SINGULAR VALUE DECOMPOSITION AND SPACE/SPATIAL-FREQUENCY ANALYSIS APPLIED TO DIGITAL WATERMARKING

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ABSTRACT

A watermarking procedure in the space/spatial-frequency domain, based on the singular value decomposition is proposed. Singular values are used for selection of pixels suitable for watermarking. Generally, regions for watermarking are characterized as: busy and moderate busy. For pixels from busy regions the entire middle frequency content in the space/spatial frequency domain is used for watermarking. In the case of moderate busy pixels the left and right singular vectors are used to shape watermark embedding region. Watermark modeling, embedding, and detection are performed in the space/spatial-frequency domain. The efficiency of the proposed procedure is tested under various attacks.

Index Terms— *singular value decomposition, space/spatial-frequency analysis, digital image watermarking, space-varying filtering*

1. INTRODUCTION

In the last two decades digital watermarking has become a popular technique for protection of digital data, such as: audio, image and video. Most of the image watermarking procedures are performed in spatial or in the frequency domain separately. However, the joint space/spatial-frequency domain is more efficient from the aspect of imperceptibility and robustness requirements [1]-[5]. The space/spatial-frequency representation proposed in [4] is firstly used to select busy regions (suitable for watermarking), and then to determine middle frequency components for watermark embedding. In [5], the selection of busy pixels is based on the eigenvalue decomposition of space/spatial-frequency representation, while shaping of middle frequency content for watermark embedding is done as in [4].

A watermarking procedure based on the singular value decomposition of space/spatial-frequency representation is proposed in this paper. In the proposed method, pixels selected using singular vectors are classified in three categories: flat, moderate busy and busy. Introducing the moderate busy category will provide better imperceptibility, since in this region watermark with lower strength, compared to busy pixels, will we embedded. Also, it will allow us to select much more pixels suitable for watermarking compared to methods [4] and [5]. For busy regions the whole middle frequency content in the space/spatial-frequency domain is used for watermark embedding, while for moderate busy region shaping of watermark is performed using variations of singular vectors. Detection of watermark is performed in the space/spatial-frequency domain, as well. It will increase detection performances, which is shown experimentally.

2. THEORETICAL BACKGROUND

A brief overview of the space/spatial-frequency representations used in the proposed watermarking procedure is given in this Section. A concept of the space-varying filtering used for watermark modeling is described. The singular value decomposition method applied to the space/spatial-frequency representation is discussed, as well.

2.1. Space/spatial-frequency representations

The simplest and the most commonly used space/spatial-frequency transform is short-time Fourier transform (STFT). The STFT for 2D signals can be written as:

$$STFT(n_1,n_2,\omega_1,\omega_2)=\sum_{m_1=-N/2}^{N/2-1}\sum_{m_2=-N/2}^{N/2-1} I(n_1+m_1,n_2+m_2)w(m_1,m_2) \times e^{-j2\pi(m_1\omega_1+m_2\omega_2)/N}, \quad (1)$$

where n_1, n_2, and ω_1, ω_2 are spatial and frequency coordinates, respectively, while $w(m_1,m_2)$ represent the 2D window function in the spatial domain.

The energetic version of the 2D STFT is called 2D spectrogram, and it is defined as:

$$SPEC(n_1,n_2,\omega_1,\omega_2)=\left|STFT(n_1,n_2,\omega_1,\omega_2)\right|^2. \quad (2)$$

Note that the 2D spectrogram for 2D signals (such as images) produces 4D space/spatial-frequency representation. It means that for each pixel we have 2D representation, which contains spectral characteristics of a surrounding region.

2.1. Space-varying filtering

The concept of space-varying filtering was originally introduced for filtering of 2D noisy signals, but also it can be used for retrieving the signal from its space/spatial-frequency representation [4]. The space-varying filtering function for 2D signal $I(n_1,n_2)$ can be written as:

$$H_s(n_1,n_2)=\frac{1}{4\pi^2}\int_{\omega_1,\omega_2} L_H(n_1,n_2,\omega_1,\omega_2) \times STFT_s(n_1,n_2,\omega_1,\omega_2)d\omega_1 d\omega_2 \tag{3}$$

where the support function $L_H(n_1,n_2,\omega_1,\omega_2)$ is defined as Weyl symbol mapping into the space/spatial-frequency domain [4]. In the discrete form, the space-varying filtering function can be written as:

$$H_s(n_1,n_2)=\frac{1}{2\pi}\sum_{\omega_1}\sum_{\omega_2} L_H(n_1,n_2,\omega_1,\omega_2)STFT_s(n_1,n_2,\omega_1,\omega_2). \tag{4}$$

The support function can be defined as:

$$L_H(n_1,n_2,\omega_1,\omega_2)=\begin{cases}1 & \text{for} \quad (n_1,n_2,\omega_1,\omega_2)\in R_f\\ 0 & \text{for} \quad (n_1,n_2,\omega_1,\omega_2)\notin R_f\end{cases} \tag{5}$$

where it is assumed that signal components are within the region R_f.

2.1. Singular value decomposition

Singular value decomposition (SVD) method has been used in numerous signal and image processing applications. For a matrix A, of size *MxN*, SVD is defined as:

$$\mathbf{A}=\mathbf{USV}^T \tag{6}$$

where **S** is diagonal matrix of size *MxN*, called singular values matrix. The matrices **U** (of *MxM* size) and **V** (of *NxN* size) are orthogonal matrixes that contain left and right singular vectors, respectively. The singular values matrix **S** is sorted in decreasing order along the main diagonal, meaning that the first singular value correspond to the most significant left and right singular vectors. This fact can provide the information about characteristics of space/spatial-frequency representation of an image.

3. WATERMARKING PROCEDURE

The image watermarking procedure based on the space/spatial-frequency representation and singular value decomposition is proposed in this Section. Algorithm which describes proposed procedure can be summarized through the following steps:

i) selection of pixels suitable for watermarking
ii) space/spatial-frequency regions shaping
iii) watermark modeling by using space/spatial-frequency mask
iv) watermark embedding
v) watermark detection

Each of these steps is described in more details in the following subsections.

3.1 Selection of pixels suitable for watermarking

One of the basic requirements of each watermarking procedure is imperceptibility of embedded watermark. Therefore, the first step of watermarking procedure is selection of pixels that are suitable for watermark embedding.

It is known that pixels that belong to flat image regions are not suitable for watermarking. Namely, watermark embedded in these pixels can degrade image quality and can be easily removed. Thus, suitable image regions should contain details, edges and luminance changes. Here, we will distinguish two categories:

- busy regions - with dense details, edges, and high luminance variations;
- moderate busy regions – with still enough details and luminance variations.

In order to characterize the image regions we will use singular values obtained by applying SVD method to the space/spatial-frequency representation. That procedure can be described through the following steps:

1) Select a certain pixel $I(n_1,n_2)$;
2) Consider a region of 2K+1 width in both directions, around the pixel;
3) Calculate 2D spectrogram;
4) Apply a SVD method on the spectrogram.

Note that the spectrogram is interpolated to the double size before SVD calculation, in order to provide more precise characterization.

The elements of singular values matrix **S** are further analyzed in order to characterize the image pixels based on the following procedure:

$$\begin{cases} if\ No(S>Tr)\le 2 & \rightarrow \ \text{flat region}\\ if\ No(S>Tr)\ge 5 & \rightarrow \ \text{busy region}\\ otherwise & \rightarrow \ \text{moderate busy region}\end{cases} \tag{7}$$

where function *No*() returns the number of singular values that are higher than the adaptive threshold $Tr=\lambda\max(S)$. The parameter λ determines which portion of the maximum is considered. An example of flat, moderate busy, and busy regions, and corresponding singular values is given in Table 1. An example of the proposed region classifications is shown in Fig 1. for Lena image, where flat regions are marked in black, moderate busy in gray, and busy regions in white. In the previous example values λ=0.1 and 2*K*+1=21 are used.

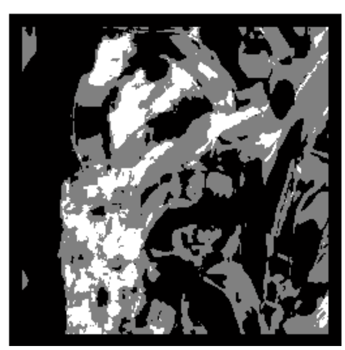
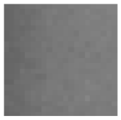
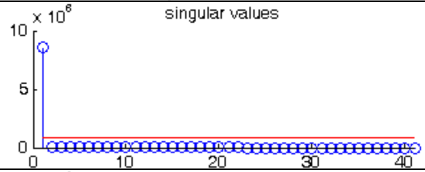
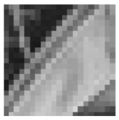
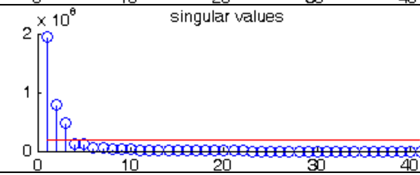
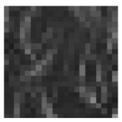
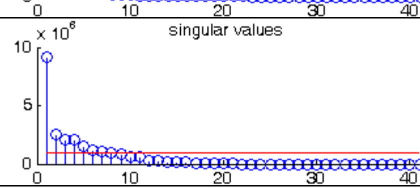


Fig. 1 Region characterization for image Lena: black – flat region, gray – moderate busy regions, and white – busy regions.

Table I. Singular value decomposition applied for characterization of image regions

Image region	Singular values	Characteristic
		$No(S > Tr) = 1$ Flat region
		$No(S > Tr) = 3$ Moderate busy region
		$No(S > Tr) = 7$ Busy region

The adaptive threshold is used because it provides adjustment to each region separately. Consequently, it will provide better results than the procedure based on fixed threshold value used in [5].

3.2 Selection of space/spatial-frequency regions suitable for watermark embedding

In the proposed watermarking scheme, the watermark is embedded in the space/spatial-frequency domain. Therefore, it is necessary to consider two basis requirements that watermark should satisfy: robustness and imperceptibility. Namely, the watermark embedded in the low frequency region provides high robustness, but may cause degradation of image quality. On the other side high frequency region is of low energy and watermark embedded in this region can be easily removed. Thus, the middle frequency region (R) is most appropriate for watermark embedding. An illustration of the middle frequency region is shown in Fig 2.

In the previous space/spatial-frequency based methods the whole middle frequency region is used for watermarking [4], [5]. In the proposed method we do the same for the busy region, but moderate busy region require additional shaping of the middle frequency content. For this purpose the singular vectors are used. The first two singular vectors are not considered since they contain information about variations of most significant components that are usually located in the low frequency region. Therefore, the third singular vectors (from **U** and **V**) are used to determine a part in the middle frequency region, i.e., the support function L_H is obtained as it is shown in Fig 3. Namely, left singular vector (LSV) and right singular vector (RSV) contains information about variations along horizontal and vertical axis, respectively. Thus, the middle frequency components for which the absolute values of both LSV and RSV are above some predefined threshold β will be used for watermarking:

$$L_H(\omega_1,\omega_2) = \begin{cases} 1 \quad if & \begin{cases} LSV(\omega_1) > \beta \; and \; RSV(\omega_2) > \beta \\ for \quad (\omega_1,\omega_2) \in R \end{cases} \\ 0 & otherwise \end{cases} \tag{8}$$

3. 3 Watermark modeling, embedding and detection

The watermark modeling procedure is performed in the space/spatial-frequency domain. For that purpose the 2D STFT of a 2-D pseudo-random sequence p is used to obtain the watermark. The space/spatial frequency characteristics of watermark are modeled according to the support function L_H:

$$STFT_{wat}(n_1,n_2,\omega_1,\omega_2) = \\ = \alpha \cdot L_H(\omega_1,\omega_2) \cdot STFT_p(n_1,n_2,\omega_1,\omega_2), \tag{9}$$

where the indexes $_p$ and $_{wat}$ denotes pseudo-random sequence and watermark, respectively. The parameter α controls the watermark strength.

Watermark embedding procedure is performed in the space/spatial frequency domain as:

$$STFT_{Iw}(n_1,n_1,\omega_1,\omega_2) = STFT_I(n_1,n_1,\omega_1,\omega_2) + \\ + STFTwat(n_1,n_1,\omega_1,\omega_2) \tag{10}$$

where $STFT_I$ is the short-time Fourier transform of original image, while $STFT_{Iw}$ is short-time Fourier transform of watermarked image.

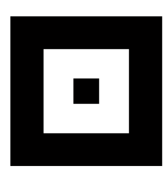


Fig 2. Illustration of middle frequency region R

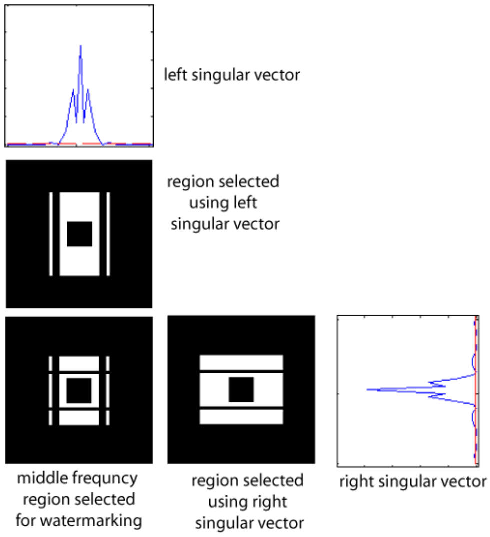


Fig 3. Selection of middle frequency regions based on LSV and RSV

Namely, the space/spatial-frequency mask is used to obtain watermarked image pixel on the position (n_1,n_2), as [4], [5]:

$$Iw(n_1,n_2)=\frac{1}{2\pi}\sum_{\omega_1}\sum_{\omega_2} STFT_{Iw}(n_1,n_2,\omega_1,\omega_2)L(\omega_1,\omega_2). \quad (11)$$

Watermark detection procedure is also performed in the space/spatial-frequency domain by using standard correlation detector:

$$D(w_{key})=\sum_{\omega_1}\sum_{\omega_2} STFT_{I_w}(n_1,n_2,\omega_1,\omega_2)STFT_{w_{key}}(n_1,n_2,\omega_1,\omega_2) \quad (12)$$

where $STFT_{w_{key}}$ can be the STFT of the watermark. Note that the detection is performed within entire middle frequency region, and thus the presence of the support function (which is different for each pixel) is not required during detection A large number of coefficients available in the space/spatial-frequency domain will improve the detection results.

The detection performance is tested by using the measure [4], [8], [9]:

$$R=\frac{\overline{D}(w_{key})-\overline{D}(w_{wr})}{\sqrt{\sigma^2_{w_{key}}+\sigma^2_{w_{wr}}}}, \quad (13)$$

where $\overline{D}(w_{key})$ and $\overline{D}(w_{wr})$ are the mean values of detector responses for a number of watermarks (right keys) and wrong keys (trials), respectively. Standard deviations of the detector responses for right and wrong keys are denoted as $\sigma^2_{w_{key}}$ and $\sigma^2_{w_{wr}}$, respectively. The probability of detection error P_{err} can be easily calculated by using measure R, as follows:

$$P_{err}=0.5erfc(R/\sqrt{2}). \quad (14)$$

where normal distribution of detector's responses is assumed. By increasing the value of measure R the probability of detection error decreases, i.e. lower probability of false alarm and miss detection is obtained. For example, for R=5, probability of detection error is $P_{err} \sim 10^{-7}$, and for R=8 we have $P_{err} \sim 10^{-15}$.

4. RESULTS

In this Section the performances of the proposed procedure are tested and compared with two image watermarking procedures in the space/spatial-frequency domain ([4] and [5]). It has been shown (in [4] and [5]) that space/spatial-frequency based procedures have the best performances in terms of robustness and PSNR compared to the other state-off-the-art image watermarking procedures. Therefore, proposed procedure is benchmarked only with procedures presented in [4] and [5].

In the proposed procedure, the 2D STFT of original image is calculated by using rectangular window of width 21x21. Selection of pixels is based on the singular values, according to (7). For moderate busy regions, the third LSV and RSV are used for an appropriate shaping of middle frequency region, according to (8). Threshold value β=0.01 is used. Original and watermarked images Lena and Boat are shown in Fig 4.

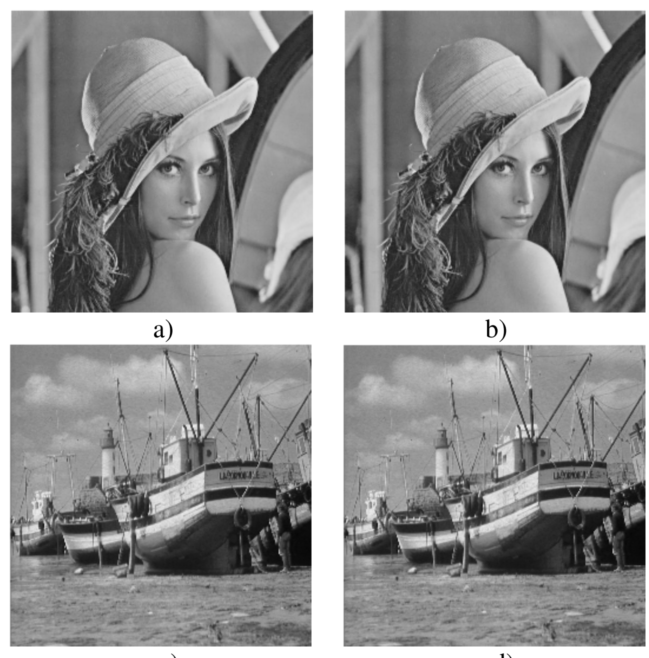


Fig 4. a) original, b) watermarked image Lena; c) original and d) watermarked image Boat

Let us first compare the performances of the proposed procedure without attack. The same PSNR=50 dB is considered for all tested procedures. For all images, the watermarking procedure is done for 50 different right keys (watermarks). For each of the right keys 100 wrong trials are generated. The measures of detection efficiency *R* for different watermarked test images are shown in Table II. The measure *R* used in [4] and [5] is the same as one defined by (13) in this paper. Note that, in all cases, proposed procedure has higher *R*, i.e., lower probability of detection error.

Table II Detection results in terms of measure *R* and PSNR=50 dB

Image	Proposed procedure	Procedure proposed in [5]	Procedure proposed in [4]
Lena	13.05	10.5	8.27
Peppers	13.28	10.6	9.14
F16	11.8	9.1	7.95
Boat	14.21	11.5	9.21

In order to further point out performances of the proposed procedure the different attacks are applied to the set of watermarked test images. For the proposed procedure the PSNR is increased to 54 dB (average value for 50 right keys). The average measures of detection quality in the presence of different attacks are shown in Table III.

Table III Detection results in the presence of various attacks

Image Lena	Measure R for the proposed procedure PSNR=54 dB	Measure R for the procedure in [5] PSNR=50 dB	Measure R for the procedure in [4] PSNR=50 dB
No attack	10.76	10.49	8.5
JPEG 80	10.01	9.24	7.4
JPEG 60	8.66	7.76	6.75
JPEG 50	7.06	6.72	5.67
Impulse noise	9.83	8.51	7.10
Gaussian noise	9.23	8.58	7.24
Median filter	7.31	4.95	4.45
Average filter	7.66	5.25	5.11
Image Boat			
No attack	11.83	11.41	9.21
JPEG 80	10.45	9.93	8.14
JPEG 60	9.42	8.11	7.6
JPEG 50	8.85	7.38	6.75
Impulse noise	9.91	9.12	8.15
Gaussian noise	9.44	9.04	8.1
Median filter	7.45	5.37	4.75
Average filter	8.15	6.65	5.15
Image Pepper			
No attack	11.31	10.65	9.14
JPEG 80	10.21	9.15	8.14
JPEG 60	9.12	8.04	7.48
JPEG 50	8.45	7.28	6.53
Impulse noise	9.23	8.76	8.23
Gaussian noise	9.43	8.84	8.29
Median filter	7.28	5.23	5.08
Average filter	7.44	5.68	4.65

From Table III one can note that in all tested cases the proposed procedure provide better detection results compared with procedures in [4] and [5], even in the case when the achieved PSNR is higher for 4 dB.

5. CONCLUSION

The image watermarking procedure in the space/spatial-frequency domain is proposed. Selection of coefficient suitable for watermarking is performed by using singular values obtained by applying SVD on the 2D spectrogram. Pixels are classified into three categories: flat (non-suitable for watermarking), moderate busy, and busy. In the case of pixels from busy regions watermark is modeled according to whole middle frequency region, while in the case of moderate busy additional region shaping is performed by using singular vectors. It provides improved imperceptibility and better detection performance compared with similar methods in the space/space-frequency domain, which is confirmed in the example. The proposed classification can be further improved and extended for more detailed characterization of image region.

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