AN ITERATIVE DETECTION ALGORITHM FOR CODED CPFSK SIGNALS WITH IRRATIONAL MODULATION INDEX

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ABSTRACT

An algorithm for iterative demodulation and decoding of convolutionally coded and interleaved Continuous Phase Frequency Shift Keying (CPFSK) signals with an irrational modulation index is presented. The algorithm is based on integrating the Per-Survivor Processing (PSP) technique into a Soft-Input Soft-Output (SISO) CPFSK demodulator. Using PSP, the phase offsets emerging from the difference between the irrational transmit modulation index and the employed rational receive modulation index are estimated within the PSP-SISO algorithm at each state and in each time step. These phase offsets are taken into account to calculate the soft information. The phase offsets and the soft information are then refined during several iterations and used to estimate the transmit symbols. Simulations have shown that with only five trellis phase states and after only four iterations, the proposed algorithm performs at BER=10^{-5} only 0.5 dB worse than a quasi-exact algorithm which has sixteen trellis phase states. This enables a significant reduction in computational complexity in terms of the number of the trellis phase states.

***Index Terms*—** Iterative detection, Soft-Input Soft-Output, CPFSK, irrational modulation index, Per-Survivor Processing.

1. INTRODUCTION

In many communication systems the linearity of the analog stages is a limiting factor. One of the approaches to deal with this problem is to use constant envelope modulations. In a constant envelope modulation, information is carried only in the phase of the modulated signal. Continuous Phase Modulation (CPM) is one of the most promising constant envelope modulations, in which the phase of the signal is varied according to the input data symbols such that the phase continuity of the signal is maintained. By properly selecting the CPM signal parameters, both high bandwidth and high power efficiency can be achieved [1]. A special member of the CPM family is Continuous Phase Frequency Shift Keying (CPFSK), which employs a rectangular frequency pulse and is very popular, due to the simplicity of signal generation. Some prominent applications of CPM include wireless sensor networks, airborne data acquisition, and Bluetooth [2]. CPM signals with rational modulation indexes generated from independent and identically distributed (i.i.d.) data symbols can be modeled with a finite state Markov process and it can be shown that Maximum Likelihood Sequence Estimation (MLSE) is the optimum detection strategy for transmission of such signals over additive white Gaussian noise (AWGN) channels [3]. However, due to unavoidable tolerances, cheap CPM transmitters do not adhere to an exactly prescribed modulation index, which in turn could lead to an irrational index instead of a desired rational one or a rational index with a large denominator causing a high receiver complexity [2], [4]. Our simulations have shown that for MLSE demodulation of CPFSK signals even small deviations from the nominal modulation index — if no further countermeasures are taken — could lead to very high bit error rates (BERs). All these issues make it attractive to design a mechanism which is capable of efficiently detecting coded CPFSK signals with an irrational modulation index. An algorithm for approximate MLSE detection of CPFSK signals with irrational modulation indexes, which is based on Per-Survivor Processing (PSP), has been proposed in [5]. In its original form, PSP is a technique for enabling MLSE using the Viterbi algorithm (VA) in uncertain environments [6]. Furthermore, it has been shown that if a CPFSK modulator is serially concatenated with a channel coding module via an interleaver, very good BER performance can be achieved using iterative decoding. An extensive study of iterative demodulation and decoding for serially concatenated CPFSK (SC-CPFSK) signals and its BER performance can be found in [7]. However, the iterative SISO CPFSK demodulation algorithm presented in [7] is based on a trellis diagram, which can only be constructed for CPFSK signals with rational modulation indexes [4]. In general, so far little attention has been paid to the problem of iterative demodulation and decoding of SC-CPFSK signals with an irrational modulation index. In this paper, an algorithm is proposed which combines the PSP technique with a SISO CPFSK demodulation algorithm using a trellis structure based on a rational modulation index at the receiver. The PSP-SISO CPFSK demodulation algorithm introduced here allows the simultaneous estimation of the phase difference emerging from a deviation between the transmit and receive modulation indexes and iterative SISO demodulation of the CPFSK signal.

The paper is organized as follows. In Section II, the system model is described. The developed PSP-SISO CPFSK demodulation algorithm is introduced in Section III. Simulation results are given in Section IV. Finally, conclusions are drawn in Section V.

2. SYSTEM MODEL

The discrete-time block diagram of the considered system in the equivalent complex baseband (ECB) is shown in Fig. 1. Binary data symbols $a[k'] \in \{0, 1\}$ are first fed to a convolutional channel encoder, which includes also a mapper. $k' \in \{(\xi - 1)N, \ldots, \xi N - 1\}$ is the uncoded symbol index, where N is the block size and $\xi \in \{1, \ldots, B\}$ is the block in-

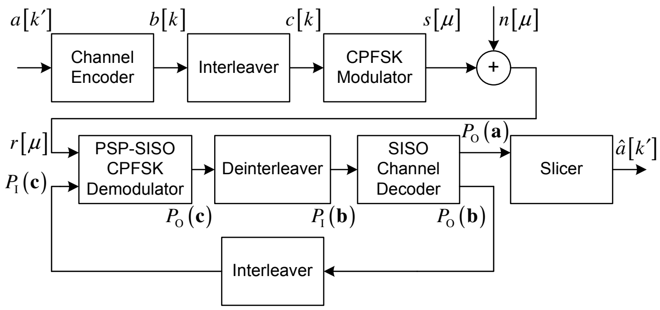

Fig. 1. System model.

dex with B as the number of blocks. The coded and mapped symbols $b[k] \in \{-1, +1\}$ are then bit-interleaved and passed to the CPFSK modulator. $k \in \left\{\frac{(\xi-1)N}{R_c}, \ldots, \frac{N\xi}{R_c} - 1\right\}$ is the coded symbol index with $R_c \in \left\{\frac{1}{2}, \frac{1}{3}, \ldots\right\}$ as the code rate of the convolutional encoder. The oversampled CPFSK modulated signal $s[\mu]$ is defined as [1]

$$s[\mu] = \sqrt{\frac{E_\mathrm{s}}{T}} \cdot \exp\left(j\phi[\mu]\right), \tag{1}$$

where μ is the sample index with $kN_\mathrm{s} \leq \mu < (k+1)N_\mathrm{s}$ for the k-th symbol [4]. N_s is the number of samples per symbol; E_s is the symbol energy and T denotes the symbol duration. $\phi[\mu]$ is the instantaneous carrier phase of the CPFSK signal at the μ-th sample and is defined by

$$\begin{aligned}\phi[\mu] &= \pi h \sum_{l=0}^{k-L} c[l] + 2\pi h \sum_{l=k-L+1}^{k} c[l] q\left[\mu - lN_\mathrm{s}\right] \\ &= \Theta[k] + \theta[\mu],\end{aligned} \tag{2}$$

where $k = \left\lfloor \frac{\mu}{N_s} \right\rfloor$ and $\Theta[k]$ is the accumulated phase of the CPFSK signal in the k-th symbol interval [4]. $L \in \mathbb{N}$ is the duration of the frequency pulse and h stands for the modulation index. In this paper we consider only full response CPFSK signals with $L = 1$. However, our proposed algorithm can be extended also to signals with larger L and different frequency pulses than rectangular one. For $L = 1$, the second term in (2) reduces to $\theta[\mu] = 2\pi h\ c[k] q\left[\mu - kN_\mathrm{s}\right]$, which represents the phase variation in the k-th symbol interval caused by the input symbol $c[k]$. $q[\mu]$ is the phase pulse related to a rectangular frequency pulse,

$$q[\mu] = \begin{cases} 0 & \mu < 0 \\ \frac{\mu}{2N_\mathrm{s}} & 0 \leq \mu \leq N_\mathrm{s} \\ \frac{1}{2} & \mu > N_\mathrm{s} \end{cases} \tag{3}$$

The channel model used here is an additive white Gaussian noise (AWGN) channel with noise samples $n[\mu]$ of double-sided power spectral density $N_0/2$. The received signal $r[\mu]$ is given by $r[\mu] = s[\mu] + n[\mu]$. All the modules of Fig. 1 work block-wise. The received samples $r[\mu]$ are fed to the PSP-SISO CPFSK demodulator, and the demodulator output is passed to the SISO channel decoder. After a certain number of iterations the a posteriori symbol probabilities delivered by the SISO channel decoder are used to determine the estimates $\hat{a}[k']$. In the following section, the functionality of the SISO algorithms used at the receiver is described.

[1]Since in principle the CPFSK signal has an infinite bandwidth, its discrete-time representation is only an approximation but a very accurate one if N_s is chosen sufficiently large.

3. PSP-SISO CPFSK DEMODULATION

In order to iteratively demodulate and decode convolutionally coded CPFSK signals with an irrational modulation index, a PSP-SISO algorithm is developed. This section describes the underlying PSP technique and its integration into the SISO CPFSK demodulator.

3.1. Per-Survivor Processing

If the CPFSK signal has a rational modulation index, i.e., $h = U/V$, where U and V are irreducible and relatively prime numbers, it can be represented by a trellis diagram with accumulated signal phases $\Theta[k]$ as states [4]. V and $2V$ specify the number of states of the trellis diagram associated with the CPFSK signal, if U is an even and odd number, respectively [4]. For such a case, the algorithm by Bahl, Cocke, Jelinek, and Raviv (BCJR) [8] can be adapted to be used as a SISO algorithm for CPFSK signal demodulation in an iterative scheme [7]. However, in case of CPFSK signals with an irrational modulation index, there is an infinite number of states, which makes it impossible to construct a phase trellis at the receiver. Our approach to deal with this problem is to construct a trellis diagram assuming a rational modulation index at the receiver, estimate the phase difference caused by the different transmit and receive modulation indexes in each state and at each time step, and take these phase differences into consideration when calculating the metrics. A general form of this approach is an algorithm called Per-Survivor Processing (PSP) [6], which has been proposed in its original form for simultaneous MLSE detection and parameter estimation. In Fig. 2 a trellis diagram used for the PSP technique is shown. With each state and each time step, an estimate for an unknown parameter is associated. This parameter is updated according to its estimate at the survivor predecessor state and the current input. The survivor branches in Fig. 2 are depicted with solid lines. The general adaptation rule of the PSP technique for the state p_2 and in the time step k can be summarized as follows:

$$\hat{\chi}_{(p_2)}[k] = f\left(\hat{\chi}_{(p_1)}[k-1], \mathbf{r}[k]\right), \tag{4}$$

where $\mathbf{r}[k] = [r[kN_s], \ldots, r[(k+1)N_s - 1]]$ and $\hat{\chi}_{(p_2)}[k]$ is the estimate of the unknown parameter in the state p_2 and at the time step k. Here, estimation is expressed via the function $f(\cdot)$ of the estimate of the unknown parameter $\hat{\chi}_{(p_1)}[k-1]$ in the survivor predecessor state p_1 and in the time step $k-1$ and the received signal during the time epoch k. The above formula can be adapted for demodulation of a CPFSK signal with a rational or irrational modulation index [5] using a trellis diagram based on a certain rational modulation index:

$$\hat{\delta}_{(p_2)}[k] = \hat{\delta}_{(p_1)}[k-1] + \check{c}_{(p_1,p_2)}[k] \cdot \Delta h \cdot \pi, \tag{5}$$

where $\hat{\delta}_{(p_2)}[k]$ is the estimate for the phase difference at the state p_2 and in the time step k, and $\hat{\delta}_{(p_1)}[k-1]$ is the estimate for the phase difference in the survivor predecessor

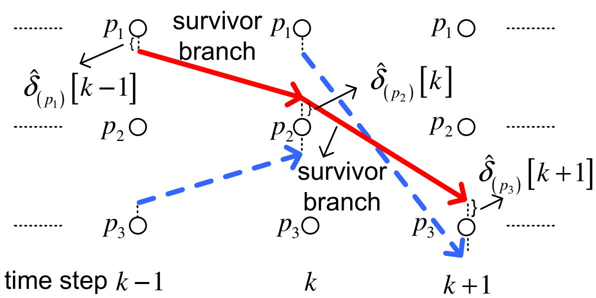

Fig. 2. Trellis diagram with Per-Survivor Processing.

state which is in this example p_1. $\Delta h = h_{\mathrm{TX}} - h_{\mathrm{RX}}$ is the difference between the transmit modulation index h_{TX} and the receive modulation index h_{RX}, and $\check{c}_{(p_1,p_2)}[k]$ is the sent symbol in the time step k and associated to the survivor state transition $p_1 \to p_2$. It is evident from (5) that the current estimate for the phase difference at a specific state is the value at the survivor predecessor state plus the phase difference caused by the survivor transition. The PSP-SISO algorithm requires knowledge of the transmit modulation index. There are algorithms for transmit modulation index estimation for CPFSK [9]. Here it is assumed that h_{TX} is perfectly known at the receiver. Now the modified phase state $\hat{\Theta}_{(p)}[k]$ at the state p and in the time step k can be calculated as

$$\hat{\Theta}_{(p)}[k] = \left(\Theta_{(p)} + \hat{\delta}_{(p)}[k]\right) \bmod 2\pi, \tag{6}$$

where $\Theta_{(p)} = \pi\, p\, h_{\mathrm{RX}}$ are the original phase states with $h_{\mathrm{RX}} = \frac{U}{V}$. U is an even number and $p \in \{0, 1, \ldots, V-1\}$ [4].

3.2. Integration of the PSP Technique into the SISO CPFSK Demodulator

The PSP-SISO CPFSK demodulator presented in this paper is based on the BCJR algorithm [8]. The first step is the calculation of likelihood values of the received samples:

$$P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p,q)}[k]\right] = p\left(\mathbf{r}[k] | \check{\mathbf{s}}_{(p,q)}[k]\right) = \prod_{\mu=kN_{\mathrm{s}}}^{(k+1)N_{\mathrm{s}}-1} \frac{1}{\pi\sigma_n^2} \mathrm{e}^{-\frac{\|r[\mu]-\check{s}_{(p,q)}[\mu]\|^2}{\sigma_n^2}}, \tag{7}$$

with the noise variance $\sigma_n^2 = f_s \cdot N_0$ and the sampling rate $f_s = N_{\mathbf{s}}/T$. The subscript $\mathbf{I}$ stands for input and $\check{\mathbf{s}}_{(p,q)}[k] = \left[\check{s}_{(p,q)}[kN_s], \ldots, \check{s}_{(p,q)}[(k+1)N_s - 1]\right]$ denotes the sequence of the hypothetically sent CPFSK samples at the symbol interval k and during the state transition $p \to q$. $\check{s}_{(p,q)}[\mu]$ can be described as

$$\check{s}_{(p,q)}[\mu] = \exp\left(j\left(\hat{\Theta}_{(p)}\left[\left\lfloor \frac{\mu}{N_s} \right\rfloor\right] + \check{\theta}_{(p,q)}[\mu]\right)\right). \tag{8}$$

It can be seen from (8) that the phase of the hypothetically sent CPFSK signal consists of two terms: $\hat{\Theta}_{(p)}[k] = \hat{\Theta}_{(p)}\left[\left\lfloor \frac{\mu}{N_s} \right\rfloor\right]$, which is the estimated accumulated phase at the state p and time step k and can be determined using (6) and $\check{\theta}_{(p,q)}[\mu] = \pi\, h_{\mathrm{TX}}\, \check{c}_{(p,q)}[k]\mu/N_s$, which is the phase variation upon the state transition $p \to q$. As mentioned in the last section, the demodulation algorithm is block oriented, which means that each block is first demodulated and decoded in a certain number of iterations before the next block is processed. The initial values of the phase offsets at the beginning of each block are to be determined. The procedure proposed here is to use the estimated phase differences at the last time step of a block as the initial values for the phase difference estimates at the first time step of the next block: $\hat{\delta}^{(\lambda=1)}_{(p)}[\frac{\xi N}{R_c}] = \hat{\delta}^{(\lambda=\Lambda)}_{(p)}[\frac{\xi N}{R_c} - 1]$ with λ and Λ as the iteration index and the total number of iterations, respectively. As can be seen, the estimated values after the last iteration are used because they provide the highest reliability. The next step is the calculation of the state probabilities during the forward recursion computed for the state q and in the time step $k+1$ [10], [11]:

$$\alpha_{(q)}[k+1] = p\left(\Psi[k+1] = q, \mathbf{r}_{<k+1}\right) = A[k+1] \sum_{p=0}^{V-1} \alpha_{(p)}[k] P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p,q)}[k]\right] P_{\mathbf{I}}\left[\check{c}_{(p,q)}[k]\right],$$

where $\mathbf{r}_{<k+1} = [\ldots, \mathbf{r}[k-1], \mathbf{r}[k]]$ and $\Psi[k+1]$ is the index of the trellis state which the CPFSK receiver considers in the time step $k+1$. $A[k+1]$ is a normalizing factor which is calculated such that in the time step $k+1$ the sum of α's is equal to one. $P_{\mathbf{I}}\left[\check{c}_{(p,q)}[k]\right]$ is used as a priori information in the CPFSK demodulator for a more reliable calculation of soft information and is determined as follows:

$$P_{\mathbf{I}}\left[\check{c}_{(p,q)}[k]\right] = \begin{cases} P_{\mathbf{I}}\left[c[k]\right] & \text{for } \check{c}_{(p,q)}[k] = 1 \\ 1 - P_{\mathbf{I}}\left[c[k]\right] & \text{otherwise} \end{cases}$$

with $P_{\mathbf{I}}\left[c[k]\right] \in P_{\mathbf{I}}\left[\mathbf{c}\right] = \left[P_{\mathbf{I}}\left[c[\frac{(\xi-1)N}{R_c}]\right], \ldots, P_{\mathbf{I}}\left[c[\frac{\xi N}{R_c} - 1]\right]\right]$ denoting the bit-interleaved version of $P_{\mathbf{O}}\left[\mathbf{b}\right]$ defined as $\left[P_{\mathbf{O}}\left[b[\frac{(\xi-1)N}{R_c}]\right], \ldots, P_{\mathbf{O}}\left[b[\frac{\xi N}{R_c} - 1]\right]\right]$, where the subscript $\mathbf{O}$ stands for output. $P_{\mathbf{O}}\left[\mathbf{b}\right]$ is the extrinsic information computed by the SISO channel decoder, which is represented later in this section. Furthermore, it is assumed that the modulator starts at the zero state in the first time step of the first block. Therefore, the following initial conditions for α can be assumed:

$$\alpha_{(q)}[0] = \begin{cases} 1 & \text{for } q = 0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Now in order to determine the survivor predecessor state of each state at a certain time step and iteration, the product of $\alpha_{(p)}[k]$ of the predecessor states and $P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p,q)}[k]\right] P_{\mathbf{I}}\left[\check{c}_{(p,q)}[k]\right]$ of the branches connecting the considered state with its predecessor states are compared. For example, if the state q has the predecessor states p_1 and p_2, then p_1 would be the survivor predecessor state if

$$\alpha_{(p_1)}[k-1] P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p_1,q)}[k-1]\right] P_{\mathbf{I}}\left[\check{c}_{(p_1,q)}[k-1]\right] > \alpha_{(p_2)}[k-1] P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p_2,q)}[k-1]\right] P_{\mathbf{I}}\left[\check{c}_{(p_2,q)}[k-1]\right].$$

As can be seen, for determination of the survivor states the a priori probabilities $P_{\mathbf{I}}\left[\check{c}_{(p,q)}[k]\right]$ are taken into account, too, which leads to a more reliable survivor state determination. After the survivor predecessor states are determined for all states of a time step, the phase offsets for these states are calculated using (5) and (6). These phase offsets are then used to compute $P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p,q)}[k]\right]$ in the corresponding time step according to (7) and (8). After computing all α's at the last time step of a block, the backward recursion starts. During the backward recursion the following state probabilities are calculated [10], [11]:

$$\begin{aligned}\beta_{(p)}[k] &= p\left(\mathbf{r}_{>k-1}|\Psi[k]=p\right) \\ &= B[k]\sum_{q=0}^{V-1}\beta_{(q)}[k+1]P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p,q)}[k]\right]P_{\mathbf{I}}\left[\check{c}_{(p,q)}[k]\right],\end{aligned} \tag{10}$$

where $\mathbf{r}_{>k-1} = [\mathbf{r}[k], \mathbf{r}[k+1], \ldots]$. $B[k]$ is a normalizing factor and is calculated such that in the time step k the sum of β's is equal to one. There are several methods for initialization of β's. One of these methods assumes that the states at the last time step of the received block are equiprobable:

$$\beta_{(p)}\left[\frac{\xi N}{R_c}-1\right] = \frac{1}{V}, \qquad p \in \{0, 1, \ldots, V-1\}\,.$$

After the last iteration in a block, the β's of the last time step are taken and used as α's for the first time step of the next block: $\alpha_{(p)}^{(\lambda=1)}[\frac{\xi N}{R_c}] = \beta_{(p)}^{(\lambda=\Lambda)}[\frac{\xi N}{R_c}-1]$. This procedure makes it also possible that the CPFSK modulator has to start at the zero state only in the first time step of the first block and does not need to go back to the zero state at the first time step of the other blocks. This makes the proposed procedure less complex than a sliding window procedure at an only slight degradation in performance. The next step is to determine the extrinsic information [11]:

$$P_{\mathbf{O}}\left[c[k]\right] = \sum_{(p,q):\check{c}_{(p,q)}[k]=1}\alpha_{(p)}[k]P_{\mathbf{I}}\left[\check{\mathbf{s}}_{(p,q)}[k]\right]\beta_{(q)}[k+1].$$

These probabilities are then bit-deinterleaved and passed to the SISO channel decoder, where they are used as a priori probabilities. Now similar calculations are done to compute α's and β's in the channel decoder in order to determine the extrinsic probabilities $P_{\mathbf{O}}\left[b[k]\right]$ and the a posteriori symbol probabilities $P_{\mathbf{O}}\left[a[k']\right]$ as follows [11]:

$$P_{\mathbf{O}}[b[k]] = \sum_{(p,q):\check{b}_{(p,q)}[k]=1}\alpha_{(p)}[k']P_{\mathbf{I}}\left[\check{a}_{(p,q)}[k']\right]\beta_{(q)}[k'+1],$$

where $k' = \lfloor R_c k\rfloor$ and $P_{\mathbf{I}}\left[\check{a}_{(p,q)}[k']\right]$ are set to 0.5 because it is assumed that all values of $a[k']$ are equiprobable. The calculated extrinsic probabilities $P_{\mathbf{O}}\left[b[k]\right]$ are then bit-interleaved and fed back to the PSP-SISO CPFSK demodulator. The a posteriori symbol probabilities are computed as follows [11]:

$$\begin{aligned}P_{\mathbf{O}}\left[a[k']\right] = &\sum_{(p,q):\check{a}_{(p,q)}[k']=1}\alpha_{(p)}[k']P_{\mathbf{I}}\left[\check{b}_{(p,q)}[k]\right] \\ &\cdot P_{\mathbf{I}}\left[\check{b}_{(p,q)}[k+1]\right]P_{\mathbf{I}}\left[\check{a}_{(p,q)}[k']\right]\beta_{(q)}[k'+1]\,,\end{aligned}$$

where $k = \frac{k'}{R_c}$,

$$P_{\mathbf{I}}\left[\check{b}_{(p,q)}[k]\right] = \begin{cases}P_{\mathbf{I}}\left[b[k]\right] & \text{for } \check{b}_{(p,q)}[k]=1\\ 1-P_{\mathbf{I}}\left[b[k]\right] & \text{otherwise}\end{cases}$$

with $P_{\mathbf{I}}\left[b[k]\right] \in P_{\mathbf{I}}[\mathbf{b}] = \left[P_{\mathbf{I}}\left[b[\frac{(\xi-1)N}{R_c}]\right], \ldots, P_{\mathbf{I}}\left[b[\frac{\xi N}{R_c}-1]\right]\right]$ representing the bit-deinterleaved version of $P_{\mathbf{O}}\left[\mathbf{c}\right]$, which is defined as $\left[P_{\mathbf{O}}\left[c[\frac{(\xi-1)N}{R_c}]\right], \ldots, P_{\mathbf{O}}\left[c[\frac{\xi N}{R_c}-1]\right]\right]$. Finally, after an appropriate number of iterations, the a posteriori symbol probabilities are used by a slicer to deliver decoded data symbols.

4. SIMULATION RESULTS

To measure the performance of the proposed iterative PSP-SISO CPFSK demodulation algorithm, simulations of BER versus E_b/N_0, E_b being the received signal energy per information bit, have been performed. The number of samples per symbol was set to $N_{\mathrm{s}} = 8$ in all simulations. The channel code used in our investigations was a non-recursive, non-systematic convolutional code with a code rate of $R_c = 1/2$ and a generator matrix of $G = \left[1+D+D^2,\ 1+D^2\right]$. We chose this code because of its simplicity and its good free distance which is the crucial property of the channel code with respect to the BER performance of the SC-CPFSK systems [7]. The used interleaver and deinterleaver were random. First, the BER performance of the iterative PSP-SISO CPFSK demodulator with an irrational transmit modulation index of $h_{\mathrm{TX}} = \pi/5$ has been investigated. In order to have a performance reference, we have also simulated the BER performance of a CPFSK transmission with a rational modulation index of $h_{\mathrm{TX}} = 5/8$ which is very close to the irrational modulation index mentioned before. CPFSK modulated signals with transmit modulation indexes $h_{\mathrm{TX}} = \pi/5$ and $h_{\mathrm{TX}} = 5/8$ have, due to the proximity of $\pi/5 \approx 0.6283$ and $5/8 = 0.625$, almost the same minimum Euclidean distance and hence the same BER performance [4]. Therefore, the exact iterative SISO algorithm with $h_{\mathrm{TX}} = h_{\mathrm{RX}} = 5/8$ can be regarded as quasi-reference for a performance comparison with the iterative PSP-SISO algorithm in our investigations. It can be concluded from Fig. 3 that our proposed iterative PSP-SISO algorithm with $h_{\mathrm{TX}} = \pi/5$ and $h_{\mathrm{RX}} = 2/5$ performs at $\mathrm{BER} = 10^{-5}$ only about 0.5 dB worse than the exact iterative SISO algorithm with $h_{\mathrm{TX}} = h_{\mathrm{RX}} = 5/8$. It has to be mentioned that the BER performance of the iterative PSP-SISO algorithm has been achieved with only 5 trellis states and after only 4 iterations which proves the efficiency of the reduced-state PSP-SISO algorithm. This enables a drastic reduction in computational complexity from 16 states to 5 states. From the same figure it can be observed that the difference in BER performance between the BCJR algorithm without PSP and the BCJR algorithm with PSP which is the PSP-SISO algorithm with no iteration is less than 0.3 dB at $\mathrm{BER} = 10^{-3}$. Again, the BCJR algorithm with PSP has a reduced number of states of 5 compared to 16 states of the BCJR algorithm without PSP.

The second series of simulations have been done to investigate the influence of the receive modulation index of the PSP-SISO CPFSK demodulator and the block length on the BER performance, when a rational transmit modulation index of $h_{\mathrm{TX}} = 22/31$ is used. As can be seen from Fig. 4, the system with $h_{\mathrm{RX}} = 22/31$ and $N = 8192$ shows the best BER

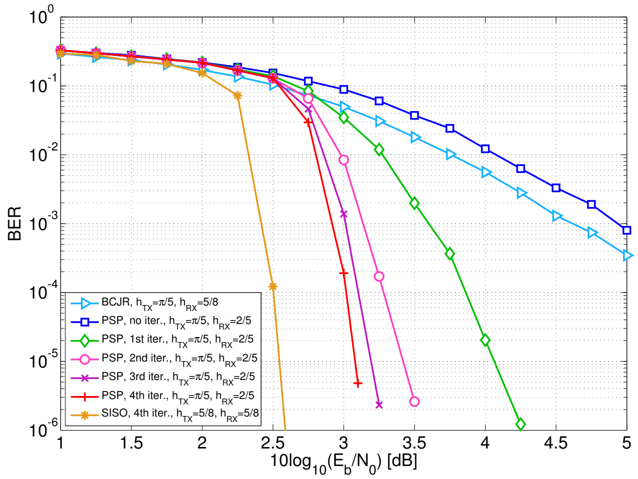

Fig. 3. BER performance of the iterative PSP-SISO algorithm (PSP) compared to the BCJR algorithm without PSP (BCJR) and the quasi-exact iterative SISO algorithm (SISO); block length: $N = 8192$.

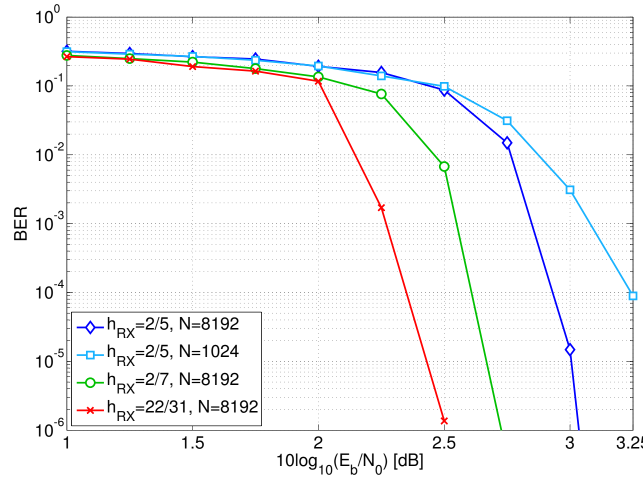

Fig. 4. Influence of the receive modulation index and the block length on the BER performance of the iterative PSP-SISO CPFSK demodulator for $h_{\mathrm{TX}} = 22/31$, number of iterations: 4.

performance which is obvious because of the equality of the transmit and receive modulation indexes. The next best performance is obtained by the scheme with $h_{\mathrm{RX}} = 2/7$ and $N = 8192$, although, $h_{\mathrm{RX}} = 2/5$ is closer to the transmit modulation index. The reason is that $h_{\mathrm{RX}} = 2/7$ provides 7 states in contrast to $h_{\mathrm{RX}} = 2/5$ with 5 states, respectively, resulting in a more precise phase difference estimation. It can also be noticed that the performance difference between the algorithm with optimal $h_{\mathrm{RX}} = 22/31$ with 31 states and $h_{\mathrm{RX}} = 2/5$ with only 5 states and with the same block length is quite small and about 0.6 dB at $\mathrm{BER} = 10^{-5}$ which indicates that the number of states of the proposed PSP-SISO algorithm can be reduced significantly compared to the exact algorithm for rational transmit indexes at an only slight loss in performance. Finally, the interleaver gain has been investigated. From Fig. 4 it can be observed that the system with block length $N = 1024$ and $h_{\mathrm{RX}} = 2/5$ exhibits only about 0.25 dB loss at $\mathrm{BER} = 10^{-4}$ compared to the system with $N = 8192$ and the same h_{RX} which indicates that the developed algorithm performs well even with relatively small interleaver lengths.

5. CONCLUSION

An algorithm for iterative demodulation and decoding of convolutionally coded CPFSK signals with an arbitrary irrational modulation index has been presented. Simulation results have shown that for $h_{\mathrm{TX}} = \pi/5$, the proposed algorithm with only 5 trellis states and 4 iterations performs only 0.5 dB worse than the quasi-reference exact algorithm with 16 states at $\mathrm{BER} = 10^{-5}$ which permits a huge complexity reduction in terms of the number of states. Using the presented algorithm, if the transmit modulation index is changed during the operation, the receiver may continue working with the old, fixed trellis and there is no need to construct a new trellis for the new modulation index. This saves a tremendous amount of complexity in hardware.

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