^{-k}$ in (2) are within a circle prescribed by $r_{\max}$, the entire allpass filter will be realizable using photonic components such as the Bragg mirror topology. We can directly translate this information into the Argument Principle by relating the function $\hat{D}(z)$ to the backward path transfer function of the allpass filter

$$\hat{D}(z) = z^N D(z) = \sum_{k=0}^{N} a_k z^{N-k} \tag{16}$$

This formulation of $\hat{D}(z)$ contains the exact same root locations as $D(z)$, but will help in managing the constants in the Argument Principle setup. To properly derive the constraint, we start by rearranging the contour integral

$$\oint_{\mathcal{C}} \frac{\hat{D}'(z)}{\hat{D}(z)} dz = \oint_{\mathcal{C}} d \ln \hat{D}(re^{j\omega}) \tag{17}$$

Decomposing into magnitude and phase yields

$$\begin{aligned} \oint_{\mathcal{C}} d \ln \hat{D}(re^{j\omega}) = \quad & \oint_{\mathcal{C}} d \ln |\hat{D}(re^{j\omega})| \\ & + j \oint_{\mathcal{C}} d\arg\hat{D}(re^{j\omega}) \end{aligned} \tag{18}$$

Since we are interested in real coefficient filters, the magnitude response of $\hat{D}(z)$ is even, making $\oint_{\mathcal{C}} d \ln |\hat{D}(re^{j\omega})|$ a closed contour integral of an odd function. The first term vanishes to zero, and we have

$$\oint_{\mathcal{C}} \frac{\hat{D}'(z)}{\hat{D}(z)} dz = j \int_0^{2\pi} \frac{d}{d\omega} \arg\hat{D}(re^{j\omega}) d\omega \tag{19}$$

Based on the setup, we require that $N_z = N$, $N_p = 0$ inside the contour $\mathcal{C}$. The constraint thus becomes

$$2\pi N = \int_0^{2\pi} \frac{d}{d\omega} \arg\hat{D}(re^{j\omega}) d\omega \tag{20}$$

Now,

$$\frac{d}{d\omega} \arg D(re^{j\omega}) = N - \frac{d}{d\omega} \arctan \frac{\mathbf{a}^\mathsf{T} \mathbf{R} \mathbf{s}(\omega)}{\mathbf{a}^\mathsf{T} \mathbf{R} \mathbf{c}(\omega)} \tag{21}$$

where $\mathbf{R} = \mathrm{diag}(1, r^{-1}, \cdots, r^{-N})$. The constraint thus simplifies to

$$\int_0^{2\pi} \frac{d}{d\omega} \arctan \frac{\mathbf{a}^\mathsf{T} \mathbf{R} \mathbf{s}(\omega)}{\mathbf{a}^\mathsf{T} \mathbf{R} \mathbf{c}(\omega)} d\omega = 0 \tag{22}$$

Taking the derivative, we get

$$\int_0^{2\pi} \frac{\mathbf{a}^\mathsf{T} \mathbf{R} (\mathbf{S}(\omega) + \mathbf{C}(\omega)) \mathbf{R} \mathbf{N} \mathbf{a}}{\mathbf{a}^\mathsf{T} \mathbf{R} (\mathbf{S}(\omega) + \mathbf{C}(\omega)) \mathbf{R} \mathbf{a}} d\omega = 0 \tag{23}$$

where $\mathbf{N} = \mathrm{diag}(0, 1, \cdots, N)$.

We can employ the same approach from the eigenfilter design algorithm to handle the fractional form. The constraint can be satisfied iteratively with the denominator from the previous iteration used as a weight for the current iteration. For notation simplicity, let $\mathbf{\Lambda}(\omega) = \mathbf{R}(\mathbf{S}(\omega) + \mathbf{C}(\omega))\mathbf{R}$, then the constraint can be best satisfied iteratively by minimizing

$$\begin{aligned} \kappa_L^{(i)} &= \mathbf{a}^{(i)\mathsf{T}} \left(\int_0^{2\pi} \frac{\mathbf{\Lambda}(\omega)\mathbf{N}}{\mathbf{a}^{(i-1)\mathsf{T}}\mathbf{\Lambda}(\omega)\mathbf{a}^{(i-1)}} d\omega \right) \mathbf{a}^{(i)} \\ &= \mathbf{a}^{(i)\mathsf{T}} \hat{\mathbf{K}}^{(i-1)} \mathbf{a}^{(i)} \end{aligned} \tag{24}$$

Note that the term $\mathbf{\Lambda}(\omega)\mathbf{N}$ causes $\kappa_L^{(i)}$ to lose its symmetry, making it difficult to combine with the eigenfilter approach. To resolve this issue, we relax the constraint by introducing an additional error term $\kappa_R^{(i)} = \mathbf{a}^{(i)\mathsf{T}} \hat{\mathbf{K}}^{\mathsf{T}(i-1)} \mathbf{a}^{(i)}$ to form the overall constraint

$$\begin{aligned} \kappa^{(i)} &= \frac{1}{2}(\kappa_L + \kappa_R) \\ &= \mathbf{a}^{(i)\mathsf{T}} \left(\int_0^{2\pi} \frac{\frac{1}{2}(\mathbf{\Lambda}(\omega)\mathbf{N} + \mathbf{N}^{\mathsf{T}}\mathbf{\Lambda}^{\mathsf{T}}(\omega))}{\mathbf{a}^{(i-1)\mathsf{T}}\mathbf{\Lambda}(\omega)\mathbf{a}^{(i-1)}} d\omega \right) \mathbf{a}^{(i)} \\ &= \mathbf{a}^{(i)\mathsf{T}} \mathbf{K}^{(i-1)} \mathbf{a}^{(i)} \end{aligned} \tag{25}$$

Note that the additional κ_R is a valid constraint by itself, since it is formulated by simply taking the transpose of the original κ_L. While more sophisticated methods to relax the constraint such as semidefinite relaxation [15] exist, a simple grouping of the terms is sufficient for the eigenfilter approach.

5. OVERALL DESIGN ALGORITHM

While the objective function and the constraint can be directly formulated as a Quadratic Constrained Quadratic Programming (QCQP) problem, we consider the simpler formulation

$$J_{EF}^{(i)} = \gamma E_{\mathrm{EF}}^{(i)} + (1-\gamma)\kappa^{(i)} \tag{26}$$

Substituting in the definition of the terms, we arrive at the following iterative minimization

$$\begin{aligned} \underset{\mathbf{a}^{(i)}}{\text{minimize}} \quad & \mathbf{a}^{(i)\mathsf{T}}(\gamma \mathbf{P}^{(i-1)} + (1-\gamma)\mathbf{K}^{(i-1)})\mathbf{a}^{(i)} \\ \text{subject to} \quad & \mathbf{a}^{(i)\mathsf{T}}\mathbf{a}^{(i)} = 1 \end{aligned} \tag{27}$$

Based on our formulation of the constraint, $\mathbf{K}^{(i)}$ is symmetric, real valued, and positive definite. We can therefore use the Rayleigh principle to find the $\mathbf{a}^{(i)}$ that minimizes $\gamma\mathbf{P}^{(i-1)} + (1-\gamma)\mathbf{K}^{(i-1)}$. The overall algorithm to solving the constrained allpass problem is shown in Algorithm 1. Note that the δ parameter in the algorithm serves as the convergence criterion. The rate of convergence highly depends on the prescribed phase requirement. A narrowband filter will require more iterations than a typical lowpass.

6. EXAMPLE DESIGNS

The constrained filter design algorithm allows for frequency selective realizations using allpass substructures in case of very narrowband operations. Let us consider the design of an allpass filter with prescribed phase response

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Algorithm 1 Constrained Allpass Filter Design

Given: prescribed phase response $\Theta_{\mathrm{pre}}(\omega)$, filter order N, weight function $W(\omega)$, γ;
$\mathbf{a}^{(0)}$ = eigenvector corresponding to the minimum eigenvalue of $\int_R W(\omega)\mathbf{S}_\beta(\omega)$;
i=1;
repeat
  formulate $\gamma\mathbf{P}^{(i-1)} + (1-\gamma)\mathbf{K}^{(i-1)}$;
  compute $\mathbf{a}^{(i)}$ = eigenvector corresponding to the minimum eigenvalue;
  i = i + 1;
until $\|\mathbf{a}^{(i)} - \mathbf{a}^{(i-1)}\| < \delta$

$$\Theta_{\mathrm{pre}}(\omega) = \begin{cases} -(N-1)\omega & 0 \le \omega \le w_p \\ -(N-1)\omega + \pi & \omega_s \le \omega \le \pi \end{cases} \tag{28}$$

We can then use the resulting allpass filter to form a lowpass filter of the form

$$H(z) = \frac{1}{2}\left(z^{-(N-1)} + A(z)\right) \tag{29}$$

where $A(z)$ is of order N. We first consider a benign example with $\omega_p = 0.1\pi$, $\omega_s = 0.2\pi$, and $N = 15$. Figure 3 shows the design results for the unconstrained approach in contrast to limiting the largest pole magnitudes 0.9, 0.8 and 0.7. The characterstics of the various designs are summarized in Table 1. Notice that while the algorithm successfully yields an allpass design with the required maximum pole magnitude, it does so at the cost of reducing passband and stopband attenuations. To compare with other least squares designs, Figure 4 shows the result from simply setting thresholds at 0.9, 0.8 and 0.7. In this setup, the unconstrained eigenfilter method is first used to solve for the allpass filter coefficients. The poles with magnitudes higher than the threshold are then scaled down to be within the constraint. It is evident that a systematic design algorithm is necessary since simple scaling of the poles greatly deteriorates the filter response.

In the second example, we consider a very narrowband lowpass filter design with $N = 15$, $\omega_p = 0.0025\pi$, $\omega_s = 0.01\pi$. These design requirements closely resemble the expected operation frequency ranges of a DTOP setup. Figure 5 shows the overall design result of the constrained approach with maximum radius set to 0.97. The resulting maximum pole radius is 0.969, with $A_p = -0.3141$dB, and $A_s =$

Requirement	$r_{\max}$	A_p **(dB)**	A_s **(dB)**
unconstrained	0.9181	−0.002717	−35.1
$r_{\max} < 0.9$	0.8879	−0.02984	−21.73
$r_{\max} < 0.8$	0.7822	−0.426	−11.67
$r_{\max} < 0.7$	0.6797	−1.287	−9.321

Table 1: Comparison results showing the passband attenuation A_p, stopband attenuation A_s and the maximum pole magnitude $r_{\max}$ for various allpass designs.

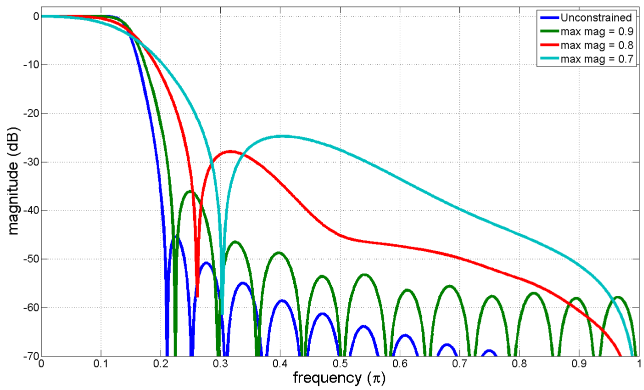

Fig. 3: Lowpass design using allpass subsections showing comparison results of unstrained approach versus constraining the maximum pole magnitudes to 0.9, 0.8 and 0.7.

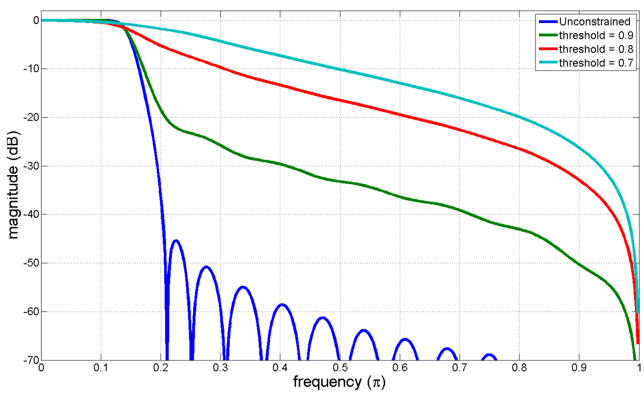

Fig. 4: Results from setting a hard threshold of 0.9, 0.8 and 0.7 on the maximum pole magnitudes. Poles with magnitudes outside of the threshold are down scaled.

-4.096dB. Note that similar design requirements with the same pole magnitude constraint were targeted in [12], but was only achieved through an *ad-hoc* method that involved a $N = 24$ filter as well as design complications such as interpolation and post filtering. The constrained eigenfilter approach is able to produce a realizable design through a generalized algorithm at lower filter order and lower complexity.

7. CONCLUSION

We present an allpass filter design algorithm that is able to incorporate a prescribed constraint on the maximum pole magnitude. We first formulate the phase error in the least squares sense, and combine it with the constraint obtained through Argument Principle. The constraint is relaxed to yield a symmetric matrix that can be easily combined with the unconstrained eigenfilter design approach. The overall result is a minimization problem that can be iteratively solved using Rayleigh's principle. The proposed algorithm will be excellent in parametrizing allpass filters realized using photonic components since it directly incorporates constraints that result from real world fabrication limitations.

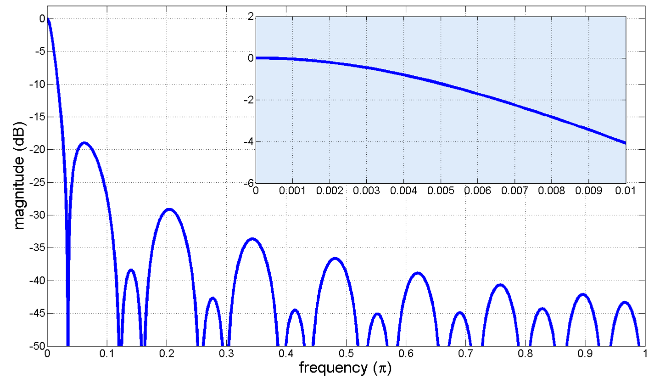

Fig. 5: Narrowband design using allpass subsections from the constrained approach with maximum pole magnitude of 0.97. A magnification of the passband and stopband region is also shown.

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